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A Study of Analytical Models

Analysis of Variance is applied to 15 models from 6 missions with 2 cameras flown over 120 targetted points.

INTRODUCTION

ANALYTICAL PHOTOGRAMMETRY provides a useful tool for studying the performance of a photographic system. With the possibility of compensating for various systematic errors introduced during the photographic process, the analytical solution has brought about a resurgent interest in the fundamental questions concerning the geometric quality of the photograph. It is evident that any improvement in the accuracy of the photogrammetric solution has to come from the improvement in, and our understanding of, the photographic process. A great deal more remains to be learned about this process, primarily about the nature of its stationary and stochastic components.

In this regard we undertook a small research project at the University of Toronto under the sponsorship of the National Research Council to investigate the variability among aerial photographs taken at varying intervals of time. The primary goal was to compare photographs exposed during a relatively short period of time, while the photographic conditions can be considered to be homogeneous, with those taken on different days. In each case the same camera and the same type of film were used.

EXPERIMENTAL DATA

A special test field containing in excess of 120 targetted points was established for this study in a suburb of Toronto. The targets consisted of white disks 6–7 inches in diameter, surrounded by a dark ring of 10–12 inches in diameter. They were painted on manhole covers, the flat roofs of factories, and apartment buildings. The targets were placed either individually or in groups of 5 or 6 over a small area of the ground. The size of the field was chosen in such a way that it was contained within a 60 per cent overlap of two pictures taken with a 6-inch lens at the scale of 1:10,000. The resulting flying altitude was 5,000 feet above the terrain.

No attempt was made to determine the positions of the targets by ground methods as the experiment was designed to study only the *differences* among the photographs.

The photography, undertaken by the Lockwood Survey Corporation of Toronto, was conducted as follows. Each photographic mission, carried on different days, consisted of at least three passes over the test field. The

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photographs were taken with 80 per cent overlap so that the whole test field was imaged in most cases on three successive pictures.

The size of the budget did not permit any extensive sampling. Altogether six missions were flown, the first three using one camera, and the last three another. The same type of film, Dupont 136 aerial film, was used throughout. Below is given the analysis of the photography obtained from the first three missions and taken with a Wild RC-5a camera equipped with a Universal Aviogon lens and a fast shutter. Altogether 15 models were obtained.

The diapositive plates, developed to a higher-than-normal density, were measured on a Wild STK-1 stereocomparator in the following way: First, the four fiducial marks were read, followed by measurement of point coordinates. Then the coordinates of each point were read four times, and all the points were read in the same predetermined sequence. Centering of the measuring marks was done monocularly on each picture and stereoscopic setting was employed only in situations when one of the images was poorly resolved or difficult to locate accurately.

ABSTRACT: *An experimental study of geometric precision of aerial photographs is based on an analysis of analytically determined models of a test field. The photography was conducted in such a way that it permits the comparison of models reconstructed from pictures taken seconds, minutes, and days apart. The test field consisted of 120 targetted points. The analysis of data is based on the principles of the method of Analysis of Variance. Practical results include half of the experimental data analyzed to date.*

DATA REDUCTION

Data reduction phase consisted of:

i. Correction of image coordinates for known or assumed errors of the photographic system;
ii. Analytical reconstruction of models;
iii. Mapping of models onto a reference model.

Correction of coordinates included correction for refraction based on the U. S. Standard Atmosphere (NASA 1963), lens distortion, film distortion as used by the U.S.-C.&G.S. (Keller, Tewinkel 1966) and correction for linear comparator errors.

The coordinates of analytically reconstructed model positions of the target points were determined (1) using the midpoint of the y-parallax and (2) using the midpoint of the shortest distance between two non-intersecting rays. The difference between the two solutions was found to be negligible.

To compare the resulting 15 models, one of them was chosen as the *reference* model and all the other models were mapped onto it. Under the hypothesis that the models differ only in orientation, scale and location, the mapping has the familiar form:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_t = \lambda R \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \qquad (1)$$

with λ the scale factor, R the rotation matrix, $(X_0\ Y_0\ Z_0)^T$ the translation vector and the subscript t denoting the transformed coordinates. The seven parameters of this transformation were estimated by the method of least squares using all the points.

DATA ANALYSIS AND RESULTS

The precision of relative orientation can be judged by the residual y-parallaxes which in this case represent the corrections to the measured y-coordinates of the photo whose relative orientation with respect to the other photo was determined by the method of least squares. These values also represent, very closely, the discrepancies between the model Y-coordinates, at photo scale, calculated from the left and right photo coordinates. Their standard errors are displayed in Table 1 in which each model is classified according to mission M and pass P from which it came. Two values, which appear in most of the cells, correspond to two models reconstructed from three successive pictures taken with an 80 per cent overlap. All the values are based on about 110 degrees of freedom.

Figure 1 shows the distribution of the targets and the residuals according to their sign for a typical model. Negative residuals are denoted by black circles, positive residuals by white circles. The large circles represent groups of 5 to 6 points, the sign being that of their average. Inspection of Figure 1 reveals certain degree of clustering of residuals of the

TABLE 1. STANDARD ERRORS OF RESIDUAL y-PARALLAXES (MICRONS)

	M_1	M_2	M_3
P_1	±3.9 3.4	3.9 4.4	4.1 4.3
P_2	3.8 —	3.2 3.8	3.4 3.8
P_3	3.2 —	4.0 —	4.0 4.3

same sign indicating possible presence of systematic errors. To help decide this issue, twelve groups of targets were painted in such a way that their images occupied an area of 1 to 2 mm² on the picture. It is not unreasonable to assume that the various photographic distortions remain constant over such small regions of the picture and, therefore, the variation of residuals within these groups furnishes an unbiased estimate of the random component of the error.

An exact statistical test is available for detection of systematic errors by comparing the variance estimated from all the residuals with the variance about the group averages. More than 70 per cent of all models tested indicated presence of systematic errors at the 5 per cent confidence level. The standard deviation within the groups yielded a value of slightly less than ±3.0 microns, but it must be emphasized that several of the groups were close to the margins of the pictures resulting in poor resolution of the targets and rather high variances for these groups. The best result was obtained from a centrally located group of six targets which yielded a standard deviation of y-parallax residuals of ±1.6 microns. This is only little more than ±1.3 microns which was the overall value of the pointing-to-targets error.

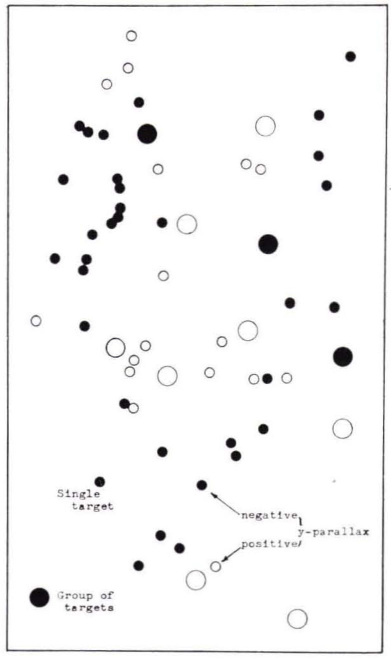

FIG. 1. Residual Y-parallaxes in Model M_1P_{12}.

To study the comparison of models, model M_1P_3, which yielded the highest precision based on the y-parallax, was chosen as the reference model. This is not to imply that this particular model was thought to give the most faithful representation of the terrain. Table 2 gives the standard errors of coordinate discrepancies (ΔX, ΔY, ΔZ) between the reference model and the models mapped onto it.

Inspection of Table 2 reveals that the ΔY discrepancies are in all cases higher than the corresponding ΔX values. The unusually high ΔZ discrepancies reflect weak Z-coordinate determinations as a result of short base in 80 per cent overlap and would be

TABLE 2. STANDARD ERRORS OF COORDINATE DISCREPANCIES AFTER MODEL MAPPINGS IN MICRONS AT PHOTO SCALE

	M_1			M_2			M_3		
	ΔX	ΔY	ΔZ	ΔX	ΔY	ΔZ	ΔZ	ΔY	ΔZ
P_1	5.8 6.3	9.1 8.0	15.2 20.3	5.6 6.0	9.4 7.4	17.5 20.2	5.3 6.1	11.3 8.2	19.3 20.1
P_2	4.8	9.3 —	20.2	6.0 6.5	8.4 7.9	19.2 15.2	9.4 5.5	12.1 9.9	18.4 21.0
P_3		Reference model —		5.1	6.4 —	20.6	5.5 9.2	8.4 10.8	18.4 18.4

expected to be half of these values with the normal 60 per cent overlap.

The X, Y, Z-coordinates of a model point are correlated quantities and must properly be regarded, statistically, as components of a three-dimensional random vector. This correlation is a result of transformation of measured photo coordinates and is readily demonstrated theoretically. At the same time a strong correlation evidently exists between the same coordinates of different points, at least locally. This latter correlation, which is mainly a result of unaccounted-for systematic errors of image positions on the photograph, is only revealed experimentally and cannot be predicted.

Because of the presence of the systematic errors, it is difficult to estimate the precision of the model Y-coordinates reliably from the residual model Y-parallaxes. The values of Table 1 can therefore be regarded as only approximate estimates of the standard errors of the Y-coordinates. The question now arises whether the Y-discrepancies in Table 2 can be explained in terms of the mapping of the models onto the reference model. To this end it is necessary to calculate the expected variance of the transformed Y-coordinates. Generally, when rotations are large, the Y-component cannot be considered independently because of its correlation with the X- and Z-components and the analysis has to be based on the multivariate methods. In our case, the models differ only slightly in scale and spatial orientation so that the mapping of Equation 1 induces little changes in coordinates. Consequently, the Y-coordinate can be treated separately for all practical purposes. Also, the variance of the transformed Y-coordinates will be expected to show only a slight increase because of the estimation of the transformation parameters and their small variance components.

For instance, the variance of the translation Y_0 is given by

$$\text{var}\ (Y_0) = s^2/n$$

where s^2 is the estimate of variance after transformation and n is the number of points—in our case about 110—from which the estimate is calculated. As the coordinates of both the reference model and the model undergoing transformation are random variables, certain amount of correlation is introduced by the mapping. In any case, the variance of the discrepancies ΔY between the Y-coordinates of the references Y_r and the transformed Y_t model is bounded by

$$2s_Y^2 \leq \text{var}\ (Y_r - Y_t) \leq 4s_Y^2$$

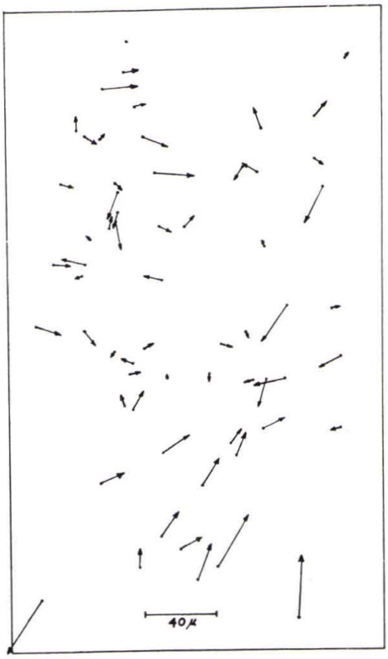

FIG. 2. Projections of residual position vectors on the XY-plane.

in which s_Y^2 represents the estimated variance of model Y-coordinates. The lower bound signifies complete independence and the upper bound total correlation between Y_r and Y_t. The corresponding bounds for the standard errors are $1.4\ s_Y$ and $2s_Y$. The standard errors of the ΔY's in Table 2 are all, except in one case, more than twice as large as the corresponding values in Table 1. This indicates that the models are not similar in terms of the mapping by Equation 1 but exhibit deformations which cause an inflation of the variance of the discrepancies. An indication of these deformations is revealed in Figures 2 and 3.

The principal goal of the experiment was to study variations among photographs taken on different occasions. In this analysis the individual models, after their transformation, are regarded as experimental units, each having specific characteristics embodied in the population from which it was drawn. In our case such a population could be thought of as all the photographs exposed under homogeneous conditions in a period of time.

In the following, the characteristics of each model is expressed quantitatively by the variance of the discrepancies between the spatial positions of corresponding points in the reference and the transformed models.

TABLE 3. NATURAL LOGARITHMS OF VARIANCES OF RESIDUAL VECTORS

	M_1	M_2	M_3
P_1	5.8551 6.2442	6.0568 6.2086	6.2710 6.2364
P_2	6.2519 —	6.1675 5.8201	6.3526 6.3421
P_3	Ref. Model —	6.1964 —	6.0638 6.2953

This leads to an analysis of variance based on the methods of univariate statistics.

The method of analysis of variance requires the observations to be independent, normally distributed variates with a homogeneous variance. As our observations are sample variances whose distribution is not normal, they have to be first transformed. The distribution of sample variance s^2 can be derived from the χ^2-distribution with the aid of the relationship

$$s^2 = \sigma^2 \frac{\chi^2}{f}$$

in which σ^2 is the population variance and χ^2 denotes sum of squares of f independent, normally distributed variates with parameters *0*, *1*). It can be shown (e.g., Kenney-Keeping 1951) that the logarithmic transformation will accomplish what is needed. Table 3 gives the natural logarithms of the variances of the residual position vectors for various models. The interpretation of the data in Table 3 is as follows. The three missions M_1, M_2, M_3 are considered a random sample from a population of missions. The three passes within each mission are regarded as random samples of passes. Finally, the two values which appear within most of the cells refer to two models reconstructed from three consecutive pictures with the middle picture common to both models. Three sources of variability can thus be distinguished: Variation between missions, variation between passes within missions and variation between models within passes within missions.

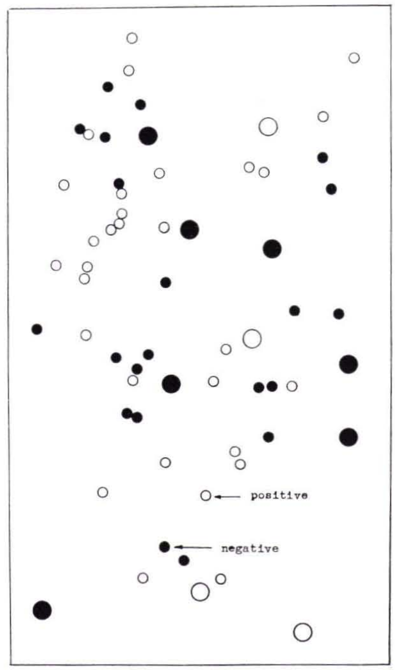

FIG. 3. Distribution of the signs of the Z-residuals after Model Mapping.

The analysis of the data in Table 3 is summarized in the Analysis of Variance Table (Table 4).

In this table the total sum of squares of the values in Table 3 is first corrected for the mean and then the residual amount is partitioned into three parts, each corresponding to the above mentioned source of variation. Similarly, the remaining fourteen degrees of freedom (after eliminating the mean) are divided to correspond to each source. The column of mean squares is obtained by dividing each sum of squares by its appropriate number of degrees of freedom.

The first question to be answered is whether there is a closer geometric resemblance between the two models obtained from photographs taken a few seconds apart than

TABLE 4. ANALYSIS OF VARIANCE TABLE

Source of Variation	*Sum of Squares*	*Degrees of Freedom*	*Mean Square*	*F Ratio*	*F Tab.*
Mean	532.7391	1	532.7391		
Between missions	0.0892	2	0.0446	1.86	$F_{2,11}=3.98$ at 5% level
Between passes within missions	0.0891	5	0.0178		
Between models within passes	0.1750	6	0.0292		

between the models belonging to different passes. This is answered by comparing the last two mean squares. It turns out, rather surprisingly, that no difference is indicated; in fact, the differences between models within passes are greater than the differences between models from different passes but, of course, not significantly greater. The last two mean squares can therefore be pooled and the resulting mean square used to test the differences between missions.

The F-ratio is thus given by:

$$F_{2,11} = \frac{0.0446 \times 11}{0.0891 + 0.1750} = 1.86.$$

From the tables of the F-distribution we find that the value to be exceeded with probability of 5 per cent is 3.98. It is therefore concluded that no significant difference can be demonstrated between models coming from different missions.

Summary of Findings

It was demonstrated that the precision of ± 3 to 4 microns of residual y-parallax could be reached consistently within a model. The random error component, resulting from identification and measuring errors, varied between ± 1.5 to 3 microns and was found to depend strongly on the quality of resolved target images. It was found that the model points exhibit locally lack of independence which is accompanied by a loss of accuracy in the coordinates. The unaccounted-for systematic errors in image positions seem to give each model a unique character of slight deformation which appears to behave without any set pattern from model to model. No differences were found to exist between models reconstructed from photographs taken at short or long time intervals.

These results should be interpreted in the light of data analyzed to date and within the framework of the conditions of this study.

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