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Empirical Observations on Relative Orientation

The Analytical Plotter offers an effective means for varying the many parameters and measuring the results.

PHOTOGRAMMETRIC RELATIVE ORIENTATION might be considered fundamentally as the effort to match a pair of stereoscopic overlapping photos so that all homologous image points have zero *y*-parallax. Many sources of errors and distortions exist which practically prevent this perfect condition of relative orientation to be attained. These errors and distortions originate in the photography, in the optical systems of the instrumentation, and can even be attributed to the operator of the stereoplotting instrument.

This problem of how to attain the optimum solution for a relative orientation is closely parallel to the equally challenging problem of attaining a perfect interior orientation and also the absolute orientation. All these phases of the general orientation problem for double resection in space are absolutely essential for the highest accuracy and the greatest economy in photogrammetric processes.

Historically, two methods of relative orientation have been developed for use with stereoplotting equipment. These are the so-called *trial-and-error* method and the *semi-numerical* or *numerical* method. The trial-and-error method is ideally suited to anaglyphic stereoprojection plotters where accurate quantitative measurements of residual *y*-parallaxes in the stereomodel are not readily made. The semi-numerical method combines some trial and error procedures with the capability to measure residual *y*-parallaxes with some degree of accuracy. Changes in the relative orientation parameters which will tend to eliminate the *y*-parallaxes can then be determined mathematically. However, because these computations are approximate, and also because the corrections to the parameters which have been computed cannot be applied with any great degree of accuracy on an analogue-type stereoplotter, the procedure fundamentally must be iterative, and still subject to some trial-and-error procedure.

The characteristics and techniques associated with both these methods are well documented and do not require further description. Practically any text book on photogrammetry describes them thoroughly.

THE INTRODUCTION of the high-precision comparators and digital computers permitted more rigorous mathematical approaches to the relative orientation procedure. However, the technique of computing the desired relative orientation of two photographs by analytic means, although by its nature being relatively accurate, could not be applied to nor-

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mal analogue stereoplotting instruments inasmuch as the computed elements of orientation could not be introduced into the stereoplotter with a degree of accuracy commensurate with the analytical determination itself.

Among the weaknesses of the trial and error and numerical procedures are the restrictions which are placed upon the location of the points where parallax is to be measured and removed. For example, elimination of y-parallax along the air base of the stereomodel is fundamental in these procedures. The numerical approaches require that the parallax be measured in areas of the model which conform very closely to predetermined coordinate values so that the coefficients used in the simplified computation are pertinent. Only the analytical approach permits the relative orientation to be made with complete freedom insofar as the location of the points at which parallax is to be removed. Of course it is to be considered that these points should represent a wide dispersion in the stereomodel for greater representation of the stereomodel as it should exist. Contrariwise, the analytical approach also permits relative orientation of partial stereomodels—that is, stereomodels where parallax clearance or measurement is limited due to incidence of water areas, cloud cover, or other natural phenomena which makes it difficult to observe homologous images.

Unfortunately, even if it is assumed that a digital computer is readily available and a precision comparator is on hand, it is not a simple matter to perform a relative orientation by analytical procedure for the reasons stated above, namely, that the mathematical derivations cannot be accurately introduced into an analogue stereoplotter. This limitation has precluded any serious study of the formation of partial stereomodels.

THE AVAILABILITY OF an Analytical Stereoplotter has somewhat changed these limitations and suggests a series of interesting experiments. The Analytical Stereoplotter, because of its unusual properties (that is, computer on-line with comparator) permits extremely rapid orientation of a stereo pair, and, further, a very precise analysis in digital terms of the quality of the orientation. Any existing procedure for performing relative orientation can be utilized on the Analytical Plotter and evaluated. Techniques such as the pure trial and error, numerical methods of various types and purely analytical procedures can all be performed and evaluated against each other.

ABSTRACT: *Procedures for relative orientation of stereo pairs when working with photogrammetric stereoplotting instruments have evolved first as trial-and-error techniques and later as techniques based upon mathematical analysis. Classical methods have been established which are now universally used and are rarely questioned as to their efficiency. Unfortunately, these classical methods are not readily analyzable with conventional stereoplotters. The use of an AP/C Analytical Stereoplotter facilitates the study of a wide variety of relative orientation techniques with variations in terrain parameters and parallax-removing station arrays. Conclusions are drawn from the empirically gathered data.*

A new program recently developed for the Analytical Plotter, Model AP/C, by the Bendix Research Laboratories was described at last year's convention to the ASP. A paper by Dr. Robert Forrest has since been published in *Photogrammetric Engineering* of November 1966. This new program permits a remarkably fast and precise orientation to be made. The orientation is based upon purely analytical techniques, but, unlike the combination of a separate precision comparator and an off-line digital computer, the results of the AP/C program computation are immediately observable to the human operator in an analogue stereoplotter type fashion. This means that the operator can scan the stereomodel immediately after the relative orientation is computed to determine whether y-parallax has been adequately removed even though the computer has already, by a digital signal, assured him of this fact.

WHILE WORKING WITH these new programs and evaluating them and their usefullness it became apparent that setting up stereomodels was no longer a tedious chore requiring a skilled operator. Any technical personnel regardless of acquaintanceship with analogue procedures can readily and quickly set up stereomodels. This encouraged us to look into the ever interesting problem of conditions for obtaining adequate, good or optimum relative orientation. The variables which affect a relative orientation process such as type of lens, altitude of photography, area of overlap, character of terrain, presence of lens distortion, differential film shrinkage could all be studied. The temptation to evaluate these variables and study them in great depth under operational conditions was quite strong; unfortunately, our time was limited and we were constrained to make a more or less exploratory survey of what areas could be studied.

The AP/C relative-orientation program requires only that the instrument operator remove y-parallax from not less than five points or not over 14 points. Less than five points is, of course, indeterminate and too many points tend to overload the computer unless special provisions are made in advance. The relative orientation program is also written so that the scale of the model is predetermined by the operator even before the computation is made. This feature does not appear to be of immediate importance to the studies we have made so far, but obviously, it would be significant for studies of relative orientation with respect to aerial triangulation procedures.

THE ACTUAL MECHANICS of performing the relative orientation requires that the operator move the floating mark to that point in the stereomodel where he desires to clear parallax. He removes the parallax by placing the floating mark on the surface he is observing and rotating the knob on his control panel. Having accomplished this he presses a button which, in effect, sends to the computer memory the coordinate data required by the computer. The operator can then repeat his observations several times which will allow the computer to average his observations, or, if he prefers, he moves on to his second parallax removal point anywhere else in the model. When the operator has annulled parallax at the desired number of points he presses a button marked "Compute." The computer then iterates the computation searching for the relative orientation parameters based upon the parallax readings made which will approximate a collinear condition with minimum residuals. Upon reaching a residual of five microns or less the computer automatically halts the computation and the parameter corrections are placed in the computer memory. This effectively creates a stereomodel. If, because of errors in the observation, distortions, or other causes, the computer cannot reach a five micron or less solution, the computer continues to oscillate and iterate around the best solution that it can obtain. It is then necessary for the operator to manually halt the computation. When the computation is thus halted the last solution is the one entered into the computer memory.

Upon completion of every computation (that is, every iteration) the instrument control panel shows by means of Nixie tubes the *figure of merit* for that computation. This figure of merit (F.M.) is essentially the square root of the sum of the squares of the residuals of the photo coordinates and can be taken as an indication of the reliability of the solution provided the number of points used is also considered.

WE STARTED OUR investigation with a study of very low-altitude, wide-angle photography over a relatively swampy area. The stereomodel was lacking in contrast and imagery was rather poor. The 6-inch focal-length, wide-angle lens was of the nominal distortion-free type, but is known to have a maximum radial distortion in excess of 10 microns. No corrections were made for systematic metric camera errors. The baseheight ratio was approximately 0.55. Table I summarizes the results and Figures 1a, 1b, 1c, 1d and 1e show the locations in the stereomodel at which parallax was removed.

An analysis of the results shown in Table I reveal that, as expected, a five point solution would result in a zero residual error. The Figure of Merit for 6, 9 and 12 points are practically equal. It could be inferred that this is possibly due to uncorrected lens distortion. However with six symmetrical points, lens distortion does not influence the y-parallax solution as it does with larger numbers of points. It would be interesting to investigate whether this limitation was due to film deformation. However, no adequate metric camera data was available to us to pursue this investigation to its quantitative end.

TABLE I. SUMMARY OF RESULTS OF SITUATIONS SHOWN IN FIGURES 1*a* · · · 1*e*. FOCAL LENGTH—6 INCHES, WIDE ANGLE; ALTITUDE—3,000 FEET; FLAT TERRAIN

Number of Points	*Time required to orient a Model (minutes)*	*Figure of Merit (Microns at photo scale)*
5	6	0
6	9	12
7	10	4
9	13	12
12	17	13

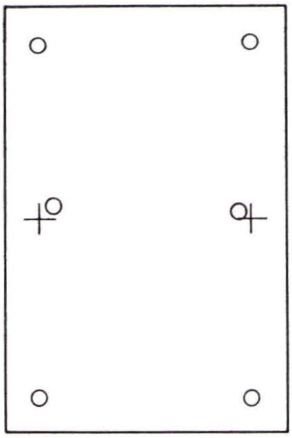

FIG. 1a. Diagram of the stereomodel in which the small circles show the locations of the points where y-parallax was removed. In this instance the points were in the 6 conventional locations, the time required for orientation was 9 minutes, and the Figure of Merit (F.M.) was 12 microns.

We did note that for the five-point solution the residual parallax at the sixth point was 40 microns. Obviously, if this sixth point had been included as in our six-point array, we could have expected results very similar to the six-point solution.

Also of interest is the seven-point array. These points were picked to surround an area of interest covering approximately one half the model. Parallax was cleared on the best possible observable points. The resultant relative orientation had a figure of merit of four microns. Examination of the model area of interest, that is, the area enclosed by the points used for parallax clearance, showed practically no observable y-parallax. However, in the areas of less interest not covered or surrounded by the array, visible parallax existed.

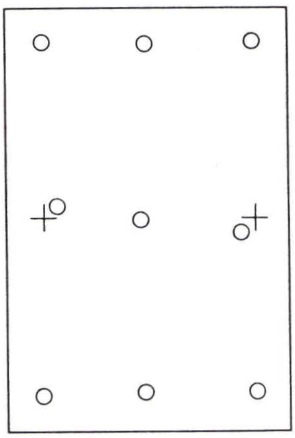

FIG. 1b. Similar to Figure 1a but where the parallax was removed at 9 locations, the time was 13 minutes, the F.M. was 12 microns.

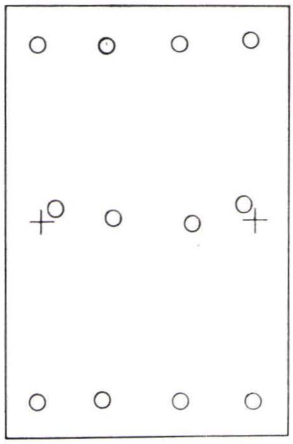

FIG. 1c. Similar to Figure 1a but where the parallax was removed at 12 locations, the time was 17 minutes, and the F.M. was 13 microns.

WE REPEATED THE relative orientation procedures described above with 5-, 6-, 9- and 12-point arrays with 20,000-feet altitude photography taken with a 6-inch focal length camera, over a relatively flat area, and then once again with the 6,000-feet photography over areas where the terrain relief was in excess of 30 per cent of the flight altitude. The cameras were again of the wide angle, nominally distortion-free type, and no corrections were made for lens or film distortion. Table II

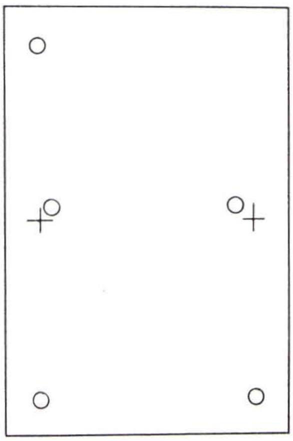

FIG. 1d. Similar to Figure 1a but where the parallax was removed at only 5 locations (the smallest number that will yield a unique solution), the time was 6 minutes, and the F.M. was zero.

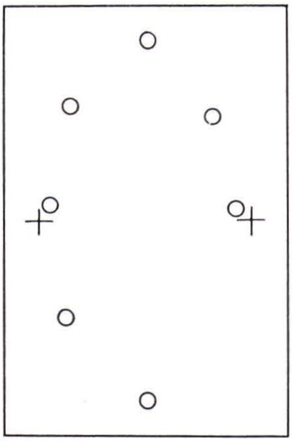

FIG. 1e. Similar to Figure 1a but where the parallax was removed at 7 locations as shown, the time was 10 minutes, and the F.M. was 4 microns.

shows results for the 20,000-feet altitude tests and Table III for the 6,000-feet photography.

In both Table II and Table III we note that the six-point array provides a zero residual. This does not necessarily mean the model is parallax free. It simply tells us that these symmetrical points have been cleared. However, when we examine the 12-point solution, it is evident that the entire model cannot be cleared with better than the accuracy indicated, unless the sources of distortion are known and corrected.

DURING OUR INVESTIGATION we heard from Mr. Lloyd O. Herd and Mr. Jennings of the Ohio Highway Department. It seemed that they had some photography in which water areas covered a good part of the center of the model for several stereo pairs of the flight. They had been experimenting with their AP/C, setting up partial models and were having a high degree of success. We thought it would be interesting to duplicate some of their studies and report on them. We started out with a conventional pair of photographs taken with the wide-angle camera, nominally distortion free-type taken at 20,000 feet over flat terrain. The model was first oriented in the classical six-point array. The same model was then re-oriented with areas being denied to the operator. This was a simulation of partial models of various patterns. Figures 2a through 2g demonstrate the partial models, location of the points at which parallax was removed, and the figure of merit for the resultant model.

The experiment was repeated with 6,000-feet altitude photography over extremely hilly terrain. Results were quite similar to those obtained at 20,000 feet over flat terrain.

The test results in both cases agreed with the findings of the Ohio Highway Department. These findings demonstrate conclusively that an Analytical Plotter will permit relative orientation of a partial stereo model with ease and with a high degree of accuracy in a very short time, even where the model is reduced to being composed of approximately one-third stereoscopic overlap as compared with the normal, usual, neat-model area. Conventional techniques of relative orientation such as trial-and-error or seminumerical methods would be impractical in such cases.

We planned to initiate the same studies with ultra-wide angle photography to ascertain whether the differences in geometry would influence the results of the orientation to any significant degree. However, we found to our chagrin that we had a limited amount of photographs of this type and no information on the lens distortion. Inasmuch as ultra-wide-angle photography cannot even be considered nominally distortion free, this would not be of sufficient interest or validity without adequate correction and, therefore, we had to forego these tests.

TABLE II. SUMMARY OF RESULTS SIMILAR TO THOSE SHOWN IN TABLE I BUT WHERE THE ALTITUDE WAS 20,000 FEET

Number of Points	*Time required to orient a Model (minutes)*	*Figure of Merit (Microns at photo scale)*
6	10	0
9	14	4
12	20	16
12*	30	7

* Three observations per point.

OUR SUPERFICIAL investigations, which have been described, have been of value in

TABLE III. SUMMARY OF RESULTS SIMILAR TO THOSE SHOWN IN TABLE I BUT WHERE THE ALTITUDE WAS 6,000 FEET AND THE TERRAIN WAS HILLY

Number of Points	*Time required to orient a Model (minutes)*	*Figure of Merit (Microns at photo scale)*
6	9	0
9	13	5
12	22	8

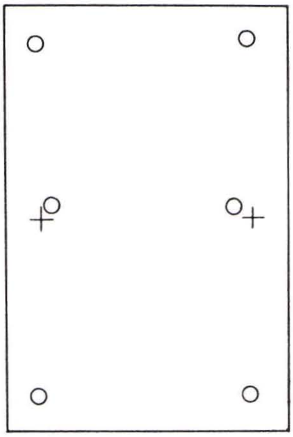

FIG. 2a. Diagram of a stereomodel oriented in the classical six-point manner, in which this solution is used as a basis for comparison with the following Figures 2b . . . 2g. The time required for the orientation was 10 minutes, and the Figure of Merit was zero.

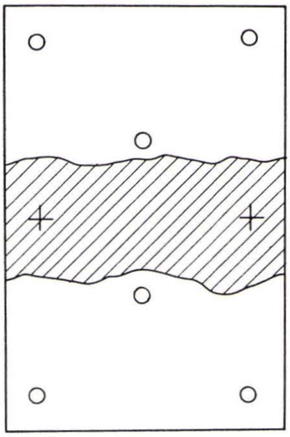

FIG. 2c. Similar to Figures 2a and 2b but where the 6 locations were distributed as shown, requiring 10 minutes, and yielding an F.M. of zero.

indicating areas which could be further studied in great detail and with accuracies heretofore not possible. Techniques of analytical photogrammetry can be evaluated on a very practical *see-for-yourself* basis. Such variables as number of parallax-removal points used, location of these points, and number of observations per point can all be evaluated against the accuracy of the model and the amount of time required to perform the orientation. The effects of type of terrain, type of photography and known distortion can also be precisely evaluated and analyzed. The Analytical Plotter provides a very objective basis for such studies because the influence of the skill of the operator to a great extent is eliminated. The skilled operator or a relatively new operator should obtain equivalent results if they both have normal vision and exercise normal care. It is expected, of course, that the operator with the greater experience on the AP/C would save some degree of time.

Of great interest in a continuing sequence of investigation along these lines would be the influence of relative orientation on absolute orientation and vice-versa. If we are concerned with individual models for plotting or cadastral work, we would naturally be primarily concerned with the fit of the model to the local ground control. Distortions in those areas of a stereomodel which would be

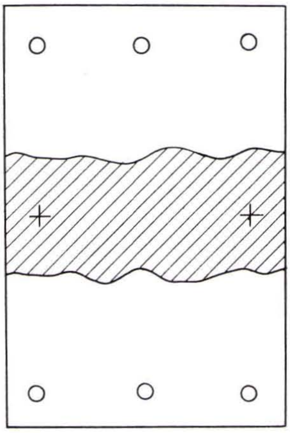

FIG. 2b. Similar to Figure 2a but where the shaded area is considered to be covered by water. The parallax was removed at the 6 locations as shown, the time was 11 minutes, and the F.M. was 21 microns.

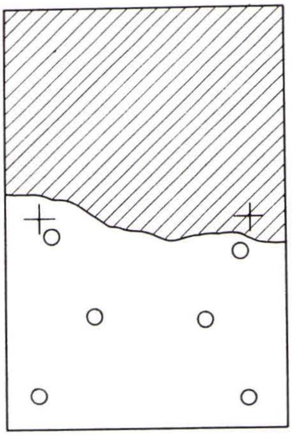

FIG. 2d. Similar to Figure 2a in which the shaded area is considered to be covered by water. The parallax was removed at 6 locations as shown, the time was 9 minutes, and the F.M. was zero.

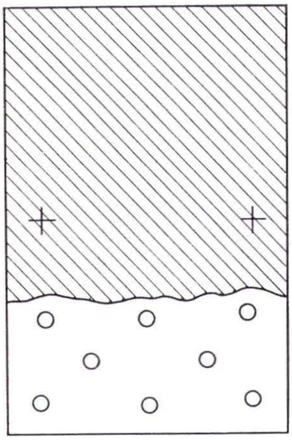

FIG. 2e. Similar to Figure 2a but where the water lies on one side of the model as shown. The number of locations used was 8, the time was 15 minutes, and the F.M. was 13 microns.

of no interest can be tolerated; therefore, a relative orientation can be patterned after this requirement regardless of the theoretical purity of the entire cone of rays. On the other hand, if we are concerned with aerial triangulation or bridging of a strip, it may well be that our primary concern is to preserve and reconstitute the original cone of rays and ignore local parallaxes in the stereomodel caused by random, local distortions.

We have only just started our investigations in these areas and we hope to pursue them further. We are very encouraged that several universities are working, or plan to work in these areas. There is much to be done.

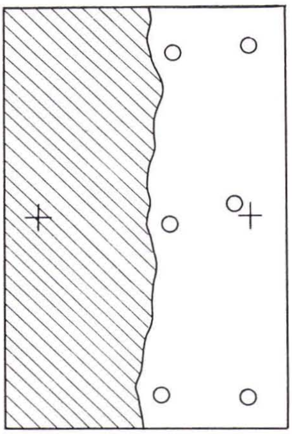

FIG. 2f. Similar to Figure 2a where the water covers part of the model as shown by the shaded area. The number of locations was 6, the time was 11 minutes, and the F.M. was 5 microns.

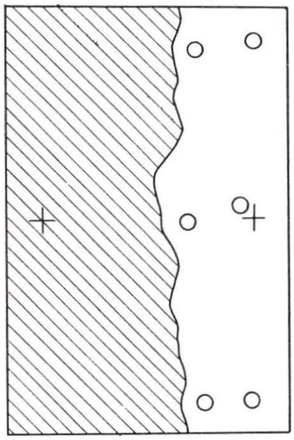

FIG. 2g. Similar to Figure 2f but where the 6 points are confined to a narrower portion of the model. The time was 10 minutes and the F.M. was 6 microns.

We at OMI are essentially a commercial group and cannot devote as much time to this type of research as laboratories or academic organizations. We do welcome any interest and will endeavor to make our AP/C Analytical Stereoplotter available to facilitate research problems along these lines.

IN CLOSING I would like to give my thanks to Mr. James R. Skidmore of our organization who has done all the laboratory work and reduced my role simply to narrating the results. I also wish to express my thanks to the personnel of the Ohio Highway Department who have been good enough to share their results with us.