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Analytical Treatment of Strip and Pan Photos

The classical projective equations apply also to continuous strip and, modified, to panoramic photographs.

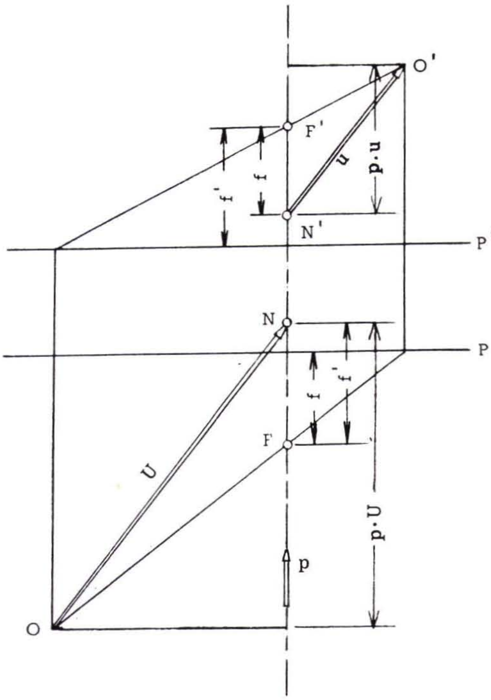

FIG. 1. Relations between an object point O and the image point O' formed by a lens with principal planes P and P'. Drawn for a positive lens, with f and f' both taken positive. For negative lenses f and f' are both negative.

INTRODUCTION

THE RELATIONS BETWEEN a ground (object) point O and the corresponding photograph (image) point O' are illustrated in Figure 1. Here N and N' are the first and second nodal points of the camera lens, P and P' are the principal planes, f and f' are the object space and image space focal lengths, and p is a unit vector normal to the principal planes. Let U be the vector $\overline{ON}$ and u the vector $\overline{N'O'}$. Then the projective conditions of geometric optics† (with atmospheric refraction and lens distortions neglected) and comparison of similar triangles lead to the following vector equations:

$$u = \frac{fU}{p \cdot U - f'} \tag{1}$$

and

$$U = \frac{f'u}{p \cdot u - f} \tag{2}$$

which are equivalent to one another.

In this paper it will be assumed that Equations 1 and 2 are valid even when there is relative motion of the lens with respect to the object and/or the image. In such cases

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† See, for example, page 152 of Born.[1]

the vectors U and u are time functions. A thorough justification of this assumption would range beyond geometric optics into physical optics, and is beyond the intended scope of this paper. The following rationalization can be developed to be at least partially satisfying.

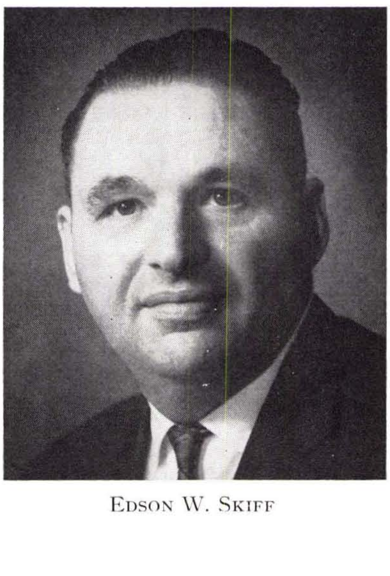
To examine the physical processes, assume a reference frame moving so the *lens* appears stationary. Thus the situation is treated as a moving image of a moving object produced by a stationary lens. Applying classical electromagnetic theory then leads to the conclusion that the imaging of any particular object *point* is in accordance with Equations 1 and 2 providing the vectors U and u are considered functions of *retarded* time. The latter takes account of the finite time required for light to travel from

ABSTRACT: *The projective equations of analytical photogrammetry, originally published by von Gruber for frame type photographs, are interpreted so as to also apply to strip and, with a slight modification, to panoramic photography. From the solutions of the projective equations, ground points can, at least in principle, be computed from measured coordinates of corresponding points on two or more photographs selected in any combination from the three types of photography. Just as fiducial marks are needed to determine the principal points of frame type photographs, so timing marks are needed to determine the locus of principal points on the other two types. Statistical adjustment of strip and panoramic photographs requires computation of certain additional parameters which are not needed for frame photographs. For maximum clarity much of the paper is written in tensor notation.*

one point to another. In this paper the retardation will be neglected, however, and the coordinates of corresponding points will be evaluated *simultaneously* in time.

Now assume an arbitrary Cartesian coordinate system (moving or stationary). Let X^a and $X_1{}^a$ be the X, Y, Z-coordinates of points O and N. Likewise let x^a and $x_1{}^a$ be the coordinates of points O' and N'. Let λ_a be the coordinates of points O' and N'. Let λ_a be the components of the unit vector p. Then Equation 1 may be written in the form:

$$x^a - x_1{}^a = \frac{f(X^a - X_1{}^a)}{f' + \lambda_b(X^b - X_1{}^b)} \cdot \qquad (3)$$



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Equation 3 is covariant with respect to transformations among Cartesian coordinate systems so long as no system approaches a velocity such that relativistic effects become appreciable. That is, Equation 1 may likewise be written in a second Cartesian coordinate system which may also be moving arbitrarily:

$$x^m - x_1{}^m = \frac{f(X^m - X_1{}^m)}{f' + \lambda_n(X^n - X_1{}^n)}\,. \tag{4}$$

We are interested, however, in using two different coordinate systems simultaneously—one for the object space, the other for the image space. Let X^a and $X_1{}^a$ be the coordinates of points O and N in the *ground* space system and let x^m and $x_1{}^m$ be the coordinates of points O' and N' in the *photograph* coordinate system. Let the direction cosines of the second system relative to the first be $C_a{}^m$. Then

$$X^m - X_1{}^m = C_a{}^m(X^a - X_1{}^a)$$

and

$$\lambda_m = C_m{}^a\lambda_a.$$

Consequently:

$$x^m - x_1{}^m = \frac{fC_a{}^m(X^a - X_1{}^a)}{f' + \lambda_b(X^b - X_1{}^b)}\,. \tag{5}$$

Although Equation 5 is valid with respect to arbitrarily moving coordinate systems, one usually takes the ground coordinate system stationary relative to the ground, and the photograph coordinate system stationary relative to the photographic film.* Thus X^a and x^m are taken constant for fixed ground points. The lens may, however, be moving both with respect to the ground and with respect to the photographic film. Thus $X_1{}^a$ and $x_1{}^m$ are, in general, time functions dependent on the motion of the camera with respect to the ground and on the Image Motion Compensation (IMC) mechanism which is built into the camera.

For aerial photographs the object distance $\lambda_b(X_1{}^b - X^b)$ is so much greater than the focal length f' that Equation 5 may be quite adequately approximated by:

$$x^m - x_1{}^m = \frac{fC_a{}^m(X^a - X_1{}^a)}{\lambda_b(X^b - X_1{}^b)}\,. \tag{6}$$

Equation 6 is to be interpreted for three different types of photography: frame, strip, and panoramic.

Statement of Equations

For frame and strip type photographs the film is exposed while confined in a geometric plane at a fixed normal distance from the camera lens. Assume that the camera motion is such that the direction normal to the film does not appreciably change direction during the exposure time for the photograph. Then the lens normal λ_b must also be constant in direction during the exposure time. Let the photograph coordinate system be oriented with its $x^{3'}-(z)$-axis normal to the film. Then $\lambda_b = C_b{}^{3'}$ and Equation 6 becomes:

$$x^m - x_1{}^m = \frac{fC_a{}^m(X^a - X_1{}^a)}{C_b{}^{3'}(X^b - X_1{}^b)}\,. \tag{7}$$

* Because it's desirable to have the formulas in terms of coordinates which can be measured directly.

This is a shorthand representation of what are often referred to as the projective equations of von Gruber. They are here seen to apply to strip type as well as to frame type photographs. X_1^a and x_1^m are constant for frame type but time functions for strip type. C_a^m (including $C_b^{3'}$) are constant for frame type and, for small regions at least, of strip type photographs.

For panoramic photographs the film is maintained in the form of half of a circular cylinder during exposure. The lens (and a slit) are rotated to produce a sweeping exposure. Hence the lens normal λ_b must be expressed in terms of the camera sweep angle $\alpha_1(=\omega t)$. Let the photograph coordinate system be oriented so the $x^{1'}$ axis is parallel to the cylinder axis and the $x^{3'}$-axis is normal to the tangent plane at the *top* of the half-cylinder. Let α_1 be zero at the $x^{3'}$-axis and increasing positively in the direction toward the $x^{2'}$-axis. Then

$$\lambda_m = C_m{}^b\lambda_b = (0, \sin \alpha_1, \cos \alpha_1).$$

Hence (using Equation 6),

$$\lambda_m(x^m - x_1{}^m) = (x^{2'} - x_1{}^{2'}) \sin \alpha_1 + (x^{3'} - x_1{}^{3'}) \cos \alpha_1 = f. \tag{8}$$

Evidently Equation 8 will be true if

$$x^{2'} - x_1{}^{2'} = f \sin \alpha_1 \tag{9}$$

and

$$x^{3'} - x_1{}^{3'} = f \cos \alpha_1. \tag{10}$$

Thus $x^m - x_1{}^m$ are *rectangular* coordinates of the circular cylinder relative to a coordinate system in which the film is stationary.

Combining Equations 9 and 10 with 6:

$$x^m - x_1{}^m = f \cos \alpha_1 \frac{C_a{}^m(X^a - X_1{}^a)}{C_b{}^{3'}(X^b - X_1{}^b)} \tag{11}$$

with

$$\alpha_1 = \tan^{-1} \frac{C_a{}^{2'}(X^a - X_1{}^a)}{C_b{}^{3'}(X^b - X_1{}^b)}\,. \tag{12}$$

Equations 11 and 12 for a panoramic type photograph correspond to von Gruber's equations for a frame photograph. $X_1{}^a$, $x_1{}^m$, and α_1 are time functions depending on the camera motion, the IMC mechanism, and the lens sweep mechanism.

Equations 7, 11, and 12 are basically correct but are of little value until the various time functions are evaluated. Evaluation of these time functions is dependent on the particular cameras and on the flight pattern. The following approximate treatment is cursory and is based on descriptions of particular cameras which have been published in the literature. Only summary statements are given as the details are available by consultation of the references cited.

FRAME TYPE PHOTOGRAPHY[2]

The film is maintained in a plane and exposed simultaneously over the whole photograph. Image motion is neglected because the exposure time is brief.

$$x^m - x_1{}^m = \frac{f_1 C_a{}^m(X^a - X_1{}^a)}{C_b{}^{3'}(X^b - X_1{}^b)} \tag{7}$$

$$x_1{}^m = x_{10}{}^m \text{ (constant)}$$

$$X_1{}^a = X_{10}{}^a \text{ (constant)}$$

$$\therefore \quad x^m - x_1{}^m = (x^{1'} - x_{10}{}^{1'}, x^{2'} - x_{10}{}^{2'}, f_1). \tag{13}$$

STRIP TYPE PHOTOGRAPHY[3,4]

The film is maintained in a flat plane but is exposed by moving it past a narrow slit which results in a sweeping exposure. The exposure time for any small area is brief but the time interval required to sweep the whole photograph is appreciable. There is a fixed angle β_1 associated with the slit-lens scanning operation. This angle is here taken positive for backward looking, or negative for forward looking, slits. Let the photograph coordinate system be oriented with its $x^{3'}$ axis normal to the film plane and its $x^{1'}$ axis parallel, but opposite, to the direction of lens-slit motion relative to the film. Assume that the time functions are linear in t:

$$x^m - x_1{}^m = \frac{f_1 C_a{}^m(X^a - X_1{}^a)}{C_b{}^{3'}(X^b - X_1{}^b)} \tag{7}$$

$$X_1{}^a \cong X_{10}{}^a + V^a t \tag{14}$$

$$x_1{}^m \cong x_{10}{}^m + v\delta_{1'}{}^m, t \tag{15}$$

$$x^m - x_1{}^m = (f_1 \tan \beta_1, x^{2'} - x_{10}{}^{2'}, f_1) \tag{16}$$

$$\therefore \quad t \cong \frac{x^{1'} - x_{10}{}^{1'} - f_1 \tan \beta_1}{v} \tag{17}$$

$$t \cong \frac{(C_a{}^{1'} - C_a{}^{3'} \tan \beta_1)(X^a - X_{10}{}^a)}{(C_b{}^{1'} - C_b{}^{3'} \tan \beta_1)V^b} \tag{18}$$

where V^a is the camera ground speed vector, and v is the velocity of IMC (here taken as a *negative* number).

PANORAMIC TYPE PHOTOGRAPHY[5,6,7]

The film is maintained in half of a circular cylinder and exposed by a slit which scans around the cylinder. The exposure time for any small area is brief, but the total scanning time is appreciable. There is a scanning angle α_1 which increases at a *rate* approximately *uniform* in time, and which is here taken positive in the direction from the $x^{3'}$ toward the $x^{2'}$-axes. Let the photograph coordinate system be oriented with its $x^{1'}$-axis parallel to the axis of the cylinder and with $\alpha_1 = 0$ at the $x^{3'}$-axis. Assume that the lens motion for IMC is in the negative $x^{1'}$ direction. Assume that $X_1{}^a$ is a linear function of time but that the IMC velocity is proportional to *cos* α_1.

$$x^m - x_1{}^m = f_1 \cos \alpha_1 \frac{C_a{}^m(X^a - X_1{}^a)}{C_b{}^{3'}(X^b - X_1{}^b)} \tag{11}$$

$$\alpha_1 = \tan^{-1} \frac{C_c{}^{2'}(X^a - X_1{}^a)}{C_b{}^{3'}(X^b - X_1{}^b)} \tag{12}$$

$$\alpha_1 \cong \omega t \tag{19}$$

$$X_1{}^a \cong X_{10}{}^a + V^a t \tag{20}$$

$$x_1{}^m \cong x_{10}{}^m + \frac{v_M}{\omega} \delta_{1'}{}^m \sin \alpha_1 \tag{21}$$

For measurement, the panoramic photograph is laid out flat. The linear coordinate in the scan direction is then

$$y - y_{10} = f_1 \alpha_1 \tag{22}$$

but no change occurs in the coordinate $x^{1'}$–$x_1{}^{1'}$—parallel to the cylinder axis. Quantitative use of a panoramic photograph is usually for calculation of derived quantities (ground coordinates, for example) as a function of measured photograph coordinates. Hence the following substitutions will usually be made:

$$x^m - x_1{}^m = \left(x^{1'} - x_{10}{}^{1'} - \frac{v_M}{\omega} \sin \frac{y - y_{10}}{f_1},\ f_1 \sin \frac{y - y_{10}}{f_1},\ f_1 \cos \frac{y - y_{10}}{f_1}\right). \quad (23)$$

These follow quite easily from the preceding equations.

Solution of the Equations

Equations 7 and 11 may *both* be rewritten in the following form:

$$X^a - X_1{}^a = \frac{C_m{}^a(x^m - x_1{}^m)}{C_n{}^3(x^n - x_1{}^n)}(X^3 - X_1{}^3). \quad (24)$$

Let a second photograph including coverage of the same ground region have the corresponding formula:

$$X^a - X_2{}^a = \frac{C_r{}^a(x^r - x_2{}^r)}{C_s{}^3(x^s - x_2{}^s)}(X^3 - X_2{}^3). \quad (25)$$

Equation 25 may be subtracted from 24 and the result rearranged in the form

$$(C_m{}^aC_r{}^3 - C_m{}^3C_r{}^a)(x^m - x_1{}^m)(x^r - x_2{}^r)(X^3 - X_1{}^3)$$
$$= C_m{}^3[C_r{}^3(X_2{}^a - X_1{}^a) - C_r{}^a(X_2{}^3 - X_1{}^3)](x^m - x_1{}^m)(x^r - x_2{}^r). \quad (26)$$

In Equation 26 set the index $a=1$ and divide by the coefficient of $(X^3 - X_1{}^3)$:

$$X^3 - X_1{}^3 = \frac{C_m{}^3[C_r{}^3(X_2{}^1 - X_1{}^1) - C_r{}^1(X_2{}^3 - X_1{}^3)](x^m - x_1{}^m)(x^r - x_2{}^r)}{(C_n{}^1C_s{}^3 - C_n{}^3C_s{}^1)(x^n - x_1{}^n)(x^s - x_2{}^s)}. \quad (27)$$

Combining Equations 24 and 27,

$$X^a - X_1{}^a = \frac{C_m{}^a[C_r{}^3(X_2{}^1 - X_1{}^1) - C_r{}^1(X_2{}^3 - X_1{}^3)](x^m - x_1{}^m)(x^r - x_2{}^r)}{(C_n{}^1C_s{}^3 - C_n{}^3C_s{}^1)(x^n - x_1{}^n)(x^s - x_2{}^s)}. \quad (28)$$

In a similar way:

$$X^a - X_2{}^a = \frac{C_r{}^a[C_m{}^3(X_2{}^1 - X_1{}^1) - C_m{}^1(X_2{}^3 - X_1{}^3)](x^m - x_1{}^m)(x^r - x_2{}^r)}{(C_n{}^1C_s{}^3 - C_n{}^3C_s{}^1)(x^n - x_1{}^n)(x^s - x_2{}^s)}. \quad (29)$$

Subtracting Equation 29 from 28:

$$X_2{}^a - X_1{}^a = \left\{\frac{(C_m{}^aC_r{}^3 - C_m{}^3C_r{}^a)(X_2{}^1 - X_1{}^1) + (C_m{}^1C_r{}^a - C_m{}^aC_r{}^1)(X_2{}^3 - X_1{}^3)}{(C_n{}^1C_s{}^3 - C_n{}^3C_s{}^1)(x^n - x_1{}^n)(x^s - x_2{}^s)}\right\}$$
$$\cdot\{(x^m - x_1{}^m)(x^r - x_2{}^r)\}. \quad (30)$$

Equations 28, 29, and 30 are equivalent to Equations 15 and 16 in the paper by Hellmut Schmid.[8] Schmid's terminology has the following correspondence to that used here.

$$X^a - X_1{}^a = [(X - X_0'), (Y - Y_0'), (Z - Z_0')]$$
$$X^a - X_2{}^a = [(X - X_0''), (Y - Y_0''), (Z - Z_0'')]$$
$$X_2{}^a - X_1{}^a = (b_x, b_y, b_z)$$
$$C_m{}^a(x^m - x_1{}^m) = (u', v', w')$$
$$C_r{}^a(x_r - x_2{}^r) = (u'', v'', w'')$$

Equation 30 for the index $a=2$ is

$$\epsilon_{abc}(X_2^a - X_1^a)C_m{}^bC_r{}^c(x^m - x_1{}^m)(x^r - x_2{}^r) = 0 \tag{31}$$

which is Equation 20 of Schmid's paper.[8] It states that points in two pictures which correspond to the same ground point must be coplanar with each other and with the flight base.

The foregoing equations are all more or less familiar in computations based on frame photographs. Evidently they may also be used for computation from strip and/or panoramic photographs. To do so one substitutes the appropriate set of Equations 14 through 17 or 19 through 23. Thus constants appropriate to the particular type of photograph are introduced, and numerical values are needed for these constants. In practice approximate values are usually known for these constants and improved values may be obtained by statistical adjustment. Note that for strip and panoramic photographs the various parameters ("constants") may have different values for different regions of the same photograph, in other words, they are only approximately constant. For this reason separate statistical adjustment over each of a number of small regions is probably more precise than adjustment over a large region using only widely scattered points.

Equations 24 and 25 are useful for approximate calculations of ground coordinates when the elevations are approximately known. Equations 28 and 29 are true algebraic solutions for ground coordinates as functions of the coordinates in two overlapping photographs. In either case calculations relative to the two different camera stations must give the same values for absolute ground coordinates—if there are no errors. In practice the errors are never completely zero and the *best values* are obtained by averaging.

Equations 14–17 and 19–23 show that the lens coordinates X_1^a and x_1^m are not constant for strip or panoramic photographs. Hence strip and panoramic photographs do not have *fixed* principal points, but have loci of principal points formed by the operation of the IMC mechanism. Thus timing marks on the edges of strip and panoramic photographs are analogous to fiducial marks on the edges of frame photographs. Victor[6] explains the use of timing marks to find the locus of principal points on a strip photograph.

Application of Statistical Techniques

Hellmut Schmid[8] has given a comprehensive treatment of the statistical adjustment of data obtained from frame photographs. His treatment distinguishes between *absolute*, *partially absolute* (2 cases), and *relative* control points and utilizes different equations for the four different situations. A corresponding treatment of data from strip and/or panoramic photographs utilizes Equations 7 or 11 for absolute control points, 28 and 29 for partially absolute control points, and 31 for relative control points. In all cases the Equations 14 through 17 or 19 through 23 are substituted where called for.

Most, if not all, of the statistical techniques which have been published for frame photographs 8, 9, 10, 11 can be modified for application to strip and panoramic photographs. The principal changes required are: (1) extension to include the additional parameters and (2) substitution of the condition Equations 14–17 or 19–23. Linearization of the equations is the same in principle but differs in details for the three types of photography. For strip and panoramic photographs differentiation of the equations with respect to the various parameters differs from that for frame type mainly in the requirement for treating the lens coordinates X_1^a and x_1^m as functions of some of the other variables. This difference does not apply to the orientation angles, however.

Differentiation with respect to the primary, secondary, and tertiary angles is just as has been described in numerous papers, 10, 12, 13 and hence will not be discussed

TABLE I. BASIC DERIVATIVES FOR STRIP PHOTOGRAPHS

	$\partial(x^m - x_1^m)$	∂X^a
∂x^n	$\delta_{2'}^m \delta_n^{2'}$	$\delta_n^{1'} V^a/v$
∂x_{10}^n	$-\delta_{2'}^m \delta_n^{2'}$	$-\delta_n^{1'} V^a/v$
∂v	0	$-(V^a/v^2)(x^{1'} - x_{10}^{1'} - f_1 \tan \beta_1)$
∂v^b	0	$\delta_b^a(1/v)(x^{1'} - x_{10}^{1'} - f_1 \tan \beta_1)$
∂X_{10}^b	0	δ_b^a
∂f_1	$\delta_{1'}^m \tan \beta_1 + \delta_{3'}^m$	$-(V^a/v) \tan \beta_1$
$\partial \beta_1$	$\delta_{1'}^m f_1 \sec^2 \beta_1$	$-(V^a/v) f_1 \sec^2 \beta_1$

here. Differentiation with respect to the other parameters is in accordance with the usual rules for differentiation with respect to variables which enter through functionals. That is, the equations are differentiated with respect to the functionals, and the functionals are differentiated with respect to the variables. The desired result is a linear combination of such derivatives formed according to well known rules. Table I (for strip type) and Table II (for panoramics) show derivatives of the relevant functions with respect to the various parameters which occur in the equations for the respective types of photography. The results are stated for the Number 1 photograph. Corresponding results for the Number 2 photograph are obtained simply by appropriate changes of indices.

Examination shows that all of the above derivatives (except those with respect to measured coordinates) are different from each other and also different from the derivatives of the functionals C_a^m with respect to the rotation angles. Hence the equations for statistical adjustment *do* have solutions for the unknown parameters.

TABLE II. BASIC DERIVATIVES FOR PANORAMIC PHOTOGRAPHS

	$\partial(x^m - x_1^m)$	∂X_1^a
∂x^n	$\delta_{1'}^m \delta_n^{1'}$	0
∂x_{10}^n	$-\delta_{1'}^m \delta_n^{1'}$	0
∂v_M	$-\delta_{1'}^m (1/\omega) \sin [(y - y_{10})/f_1]$	0
∂V^b	0	$\delta_b^a (y - y_{10})/\omega f_1$
∂X_{10}^b	0	δ_b^a
$\partial \omega$	$\delta_{1'}^m (v_M/\omega^2) \sin [(y - y_{10})/f_1]$	$-(V^a/\omega^2)(y - y_{10})/f_1$
∂f_1	(see below)	$-(V^a/\omega)(y - y_{10})/f_1^2$
∂y	(see below)	$V^a/\omega f_1$
∂y_{10}	(see below)	$-V^a/\omega f_1$

$$\frac{\partial(x^m - x_1^m)}{\partial f_1} = \delta_{1'}^m \frac{v_M}{\omega} \frac{y - y_{10}}{f_1^2} \cos \frac{y - y_{10}}{f_1} + \delta_{2'}^m \left(\sin \frac{y - y_{10}}{f_1} - \frac{y - y_{10}}{f_1} \cos \frac{y - y_{10}}{f_1} \right) + \delta_{3'}^m \left(\cos \frac{y - y_{10}}{f_1} + \frac{y - y_{10}}{f_1} \sin \frac{y - y_{10}}{f_1} \right)$$

$$\frac{\partial(x^m - x_1^m)}{\partial y} = -\delta_{1'}^m \frac{v_M}{\omega f_1} \cos \frac{y - y_{10}}{f_1} + \delta_{2'}^m \cos \frac{y - y_{10}}{f_1} - \delta_{3'}^m \sin \frac{y - y_{10}}{f_1}$$

$$\frac{\partial(x^m - x_1^m)}{\partial y_{10}} = \delta_{1'}^m \frac{v_M}{\omega f_1} \cos \frac{y - y_{10}}{f_1} - \delta_{2'}^m \cos \frac{y - y_{10}}{f_1} + \delta_{3'}^m \sin \frac{y - y_{10}}{f_1}$$

Conclusion

In this paper derivations have been presented which imply the theoretical possibility of triangulating ground point coordinates from two overlapping photographs which may be any combination of frame, strip, or panoramic types. The total number of parameters whose values must be established for the various possible combinations is shown in Table III. For strip and panoramic photographs the orientation and flight parameters are not necessarily constant over the whole photograph. Hence particular statistical adjustments may be valid over only limited regions of the photographs. Determination of values of the parameters for various adjacent regions of a strip or panoramic photograph is somewhat analogous to extending control over a whole strip of frame photographs. This situation must be taken in account if strip and/or panoramic photographs are to be fitted into a strip or block adjustment.

In one type of strip photography two (continuous) photographs are taken simultaneously with one camera, which has *two* lens—slit combinations scanning at different angles. Statistical adjustment of these two photographs may be accomplished as was described above. In this case, however, there are additional constraints available, of a type which doesn't occur with frame photographs. These constraints follow from the fact that the orientation and flight parameters are identical for the two photographs at the instant of their simultaneous exposures. Exploitation of these constraints requires identification of the *different* ground points which are thus exposed simultaneously. So far as is known this type of constraint has not been discussed in the literature.

Table III. Number of Parameters in Various Pairs of Photographic Equations

Type of Number 2 Photograph	*Type of Number 1 Photograph*		
	Frame	*Strip*	*Panoramic*
Frame	18	23	23
Strip	23	28	28
Panoramic	23	28	28

References

1. Born, M. and Wolf, E., "Principles of Optics." Macmillan New York, 1964.
2. American Society of Photogrammetry, "Manual of Photogrammetry." 1952.
3. Elms, David, "Mapping with a Strip Camera," *Photogrammetric Engineering*, Vol. XXVIII, No. 4, Sept. 1962, pp. 638–653.
4. Wohl, M. and Sickle, S. M., "Continuous Strip Photography—an Approach to Traffic Studies," *Photogrammetric Engineering*, Vol. XXV, pp. 397–403 and 740–743.
5. Itek Laboratories, "Panoramic Progress,—Parts I and II," *Photogrammetric Engineering*, Vol. XXVII, No. 5, Nov., 1961, pp. 747–754, and Vol. XXVIII, No. 1, Jan., 1962, pp. 99–107.
6. Abraham, Victor, "Relative Geometric Strength of Frame, Strip, and Panoramic Cameras," *Photogrammetric Engineering*, Vol. XXVII, No. 5, Nov., 1961, pp. 755–766.
7. Rosenau, Milton D., "Parabolic Image-Motion," *Photogrammetric Engineering*, Vol. XXVII, No. 3, July, 1961, pp. 421–427.
8. Schmid, Hellmut, "An Analytical Treatment of the Problem of Triangulation by Stereophotogrammetry," *Photogrammetria*, Vol. XIII, Nos. 2 and 3, 1956–1957, pp. 67–77 and 91–116.
9. Schut, G. H., "An Analysis of Methods and Results in Analytical Aerial Triangulation," *Photogrammetria*, Vol. XIV, No. 1, 1957–1958, pp. 16–33.
10. El-Assal, A. A., "Analytical Aerotriangulation at the University of Illinois," *Photogrammetric Engineering*, Vol. XXIX, No. 1, Jan., 1963, pp. 199–206.
11. Case, James B., "The Utilization of Constraints in Analytical Photogrammetry," *Photogrammetric Engineering*, Vol. XXVII, No. 5, Nov., 1961, pp. 766–778.
12. Lucas, James R., "Differentiation of the Orientation Matrix by Matrix Multipliers," *Photogrammetric Engineering*, Vol. XXIX, No. 4, July, 1963, pp. 708–715.
13. Harris, Wm. D., Tewinkel, G. C., and Whitten, Charles A., "Analytic Aerotriangulation in the Coast and Geodetic Survey," *Photogrammetric Engineering*, Vol. XXVIII, No. 1, March, 1962, pp. 44–69.