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Block Adjustment by Polynomial Transformations

A complete computer documentation in Fortran IV applies also to strips.

Introduction

Computer programs for the polynomial transformation of strips and of blocks have been in use since 1960 at the National Research Council of Canada.

When in July 1965 the IBM 1620 computer at the NRC laboratories was replaced by an IBM System/360, it became evident that eventually the current programs, written in the symbolic SPS language for the IBM 1620[1,2,3] would have to be replaced by a Fortran program for the IBM S/360.

Before this was done, an investigation was carried out to determine whether at the same time the polynomial transformation of strips should be replaced by a linear transformation of models. This investigation showed[4] that with the low density of ground control that is the practice in topographic mapping, the polynomial transformation of strips gives at least equally accurate results as the linear transformation of models. It does this at a much smaller cost and with much simpler requirements in the data handling.

Consequently, it has been found worthwhile to re-program the polynomial adjustment in the Fortran language. This Fortran program serves for both strip adjustment and block adjustment.

The transformation formulas have been simplified somewhat. These simplifications have an insignificant effect upon the results.

Adjustment Procedure

The adjustment of a strip is performed in steps. These steps are performed one after the other without operator intervention.

First, the strip is subjected to a (geometric) similarity transformation. This consists in scaling, rotating, and translating the strip with the help of the specified ground-control points. If a correction for longitudinal curvature or torsion shall be applied, this transformation is followed by a rotation to axis-of-flight coordinates.

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Next, the height adjustment is performed. This adjustment contains any specified corrections for longitudinal curvature, torsion, and earth curvature. It also contains final corrections to the translation in height and to the tilts. The height corrections are parabolic functions of specified degrees in the planimetric coordinates.

Finally, the planimetric adjustment is performed. This adjustment consists in a conformal transformation of the specified degree in the planimetric coordinates.

The adjustment of a block is an iterative procedure in which each strip in turn is transformed. This series of transformations is repeated the required number of times. The control points for each transformation are the specified ground-control points and the tie points with any overlapping strips. For the latter, the latest transformed coordinates from the overlapping strips are used as ground-control values.

An alternative to this iterative procedure would be the iterative Gauss-Seidel solution of the complete set of normal equations for the simultaneous solution of the transformation formulas of all strips. This alternative could be employed when, after the first series of transformations, all strips have been transformed to the ground-control system. It would produce formulas for direct transformation from the initial position in the ground-control system to the adjusted position.

ABSTRACT: *This paper contains the description of a Fortran IV program for the adjustment of single strips and of blocks of strips by means of polynomial transformations. The strip adjustment comprises an initial (geometric) similarity transformation, followed first by a height adjustment, then by a planimetric adjustment. The height adjustment is a parabolic-type of transformation with corrections of specified degrees for curvature and torsion, and the planimetric adjustment is a conformal transformation of specified degree. The block adjustment consists in an adjustment of the individual strips in an iterative procedure.*

After each iteration, the present procedure produces virtually the same result as the Gauss-Seidel solution would give if in that procedure after each iteration the transformations were actually performed. Therefore, since the Gauss-Seidel solution always converges, the present procedure converges also and with the same speed.

As a rule, ten iterations of a block adjustment are ample to obtain the desired degree of convergence.

Although the computations in this iterative procedure require considerably more time than the Gauss-Seidel solution of the complete set of normal equations, it has had the important advantage that with little time and effort the program could be increased in scope from a strip adjustment to a block adjustment.

Block adjustments as well as strip adjustments are performed in one pass on the computer, and the data decks for any number of strips and blocks may be stacked for continuous processing.

The operation of the program has been made as simple as possible by placing the least possible restrictions on numeration and on sequence of points in the card deck, and by reducing the necessary card handling to a minimum.

The time required for a strip adjustment depends upon the degrees of the transformation formulas and the number of points. For second-degree transformations, using the IBM S/360 model 50 and the Fortran compiler of June 1965, it is about 15 seconds in the case of the adjustment of a single strip, and about 5 to 8 seconds in the case of an iteration in a block adjustment.

Transformation Formulas

1. INITIAL TRANSLATIONS

In the computations, use is made of two rectangular three-dimensional coordinate systems. One is the strip coordinate system with respect to which the triangulation of a strip has been performed. The other is the "ground-control system." The latter is constructed from the planimetric map coordinates, as easting and northing, and the terrain heights.

In order to perform the computations with reasonably small numbers, the origins of the ground-control system and of the strip-coordinate system are shifted to a point inside the strip.

The origin of the ground-control system is initially shifted to the first ground-control point in the data deck of the strip. Depending on the availability of easting, northing, and height, this point may or may not be the same point for planimetry as for height. During the computation of the transformation formulas, further shifts will be added to this initial shift.

If a correction for height deformation is to be applied, the origin of the strip-coordinate system is shifted to the point midway between two points which define the strip axis. If such a correction is not to be applied, the origin is shifted to the point midway between the first two ground-control points in the data deck. In the latter case, any other point in the strip could serve the purpose equally well. The present choice required the least programming. No further shifts are applied to the origin of the strip-coordinate system.

The thus reduced ground-control coordinates and strip coordinates will be denoted by E, N, H, and X, Y, Z, respectively.

2. THE INITIAL SIMILARITY TRANSFORMATION

Disregarding here translations, which will be dealt with farther on, the initial similarity transformation of the strip consists only in scaling the strip and rotating it about the origin. This transformation can be written in matrix notation as:

$$\begin{bmatrix} X_4 \\ Y_4 \\ Z_4 \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{1}$$

where $\boldsymbol{A}$ is a matrix which is the product of a scale factor and an orthogonal matrix.

The elements of the matrix can be defined in many different ways as functions of four independent parameters. These functions are always non-linear with respect to the chosen parameters and therefore the matrix is computed in a step-by-step procedure. Three or four transformations are computed in succession, and the matrix is found as the product of the matrices of these transformations.

i. The first step is a similarity transformation of the X- and Y-coordinates. Because in general the strip will not yet be at terrain scale, a levelling of the strip is not performed at this stage.

Using complex numbers, this transformation can be written

$$(X_1 + iY_1) = (X + iY) + (e_3 + ie_4)(X + iY) \tag{2}$$

Separating the real and the imaginary parts, and multiplying the Z-coordinates by the scale factor λ of the transformation, this gives

$$\begin{aligned} X_1 &= (1 + e_3)X - e_4 Y \\ Y_1 &= e_4 X + (1 + e_3) Y \\ Z_1 &= \lambda Z, \end{aligned} \tag{2'}$$

where $\lambda = ((1+e_3)^2+e_4^2)^{1/2}$.

In matrix notation, this becomes

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = R_1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \quad \text{where} \quad R_1 = \begin{bmatrix} 1+e_3 & -e_4 & 0 \\ e_4 & 1+e_3 & 0 \\ 0 & 0 & \lambda \end{bmatrix} \tag{2''}$$

ii. The second step is a levelling of the strip. It is performed by means of the transformation

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = R_2 \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix},$$

where

$$R_2 = \begin{bmatrix} 1-\frac{1}{4}b_1^2+\frac{1}{4}c_1^2 & -\frac{1}{2}b_1c_1 & -b_1 \\ -\frac{1}{2}b_1c_1 & 1+\frac{1}{4}b_1^2-\frac{1}{4}c_1^2 & -c_1 \\ b_1 & c_1 & 1-\frac{1}{4}b_1^2-\frac{1}{4}c_1^2 \end{bmatrix} \tag{3}$$

The matrix R_2 would be orthogonal if a factor $1+\frac{1}{4}b_1^2+\frac{1}{4}c_1^2$ had not been omitted.[5] Therefore, it applies the required rotation to the strip and also a small change of scale.

The parameters b_1 and c_1 correct for longitudinal tilt and transversal tilt, respectively. They must be computed from linear formulas obtained by omitting the squares and products of b_1 and c_1 in Equation 3. Therefore, if the tilts are large, this levelling does not give the best possible fit at the control points. That is corrected in the final height adjustment, together with the introduction of any corrections for height deformation.

iii. The third transformation is again a similarity transformation of the planimetric coordinates:

$$\begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix} = R_3 \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}, \tag{4}$$

where R_3 is constructed from two parameters e_3 and e_4 in the same way as R_1.

This transformation serves to give the transformed strip the correct scale, in preparation for the final height adjustment. It is needed when, in cases of considerable tilt of the strip, the first planimetric transformation has not been able to do this. At the same time, the small unnecessary change of scale introduced in the second step is corrected.

iv. If a correction for longitudinal curvature or torsion is to be applied, the origin of the X, Y, Z coordinate system has earlier been placed at the point midway between two points which define the strip axis. After the above transformations, the origin is still at this point. If such a correction is to be applied, the strip is now rotated about the Z-axis in order to make those two points lie in the X, Z plane. In this way, the axis-of-flight coordinates are obtained which such corrections require.

Thus

$$\begin{bmatrix} X_4 \\ Y_4 \\ Z_4 \end{bmatrix} = R_4 \begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix}, \quad \text{where} \quad R_4 = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

and α is the angle between the strip axis and the X-axis after the transformation 4. *Cos* α is equal to the X_3-coordinate of one of the two points which define the strip axis divided by the distance from the point to the origin, and *sin* α is equal to the Y_3-coordinate of the point divided by the same distance.

It follows from the above that the matrix $\boldsymbol{A}$ in Equation 1 is the product of the three or four matrices $\boldsymbol{R}$:

$$\boldsymbol{A} = \boldsymbol{R}_3\boldsymbol{R}_2\boldsymbol{R}_1,$$

or

$$\boldsymbol{A} = \boldsymbol{R}_4\boldsymbol{R}_3\boldsymbol{R}_2\boldsymbol{R}_1. \tag{6}$$

During the computation of the above transformation formulas, the matrices $\boldsymbol{R}$ become known one after the other and therefore the approximations $\boldsymbol{R}_2\boldsymbol{R}_1$ and $\boldsymbol{R}_3\boldsymbol{R}_2\boldsymbol{R}_1$ of $\boldsymbol{A}$ can be computed successively. These are used to transform the original strip coordinates X, Y, and Z of the control points rather than using Equations 3 and 4. This procedure makes it unnecessary to retain the individual matrices $\boldsymbol{R}$.

3. THE HEIGHT ADJUSTMENT

Next, the final height adjustment is performed by means of a polynomial transformation of the coordinates X_4, Y_4, and Z_4. Disregarding again a Z-translation, the transformation formulas are:

$$\begin{aligned}
X_5 &= X_4 - Z_4b_1 - 2X_4Z_4b_2 - 3X_4^2Z_4b_3 - \cdots - 2X_4Z_4d_1 \\
Y_5 &= Y_4 - Z_4c_1 - X_4Z_4c_2 - X_4^2Z_4c_3 - \cdots - 2Y_4Z_4d_1 \\
Z_5 &= Z_4 + X_4b_1 + X_4^2b_2 + X_4^3b_3 + \cdots \\
&\quad + Y_4c_1 + X_4Y_4c_2 + X_4^2Y_4c_3 + \cdots + (X_4^2 + Y_4^2)d_1
\end{aligned} \tag{7}$$

The terms with b_1 and c_1 serve to give final corrections to the longitudinal and transversal tilt. Comparison with Equation 3 shows that these corrections do not constitute an exact similarity transformation. However, because at this stage the corrections are small, the departure from a similarity transformation is negligible.

The remaining terms with coefficients b and c serve to correct the strip for longitudinal curvature and torsion, respectively.

Those with coefficients b apply a parabolic correction to the X_4- and Z_4-coordinates. In the former program for the IBM 1620, a conformal transformation was used. Since X_4 and Z_4 are axis-of-flight coordinates with the origin in the centre of the strip, the difference between these two types of corrections is negligible.

Those with coefficients c apply a correction which in each plane $X = constant$ differs very little from a rotation about the X-axis. This can be seen by writing the contributions of these terms to Y_5 and to Z_5 as $-Z_4c_1'$ and Y_4c_1', respectively, where,

$$c_1' = c_1 + X_4c_2 + X_4^2c_3 + \cdots.$$

The terms with the coefficient d_1 apply the correction for earth curvature. The coefficient is computed as the reciprocal of the diameter of the earth. The correction is of the parabolic type and it is applied in the strip direction as well as across the strip.

If the correction for longitudinal curvature is to be used simultaneously with the earth curvature correction, only the curvature correction across the strip is based upon the earth curvature. The curvature correction in the strip direction is then derived from the ground control.

4. THE PLANIMETRIC ADJUSTMENT

If the transformation to axis-of-flight coordinates by means of Equation 5 has been included in the initial similarity transformation, the coordinates X_5 and Y_5 are now reconverted to the ground-control system by means of the inverse transformation

$$X_6 = \cos\alpha X_5 - \sin\alpha Y_5$$
$$Y_6 = \sin\alpha X_5 + \cos\alpha Y_5. \quad (8)$$

If that transformation has not been applied, X_6 and Y_6 are equal to X_5 and to Y_5, respectively.

Subsequently, the planimetric adjustment is performed by means of a conformal transformation of the coordinates X_6 and Y_6. Disregarding a translation e_1+ie_2, the transformation formula is:

$$(X_7 + iY_7) = (X_6 + iY_6) + (e_3 + ie_4)(X_6 + iY_6) + (e_5 + ie_6)(X_6 + iY_6)^2 + (e_7 + ie_8)(X_6 + iY_6)^3 + \cdots \quad (9)$$

A parabolic type of transformation is here not suitable because X_6 and Y_6 are not axis-of-flight coordinates.

5. THE TRANSLATIONS

X- and Y-translations e_1 and e_2 are not used in the coordinate Transformations 2″, 4, and 9; and Z-translations a_1 are not used in the coordinate Transformations 3 and 7.

However, a proper adjustment is not possible when the translations are omitted altogether, and therefore they are inserted in the transformation formulas when the coefficients in these formulas are computed. In this way, the translations are computed at the same time as the coefficients. Then, instead of employing them in the transformation formulas, they are added to the shifts of the ground-control system directly after they have been computed.

Therefore, the origin of the ground-control system undergoes the following translations. In planimetry, an initial shift to the first ground-control point in the data deck of the strip is applied and is followed by the three sets of translations e_1 and e_2 which have been omitted from the Equations 2″, 4, and 9. In height, an analogous initial shift is applied and is followed by the translations a_1 which have been omitted from the Equations 3 and 7.

As the final step in the adjustment, all these translations are now added to the coordinates X_7, Y_7, and Z_5. Thus, the transformed coordinates with respect to the ground-control system are:

$$X_{tr} = X_7 + c_E + e_1^{(2'')} + e_1^{(4)} + e_1^{(9)}$$
$$Y_{tr} = Y_7 + c_N + e_2^{(2'')} + e_2^{(4)} + e_2^{(9)}$$
$$Z_{tr} = Z_5 + c_H + a_1^{(3)} + a_1^{(7)} \quad (10)$$

Here, c_E, c_N, and c_H denote the initial shifts of the origin of the ground-control system, and the superscripts attached to the other translations denote the equations to which they belong.

6. CORRECTION EQUATIONS

For the computation of the parameters in the above transformations, each ground-control point provides one or more condition equations which state that the transformed strip coordinates of the point should be equal to the given ground-control coordinates. If the program is used for block adjustment, in addition each tie point provides three condition equations which state that its transformed coordinates should be equal to those of the corresponding point in an earlier transformed overlapping strip.

The parameters are computed by the method of least squares. Consequently, the condition equations are used for the formation of correction equations in which the parameters occur as the unknowns. The correction equations are used for the formation of normal equations and the normal equations are solved for the unknowns.

The correction equation for planimetry, derived from Equations 2 and 9, is

$$(e_1 + ie_2) + (X + iY)(e_3 + ie_4) + (X + iY)^2(e_5 + ie_6) + (X + iY)^3(e_7 + ie_8) + \cdots = (E + iN) - (X + iY) \quad (11)$$

Here, E and N represent either the ground-control coordinates of a planimetric control point or the transformed coordinates of a tie point in an overlapping strip, reduced for the shift of the origin. X and Y represent either the strip coordinates X and Y, the transformed coordinates X_2 and Y_2, or the transformed coordinates X_6 and Y_6, depending upon the transformation whose parameters are being computed.

In the case of the first two planimetric transformations, only the linear part of the correction equation is used; for the last transformation, any specified higher-degree terms are used also.

The correction equation for the heights, derived from Equations 3 and 7, is:

$$a_1 + Xb_1 + X^2b_2 + X^3b_3 + \cdots + Yc_1 + XYc_2 + X^2Yc_3 + \cdots = H - Z - (X^2 + Y^2)d_1 \quad (12)$$

Here, H represents either the height of a ground-control point or the transformed height of a tie point in an overlapping strip, each reduced for the shift of the origin. X and Y represent either the transformed coordinates X_1 and Y_1 or the transformed coordinates X_4 and Y_4. Because the coefficient d_1 is computed in advance, as explained before, the numerical value of the term with d_1 can be computed. Accordingly, this term is placed in the second part of the equation.

Here also, for the first of the two transformations only the linear part of the correction equation is used while for the last transformation also any specified higher-degree terms are used.

7. NORMAL EQUATIONS

Let a correction equation be represented by the matrix equation

$$\boldsymbol{a}_r\boldsymbol{x} = \boldsymbol{b}, \quad (13)$$

where $\boldsymbol{x}$ is the column vector whose components are the parameters in the correction equation, $\boldsymbol{a}_r$ is the row vector whose components are the coefficients of the parameters, and $\boldsymbol{b}$ is the algebraic sum of the terms in the second part of the equation. In the case of a planimetric adjustment the components of the two vectors and the term $\boldsymbol{b}$ are complex numbers; in the case of a height adjustment they are real numbers.

Let $\boldsymbol{a}_c$ be the column vector which in the case of a planimetric adjustment is the conjugate of the transpose of $\boldsymbol{a}_r$ and in the case of a height adjustment is the transpose of $\boldsymbol{a}_r$.

Then, for each correction equation a matrix $w\boldsymbol{a}_c\boldsymbol{a}_r$ and a vector $w\boldsymbol{b}\boldsymbol{a}_c$ must be computed. Here, w is the weight applied to the equation.

The matrix of coefficients of the normal equations and their vector of second parts are computed by summation of the matrices $w\boldsymbol{a}_c\boldsymbol{a}_r$ and the vectors $w\boldsymbol{b}\boldsymbol{a}_c$, respectively. Thus, the normal equations are

$$[w\boldsymbol{a}_c\boldsymbol{a}_r]\boldsymbol{x} = [w\boldsymbol{b}\boldsymbol{a}_c]. \quad (14)$$

The weight of a correction equation is thus defined as the number of times that its contribution is added to the normal equations.

In the planimetric adjustments, the matrix of the normal equations is hermitian; in the height adjustments, it is symmetric. The equations are solved by Gaussian elimination and back substitution. In the elimination procedure, successive elements on the main diagonal of the matrix are used as pivotal elements.

THE PROGRAM

1. INPUT DATA

The program employs a card reader as its input device.

The cards with all the necessary information for the adjustment of either one single strip, more than one single strip, or one block of strips are arranged in a deck, called the data deck. As explained farther on, any number of such data decks may be stacked for continuous processing.

There are three types of cards in each data deck: cards with ground-control coordinates, cards with strip coordinates, and control cards.

All cards with ground-control coordinates are placed at the beginning of the deck. For each ground-control point, a card is punched containing:

Numeric point identification,
Easting,
Northing,
Height.

If a coordinate is not known, its field is left blank.

The remainder of the cards are arranged in groups according to the strips. If there is more than one strip, the adjustment is performed in the sequence in which the strips occur in the data deck.

The first card in each group is a control card which specifies which terms in the transformation formulas are to be used. For strip adjustment as well as block adjustment, this control card must contain the following information:

A minus sign,
The degree of the scale and azimuth correction by means of Equation 9,
The degree of the longitudinal tilt correction by means of the terms with b in Equation 7,
The degree of the transversal tilt correction by means of the terms with c in Equation 7,
If the correction for earth curvature is to be applied, the radius of the earth; if not, zeros or blanks. The unit of length of the radius must be the same as the one used for ground-control coordinates.

For block adjustment, the control card must contain the following additional information:

The number of iterations required for the block adjustment. This number need be recorded only on the control card of the first strip of a block and then only if more than one iteration is required.
The weights of planimetric control points and height-control points, respectively, relative to a weight 1 for tie points. The decimal point, which is not punched, is between the second and the third digit from the right. These weights should be punched only if the weight applied to ground-control-point equations is to be different from the weight applied to tie-point equations.

If the control card specifies that a correction for height deformation is to be applied (that is, the tilt corrections are not of the first degree and/or the correction for earth curvature is to be applied), the control card must be followed by two cards with the strip coordinates of two points which serve to define the axis of the strip: one at the beginning and one at the end of the strip. Suitable points for this purpose are the first and the last principal point or points near these. Fictitious points may be used. However, care must be taken that their heights are fairly representative of the terrain heights in the strip-coordinate system.

If the tilt corrections will be of the first degree and in addition the correction for earth curvature will not be applied, these two cards must be omitted.

Next follow the cards with the strip coordinates of all points which are to be used for the computation of the transformation formulas. In the case of a block adjustment, these points include the tie points, whether or not during the first iteration of the strip required transformed coordinates from overlapping strips have already been computed.

If the strip contains no points which must be transformed only, this completes the deck of cards for the strip.

If the strip contains one or more of such points, next follows first a control card with the negative number -8 punched in field 1. Then follow the cards of the points which must be transformed only.

All the above cards with strip coordinates are punched as follows:

Numeric strip- or model-identification (positive and non-zero).
Numeric point identification.
Strip X-coordinate.
Strip Y-coordinate.
Strip Z-coordinate.

Finally, a control card in which field 1 is either blank or punched with zeros is placed at the end of the data deck.

It is possible to stack any number of such data decks, each with its own ground control, one behind the other. In this case, in each data deck except the last one the control card which is placed at the end must be replaced by a control card which has the negative number -9 punched in field 1.

2. RESTRICTIONS ON THE INPUT

The cards with ground-control coordinates may contain either easting and northing, or height, or all three of these coordinates. A northing or height which is equal to zero will be interpreted as easting and northing, or height, being unknown.

Easting, northing, and height must be expressed in the same unit of length. The centimeter is the most suitable unit. If this unit is used, the decimal point which is printed in the output indicates meters.

The three strip coordinates must be expressed in the same unit of length.

The ground-control system and the strip-coordinate system must both be either right-handed or left-handed. Therefore, with easting, northing, and height, in this sequence, representing a right-handed coordinate system, the strip-coordinate system X, Y, Z must also be right-handed. If the strip-coordinate system is left-handed, either X and Y should be interchanged on the cards or the sign of X or of Y should be changed.

All planimetric control points of a strip are used with the same weight and all its height-control points are used with the same weight. Variations in these weights are possible only by placing more than one card with the strip coordinates of the point in the data deck.

Each point that is to be used as a tie point in a block adjustment should occur in at least two strips. If it occurs in one strip only, in each iteration of the block adjustment its transformed coordinates from the preceding transformation of this strip will be used as ground-control coordinates.

The contents of the data deck is stored in arrays in the core memory of the IBM S/360. Using the present Model 50 with 200K bytes of core storage available for data, the maximum number of points that can be accommodated in the ground-control arrays for planimetry and in those for height will be restricted to 600. This number includes the points in the ground-control deck and the tie points whose transformed coordinates will eventually be added to the arrays. The maximum number of control cards and coordinate cards which may follow the cards with ground-control coordinates is then about 5,000. However, see for temporary restrictions section 5.iii.

The maximum number of points that can be used for the computation of the transformation formulas for a strip is 100. This is independent of whether easting and northing, or height, or all three of these coordinates are known.

In the data deck for a block adjustment, the strips should be arranged in such a sequence that already during the first iteration each strip has a sufficient number of control points. For instance, if only tie points with one preceding strip are available, it may not be possible to determine transversal tilt and torsion to within a few degrees. Such tie points must then be supported either by ground-control points in suitable locations or by tie points with another preceding strip. If this is not done, during the first iteration the strip may receive an excessive transversal tilt and torsion. In that case, the longitudinal tilt correction will be applied to the tilted strip. As a result, the block adjustment may require more iterations and may not give the best possible result.

The maximum size of the code numbers which specify the degrees of the terms in the polynomials for scale and azimuth correction, for longitudinal tilt correction, and for transversal tilt correction is 9. These inordinately high degrees have been allowed because their inclusion does not require any coding. All that is necessary is provision for sufficiently large stores for the coefficients in the transformation formulas, the correction equations, and the normal equations.

In the case of strips that have been triangulated analytically or on first-order plotters, the polynomials can usually be restricted to second-degree terms. With three bands of ground-control points across a strip or across a block of strips and up to 30 photographs per strip, second-degree polynomials have shown to give on the average equally accurate results as higher-degree polynomials.[4] In the case of a denser net of ground-control points, third-degree polynomials may give better results. Polynomials of higher than the third degree should be used only under exceptional circumstances.

3. OUTPUT

The program uses the on-line printer as output device.

For each strip, the transformation code punched in field 1 of its first control card is printed at the top of a page.

Next, identification, coordinates, and residuals are printed, one line for each point. For ground-control points, whether or not they have been used for the computation of the transformation formulas, the printed coordinates are the coordinates in the ground-control deck. For all other points, they are the adjusted coordinates.

The output for control points and the output for points for-transformation-only are separated by a double space.

In the case of a block adjustment, this printing occurs only during the last iteration. During the preceding iterations, the stored contents of the first control card of each strip is printed. In addition, field 2 shows the number of the iteration.

For tie points, the printed coordinates are the mean of those for the two strips in which they occur. Since each tie point should occur in (at least) two strips, its meaned coordinates will be printed twice. Since the adjustment is an iterative procedure, if complete convergence has not been reached these two sets of coordinates may differ a a little. The last set should then be accepted. Appreciable differences indicate that more iterations should be performed.

In addition to the meaned coordinates of the tie points, the deviations from the mean (half-discrepancies) are printed out.

The output includes:

Numeric strip- or model-identification.
Numeric point identification.

Easting.
Northing.
Height.

Further, for ground-control points,

Easting residual (computed minus given coordinate).
Northing residual.
Height residual.

and for tie points,

Half-discrepancy in easting (coordinate in present strip minus mean).
Half-discrepancy in northing.
Half-discrepancy in height.

4. ERROR DETECTION

The following error messages have been built into the program:

i. Too few control points. In this case, fewer control points are available than are necessary to compute the coefficients in the transformation formulas. For planimetry, the minimum number is one higher than the degree of the planimetric adjustment; for the heights, it is one higher than the sum of the degrees of the longitudinal and the transversal height adjustments.

ii. Normal equations are insolvable. During the solution of the normal equations, a pivotal element is encountered that is equal to zero. This may occur if the ground-control points, although sufficient in number, are not in suitable locations.

iii. Memory overflow. Either the number of ground-control points and tie points or the number of control cards and strip-coordinate cards exceeds the maximum that can be accommodated in the arrays in core storage.

When one of the first two messages occurs, the computation for the strip which is currently being processed is discontinued. The computations are resumed when the first control card for the next strip is encountered. If such a message occurs during the first iteration of a block adjustment while during subsequent iterations tie points will provide additional control, it can be disregarded. The result of the adjustment will then be correct and complete, except that more iterations may be required than has been anticipated.

One of these two messages may be printed also if the cards of the two points on the axis of the strip have been omitted erroneously.

A memory overflow terminates the computations. Any output of the block that is being processed should not be considered as final.

Other possible errors and their effect on the computation are:

- The first control card for the first strip transformation is either missing or is of the wrong type. In this case, the program reads cards without processing them until a proper first control card is found.
- Another control card is either missing or is of the wrong type. The program continues processing as instructed by the latest valid control card.
- The cards of the two points on the axis of the strip have been omitted erroneously. In this case, the first two control points are used to define the axis of the strip and they are not used as control points.
- For the computation of the transformation formulas, more than the maximum number of 100 points has been specified. In this case, the first 100 points will be used, and the remainder will be transformed only.
- If the ground-control points are in unsuitable locations and consequently are not sufficient to define the transformation formulas, the program proceeds with the computations. A pivotal element that is equal to zero is then usually not encountered. This is the result of rounding off. In this case, the program will give a good fit at the used ground-control points, but it can produce large errors in uncontrolled areas of the strip.

5. ADDITIONAL REMARKS ON THE PROGRAM

i. The program can cope successfully with great initial tilts of a strip. For this purpose, the Transformations 3 and 4 are included in the program.

However, if in the case of nearly vertical photographs strip coordinates have been obtained by means of triangulation in a first-order plotter or by means of analytical triangulation, the initial tilt of a strip is usually small. In that case, the result of the adjustment does not change appreciably if the Transformations 3 and 4 are omitted.

The omission of these transformations reduces the computer time needed for a block adjustment by less than one per-cent. If it is desired, it can be achieved by replacing the constant 2 by the constant 1 in the relevant Fortran statement.

ii. The Canadian topographic maps show eastings and northings in meters and terrain heights in feet. This hybrid system cannot be used in a three-dimensional transformation. However, it can be used as input and it can be obtained as output by inserting four Fortran statements in the program.

iii. The use of the program on the IBM S/360 requires, besides the program deck and the data deck, some special control cards for the Fortran compiler and the monitor. Information on these cards must be obtained at the installation where the IBM S/360 is operated.

The magnetic tape units are not used because the necessary rewinding would greatly increase the time required for the adjustment. Instead, the whole contents of the data deck is stored in the core memory.

The program has been written for the S/360 Basic Programming System Fortran compiler of June 1965. Both this compiler and the *E* level Fortran compiler of late 1966 restrict the size of the data storage in core memory to 48K bytes.

This has necessitated restricting the number of ground-control points to 300 and the number of control cards and coordinate cards which may follow the cards with ground-control coordinates to 1075.

The *H*-level compiler of 1967 will allow unrestricted use of the core storage of the Model 50 that is available for data. This will make it possible to accommodate the program (about 10,500 bytes), 600 ground-control points and possibly 5,000 measured points. This requires changing the relevant constants in several Fortran statements.

iv. The use of the program on an IBM 1620 would require conversion of the Fortran IV program to a Fortran II program.

The Fortran IV version used in this program is the Basic Programming Support Fortran IV, described in IBM, form C28-6504-2. The main differences between this simple version of Fortran IV and the Fortran II for the IBM 1620 concern the read and write statements and the double-precision statements.

The conversion may be of interest because at NRC many requests have been received for copies of the earlier IBM 1620 programs for strip and block adjustment. Often these programs could not be used because the floating-point hardware that those programs require was not available.

For floating-point arithmetic on the IBM 1620, the use of 10-digit mantissas is sufficient. The double-precision statements for the IBM S/360 provide the equivalent of about 16 decimal digits.

For an IBM 1620 with core storage only, the program will have to be reduced to a strip adjustment. Block adjustment will require storing the data on a disk or on magnetic tape.

REFERENCES

1. G. H. Schut, "The Use of Polynomials in the Three-dimensional Adjustment of Triangulated Strips." *The Canadian Surveyor*, Vol. XVI, No. 3, May 1962.
2. ——— "Development of Programs for Strip and Block Adjustment at the National Research Council of Canada." PHOTOGRAMMETRIC ENGINEERING, Vol. XXX, No. 2, March 1964.

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and forming the coefficients of the observation equations.

- Although I believe this C&GS system to be mathematically rigorous, I am aware that other contemporary systems may possess economical advantages through the application of slightly less rigor to yield results which are nevertheless acceptable.

Our block adjustment program is a "skeleton" system to which we plan to add several features, such as the inclusion of auxilliary control data. We do not yet derive the inverse matrix; we plan to do so in the near future in order to study errors and weights more effectively. We have not included a "summation" column in the equation solution. Consequently we are not sure of the effects of computational roundoff. However, recent tests with 180 photographs indicated that the final corrections were correct to at least three significant digits, which is ample to ensure the convergence of the solution. I have a list of more than a dozen items that we plan to incorporate eventually.

From time to time during the development we experienced set-backs that are amusing to us as we look back. Our first program in 1960 failed to converge to a solution. We drove hurriedly to Aberdeen to ask Dr. Schmid the reason: I had omitted some essential terms which I thought were insignificant—they seemed to me to be too difficult to include—"Doc" showed us an easy way to include them. Last fall the first 180-photo block came back from the computer with the message that after working all night without a solution, the computer gave up; we spent a month reprogramming the equation solution in order to make the disk look-up more efficient. In spite of our preliminary programs to eliminate human blunders, they continued to "goof up" every solution. Finally we programmed still another preliminary program to detect the blunders that escaped the previous programs.

As Morry has mentioned, I believe in publishing our program documentations for anyone to use, in whole or in part, if he wishes. After all, I am supported by your taxes, and it seems that what I develop is public property. Also, I have received several constructive criticisms which has helped me in subsequent work, as well as in helping others use it. I also like to publish our efforts because it gives me added personal pride. Facetiously, nevertheless true, I believe that the Awards Committee might not have noticed me if I had not deluged the literature as I have for the past twelve years!

Mr. Morris M. Thompson of the U. S. Geological Survey receives a replica plaque from Mrs. Clarice L. Norton for his retention as the 1966 winner of The Photogrammetric Award.

Incidentally, some of you may have noticed that Morry and I have a good thing going. Several years ago I talked him into voting for me for Society president. I then appointed him to direct the Third Edition of the MANUAL OF PHOTOGRAMMETRY. For this he received this Photogrammetric Award last year. Now as chairman of the committee he conveniently reciprocates! I'm not sure where this is going to end, but as of now I'm ahead: he really had to *work* for this!

Again, I wish to thank all of you for this coveted Award.

—*G. C. Tewinke*

(*Continued from page 1053*)

3. ——— "Operating Instructions for Programs for Strip and Block Adjustment by Polynomial Transformation." Publication AP-PR 28 (NRC 7906) of the Division of Applied Physics, March 1964.
4. ——— "Polynomial Transformation of Strips Versus Linear Transformation of Models: A Theory and Experiments." Paper presented at the Symposium on Spatial Aerotriangulation of Commission III of the International Society for Photogrammetry in Urbana, Illinois, February–March 1966 and to be published in *Photogrammetria*.
5. ——— "Construction of Orthogonal Matrices and their Application in Analytical Photogrammetry." *Photogrammetria*, Vol. XV, No. 4, 1958–59.