

SANJIB K. GHOSH

Ohio State University

Columbus, Ohio 43210

EVERETT H. RAMEY

U. S. Coast & Geodetic Survey

Rockville, Maryland 20852

# Super Wide-Angle and the AP/C

**ABSTRACT:** *The Nistri Analytical Plotter AP/C was tested with regard to horizontal and vertical errors obtained using super wide-angle photographs in strip triangulation. Although the AP/C tests were limited in quantity, it was evident that very high quality measurements can be obtained and that the system provides an effective means to control systematic errors. The results were superior to what can normally be expected of an analogue system; e.g., the vertical error was 1/17,000 H. As a control reference for comparison, the tests were also conducted with a Wild Autograph A7 which was not equipped for super wide-angle photography. The photography was handled satisfactorily with the A7 by applying an affine deformation.*

## INTRODUCTION

THE SUPER-WIDE-ANGLE (SWA) single-lens aerial camera is a recent development in photogrammetry. With its development has come a new family of instruments and procedures to manipulate the photography with angular coverage of 120 degrees. In some instances existing instruments and procedures have been modified to accommodate use of super-wide-angle photography (Figure 1).

The SWA system was studied by the photogrammetry research group at The Ohio State University particularly with its use in high-way mapping at medium and large scales. The objectives of this study were to evaluate SWA photography with special reference to the equipment presently available with the State of Ohio Department of Highways (particularly the Nistri-Bendix Analytical Plotter AP/C) with a view to improving the present mapping system.

The study comprised three broad aspects:

1. A general study of the SWA system and a theoretical analysis of the obtainable observational accuracy as compared to the standard wide-angle (WA) system of photography.

2. A study and analysis of the AP/C system of handling photogrammetric problems. This study was required in view of the very recent developments of the AP/C which had not been, thus far, thoroughly evaluated by any agency. This led to establishing a procedure for handling SWA photography with the AP/C.

3. A test of the procedure by using SWA photography taken on a controlled test area with an RC9 camera (manufactured by Wild Heerbrugg, Ltd.) which is the only SWA camera available commercially in the Western World. A comparative study was made between results obtained from aerotriangulations with the AP/C and with the Wild A7 stereo-instrument.

## COMPARATIVE STUDY BETWEEN SWA AND WA SYSTEMS

In a series of tests conducted by USAE/GIMRADA<sup>4</sup> empirically obtained results of two systems were compared. In this study, comprising (a) grid model flatness test, (b) terrain model flatness test, and (c) stereo-triangulation test, it was established that the SWA system is better than the standard WA system. This study, however, did not include a thorough theoretical analytical comparison of the accuracies of relative orientation in each system. This was deemed necessary to complete the evaluation because the accuracy of the compiled map in a photogrammetric system is directly related to the accuracy of orientation (particularly, relative orientation) of individual stereo models.

This study was made in two modes (1) with six model points for orientation; and (2) with nine model points for orientation. Comparative studies were made by forming variance-covariance matrices in each case. These studies indicated that SWA photog-

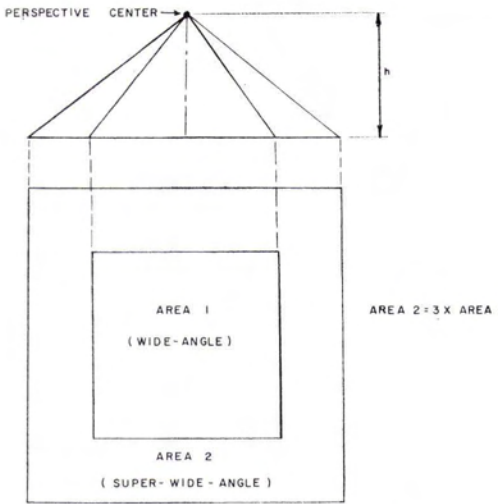


FIG. 1. Comparison of Areas of RC9 and Wide-Angle Photographs

raphy is definitely capable of yielding better accuracy and better efficiency than the standard WA photography.

THE ANALYTICAL SYSTEM

An analytical system offers distinct advantages over analogue systems in controlling error sources. This is accomplished by improving the photogrammetric measurements by use of simple, precise measuring devices which have been calibrated. Also corrections are applied better and easier for known systematic errors such as lens distortion, film distortion, earth curvature and atmospheric refraction.

In the present studies use was made of the Nistri-Bendix Analytical Plotter, Commercial (AP/C) with the State of Ohio, Department of Highways. There are three basic component parts of the AP/C (See Figure 2):

- The computer with its associated electrical equipment;
- The optical-mechanical viewing unit, essentially a two-plate comparator;
- The plotting table.

With the computer unit is a typewriter with a punched tape reader and punch-out routines. The computer is small (approximately 2,000 word memory), fully transistorized, and very fast.

The functions of the system as depicted in Figure 2 are described below:

The computer receives input data as model coordinates from the hand wheels ( $X_m, Y_m$ ) and the footwheel ( $E_m$ ), from the operator's control panel (data commands), and from the

typewriter and tape unit (generally, programs and subprograms are entered). In turn, the computer causes print-out of data at the typewriter and tape unit. It sends commands to servos to keep the photographs properly positioned and oriented for viewing by the operator. It might also display various data at the operator's control panel. Finally, the computer commands servos for plotting at the coordinatograph.

The photogrammetric solutions on AP/C can perhaps be shown by describing a typical operation procedure. First refer to Figure 3 for a presentation of the coordinate systems used. The usual case in photogrammetry might begin with photograph coordinates; then, after applying corrections and transformations, model coordinates are obtained. However with the AP/C the procedure is handled in the inverse manner as the computer reacts to commands from hand wheels and the footwheel—the model coordinates  $X_m, Y_m, E_m$  for model point  $P$ . These coordinates are translated to the perspective center  $O_1$  by airbase components  $b_x, b_y, b_z$  to the coordinate system  $X_c, Y_c, Z_c$ . Orientation elements  $\kappa, \omega, \phi$  are next applied sequentially in this given order to rotate to the coordinate system  $X''', Y''', Z'''$  still at model scale. A scaling by the factor  $f/z'''$  converts them to photograph coordinates  $x, y, z (=f)$ .

The systematic corrections to coordinates are applied in a similar manner as the transformations above. First the model coordinates are corrected for earth curvature which depends on  $X_m$  and  $Y_m$  and some scale constant  $S_G$  to correct the model coordinate  $E_m$ . The atmospheric refraction depends on the  $X_c, Y_c$  from the nadir point and on  $E_m$ . Earth curvature and refraction corrections are added to obtain  $Z_c$ . Finally, corrections for lens distortion and film shrinkage are

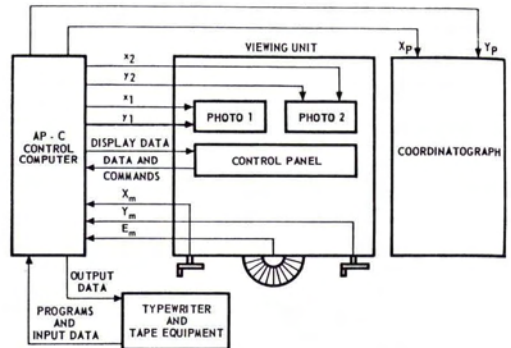


FIG. 2. System Diagram for AP/C

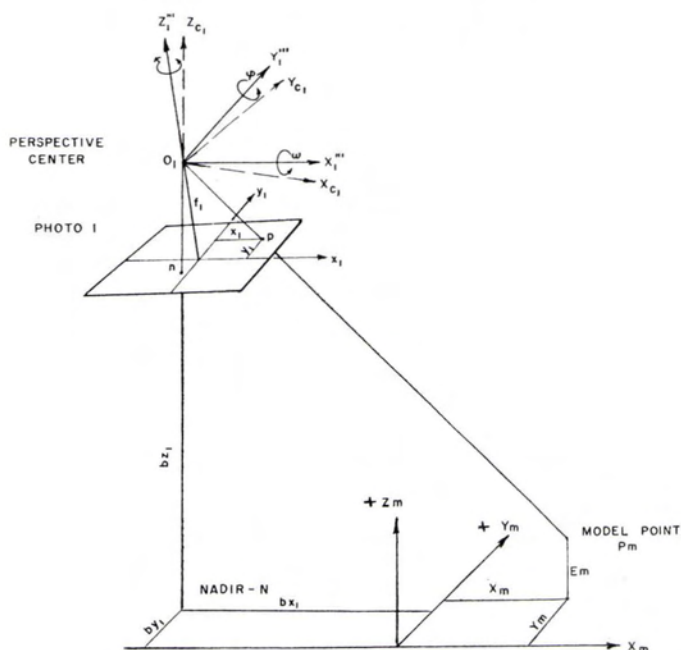


FIG. 3 Coordinate Systems in AP/C

applied to the photo coordinates to furnish the operator his correct stereoscopic model for viewing.

The above cycle is performed for each photograph as the operator moves about the model space either with hand wheels and footwheel, or with the Vetropolo slewing device (which actually permits slewing accurately in any desired azimuth). These computations comprise the biggest effort of the computer and are designated as the *real-time* Program. Intermixed with the cycles for the real-time program are computations with several subroutines. The real-time program is executed at the rate of 30 times per second and the operator's control panel is also interrogated by the computer at 30 times per second. The subroutines (display date, lens and film corrections for each photo, and model corrections for each photo) cycle 5 to 6 times per second. Thus, many computations, such as for relative orientation, seem to be instantaneous to the operator because of the speed of execution.

#### THE TEST PROCEDURE

It was fortunate for this study that the U. S. Geological Survey furnished diapositives on one of its RC9 cameras for a test area of Arizona. The scale was approximately 1:17,000 ( $h = 5,000$  ft.) which approaches the

desired scale for highway mapping. Also, an abundance of premarked control stations cover five of the six photographs. (See Figure 4.)

The control layout of the area was designed for photogrammetric use and was established as a joint effort of the Army Map Service and the U. S. Geological Survey. Two of the models of this strip are each controlled by four horizontal control stations and over 40 vertical control stations. These approximately cover the full extent of the models and are nearly ideal to study the elevation errors in the entire model area. All control was targeted in advance of photography by either 6 or 9-foot square panels.

The objectives of the test were to measure the inherent or residual errors in the RC9-AP/C system, and later to compare these results with those obtained from the RC9-Wild-A7 system. The test was done with absolute orientation for each model in the high-density controlled area. This confined the error to within each model so as not to introduce any propagated error.

As a secondary step, the entire strip was done to test for propagated error in aerotriangulation. Actually, this is a small number of models for this test but it should show some pattern of error propagation as measurements were made to microns.

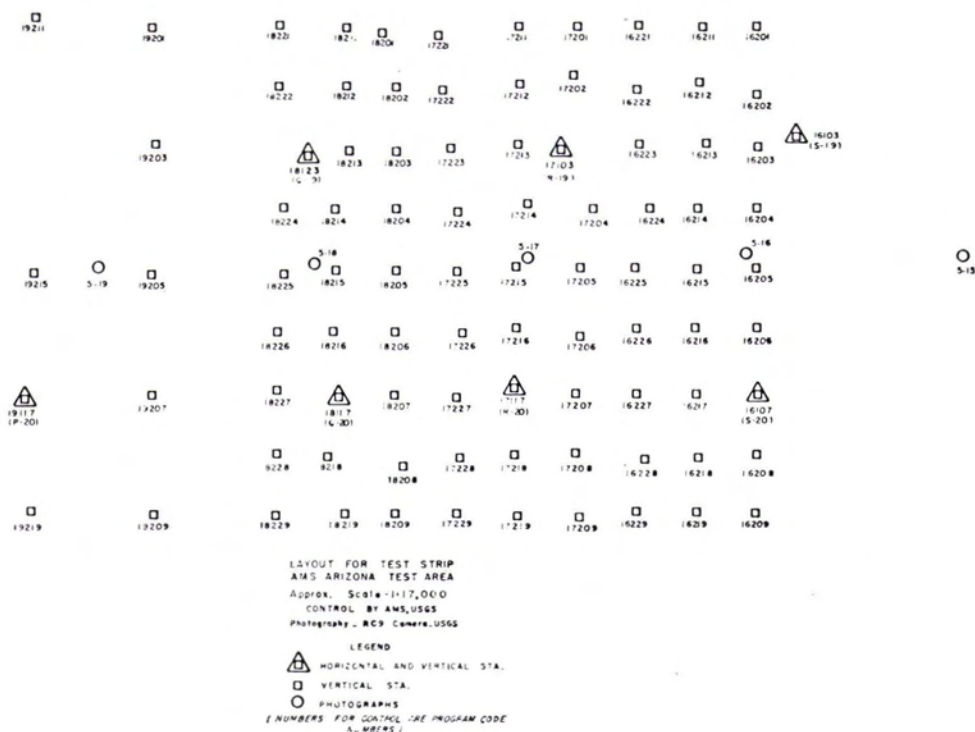


FIG. 4 Layout for Test Strip, AMS Arizona Test Area

The diapositives for this test were printed with corrector plates for symmetrical lens distortion, earth curvature, and atmospheric refraction. The latter corrector plate used was for 10,000-foot altitude whereas the flight height was 5,000 ft. A computed correction was thus made in the AP/C system to compensate for this overcorrection of the plate.

The photographs were prepared for these tests in the following manner. First, a row of three pass points were marked with a Wild PUG approximately on a line normal to the flight line through the photograph center. A tippet was used to help maintain symmetry in the location of these pass points. Coding of the photograph points was accomplished by using a five-digit number as follows: first two digits for the number of the photograph on which the point was identified, the next digit for classifying the point, and the last two digits for denoting the serial number within these groupings. (See Figure 4.)

#### INTERIOR ORIENTATION

The interior orientation on the AP/C is the first step in the photogrammetric solution.

The operator is led by the computer in a semi-automatic routine to the vicinity of each fiducial mark where the coordinates are manually measured and stored in the computer. From this, the computer unit determines the centroid of the measurements and furnishes the operator a display of coordinates for each mark.

Corrections affecting the images (the photographs) and handled as part of the interior orientation problem are lens distortion, film shrinkage, earth's curvature, atmospheric refraction, and principal point offset. These are discussed separately.

For calibration in this test, a master glass negative was used which was exposed in the laboratory to show only the fiducial marks in their correct relationship in the camera. Using this, a set of five measurements on each leg of the fiducial marks, and five measurements on the substitute marks were made. The equation for the mean line defined by opposite legs of the fiducial marks were determined by a least-squares procedure using the ten observations. The intersection of the lines was obtained by the simultaneous solution of the two equations.

The principal point of the photograph is generally the origin of the photograph coordinate system and is determined by the intersection of the lines connecting opposite fiducial marks. But in the AP/C system the centroid of the fiducial marks is determined in interior orientation. Thus, it is always necessary to determine the offset of the principal point from this centroid position. In this test, the principal point offset from the centroid of the substitute fiducial system was determined. (See Figure 5).

#### FILM DISTORTION

Film distortion is corrected by the AP/C by maintaining the relative dimensions in the photograph coordinates by use of ratios of measured to calibrated coordinates for the fiducial marks. A constant for correcting for differential film shrinkage is computed for each photograph. It is based on the ratio of measured values between  $x$  and  $y$ -coordinates, to the ratio determined by the calibration routine, as outlined earlier. The correction is applied only to  $y$ -coordinates, which brings them to the correct ratio with the  $x$ -coordinates. Some further change in coordinates is possible with the  $b_z$ -element. However, this introduces incorrect geometry in the photograph and could have a systematic effect in the aerotriangulation. A better procedure would be to change the focal length of the cameras. The measured coordinates of the fiducial marks could give data for this. Then the focal length entered in the computer would be

$$= \frac{(\Delta x) + (\Delta y) \text{ (measured)}}{(\Delta x) + (\Delta y) \text{ (calibrated)}} \cdot f \text{ (calibrated)}.$$

The data for computing film distortion correction are obtained at the centering phase of interior orientation. The computation is done by desk calculator for each photograph of a strip and is based on the  $\Delta x$ 's and  $\Delta y$ 's between the measured fiducial marks of the photographs.

#### LENS DISTORTION

In the AP/C system, corrections for lens distortion are computed by use of a stored function table in which the distortion is expressed as a function of the radial distance from the principal point of the photograph. The correction equations are:

$$\text{In } x, \quad \Delta x_L = x \cdot d(r/D),$$

$$\text{In } y, \quad \Delta y_L = y \cdot d(r/D),$$

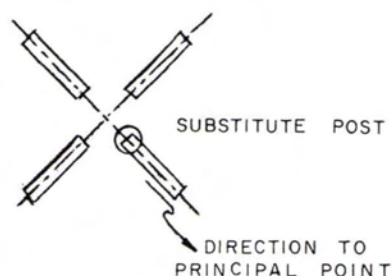


FIG. 5. Substitute Fiducial Mark

where

- $r$  is the radial distance
- $D$  is a scale factor
- $x, y$  are photo coordinates
- $d(r/D)$  is the distortion function.

Corrections are applied only for symmetric radial distortion, asymmetric and tangential distortion being ignored. Figure 6 shows the actual calibration distortion curve for this test and the series of linear approximations as used by the computer.

The method of approximating the lens distortion function might introduce a slight systematic error. The linear segments in this example generally approximate the actual graph (mean curve) within about 5 microns. It is more significant that camera No. 367 (used in this test) had asymmetric distortion as great as 20 microns. These distortions cannot be corrected with the present procedure of the AP/C. Also possibly significant are distortions related to errors in the principal-point offset.

#### EARTH CURVATURE AND ATMOSPHERIC REFRACTION

The distortions resulting from earth's curvature and atmospheric refraction are removed by correcting the model coordinate  $E_m$  (see Figures 3, 7 and 8). The distortions are corrected using the line of camera station to nadir point as reference and in shifting the direction ray to the corrected  $E_m$  in model space.

The corrections in each case are applied using model scale. Here, a scale factor  $S_G$  is entered initially during interior orientation which sets up a table of parameters for the correction equations. The earth's radius and the flight height are reduced to model scale.

The correction (of negative sign) for earth curvature (see Figure 7) is computed using the radial distance in the photogrammetric model.

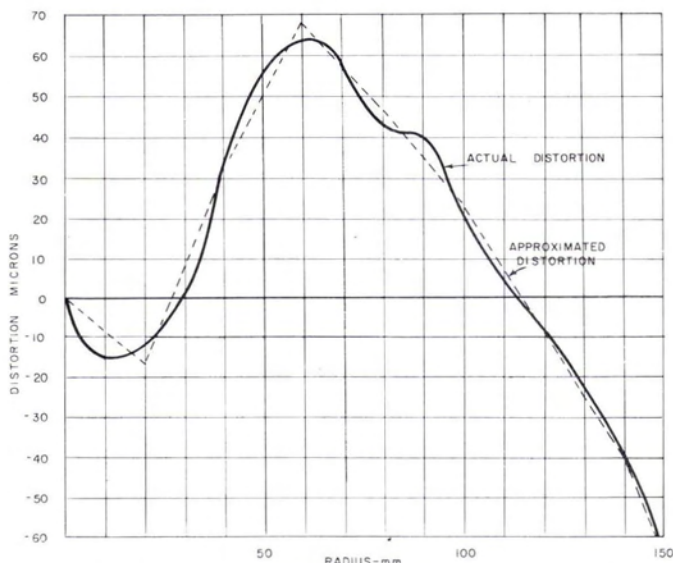


FIG. 6 Distortion Curve with Approximation for AP/C

Atmospheric refraction is similarly computed in model space but using the lens-nadir line for reference. (See Figure 8.) The equation used for the purpose is<sup>1</sup>

$$D_{m_i} = [A_i - B_i \cdot E_m + f(E_m)][1 + \tan^2 \theta_n]$$

where

$$f(E_m) = LE_m^2 + ME_m + N$$

$$\tan^2 \theta_n = (X_{c_i}^2 + Y_{c_i}^2) / Z_{c_i}^2$$

The correction terms are computed by a stored computer program. The parameters

$A, B, L, M$  and  $N$  are based on a standard atmosphere. The function  $f(E_m)$  is the parabolic representation for the refraction effect due to the ground elevation as shown by Laurila.<sup>14</sup> The angle  $\theta$  is the angle between the directions to the nadir and the model point.

Both the atmospheric refraction and earth curvature corrections are combined with the  $b_z$  to give the model correction term,<sup>1</sup> from which a corresponding shift in the photo point is computed:

$$\Delta Z_{m_i} = \Delta D_{m_i} + \Delta Z_{c_i} - b_{z_i}$$

USE OF CORRECTOR PLATES

The use of corrector plates and atmospheric refraction to correct for distortions for lens, earth curvature, has limitations. To economize, agencies operate with a number of

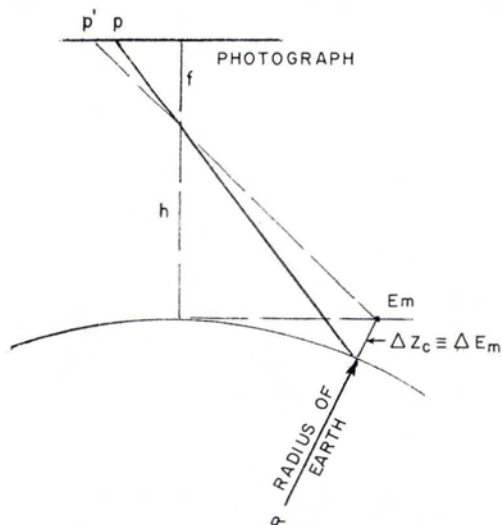


FIG. 7. Earth Curvature Effects

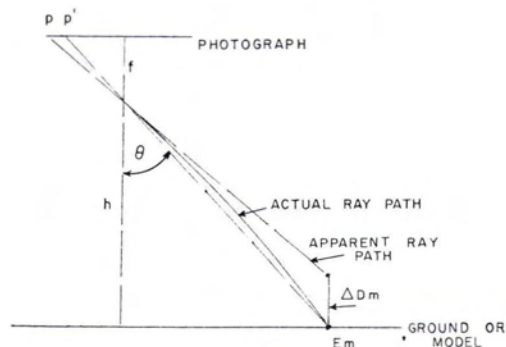


FIG. 8. Atmospheric Refraction Effects

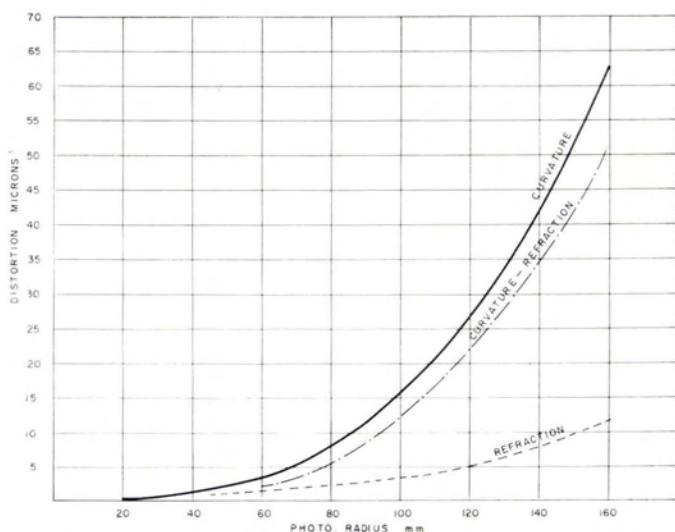


FIG. 9 Residual Effects from Improper Corrector Plate

plates covering minimum flight altitude increments. Thus for this test, the corrector plate was for a flight altitude of 10,000 ft. whereas the actual flight height was 5,000 ft. Such an application introduces significant systematic effects which were corrected for this test.

The residual effects from use of the corrector plate for this test were computed and entered as corrections to the lens distortion curve, shown in Figure 6. This is a valid, though simple, procedure for near-vertical photography but it might introduce a significant error for convergent photographs. It is readily apparent from Figure 9 that the effects were very significant for this case.

Of further significance in this test example are the residual errors in the use of the corrector plates themselves. Based on some calibration data of the U. S. Geological Survey in which grids were used, these residual errors are as large as 15 microns. Such corrections can be made in the AP/C if calibration data is available.

#### RELATIVE ORIENTATION

The solution of relative orientation in the AP/C supplies a set of corrections for updating the orientation elements for one or both of the photographs of the pair. These corrections are obtained by an iterative procedure employing a least squares solution with parallax data from six points (see Figure 10).

The computation procedure requires an input of scaled values for  $b$  and  $d$  and focal length. A single set of these values will suffice

for an individual strip but a separate program tape is required for the *base-in* and the *base-out* modes of operation.

Each numerical orientation computation is practically an instantaneous operation with the AP/C and is a very convenient facility. It requires rough approximate orientation to begin the solution and this is best handled by the operator using similar procedures as with other stereoplotters. The corrections, however, are applied by the computer to the orientation element selected by the operator.

#### ABSOLUTE ORIENTATION IN THE AP/C

The absolute orientation, comprising scaling and leveling the model, is accomplished

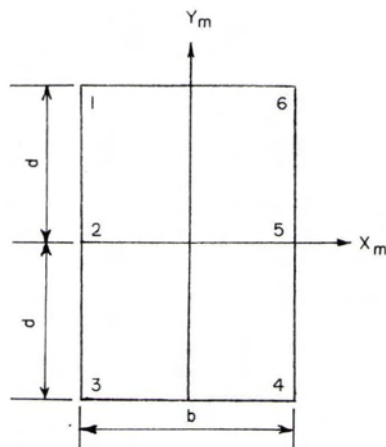


FIG. 10. Designation of Model Points in AP/C System

using a least squares technique. It requires at least three control points, all with  $X$ ,  $Y$  and  $Z$  coordinates, but it can have a possible input of many control points. In order to prevent an overflow in the summation of coefficients for the normal equations, the practical upper limit of the number of control points is thirty. In operation, the ground coordinates must first be reduced to the working model scale.

The program, as at present, seems limited on the minimum condition because it does not provide for inclusion of partial control—that with only horizontal (plan), or only vertical (elevation) coordinates. However, the operator can level the model by conventional methods. For this the automatic computer is not to be used initially.

Absolute orientation does not comprise any translation of coordinates; the model coordinates get changed due to scaling and leveling but they remain in the same model system as before absolute orientation was begun. Next, the model may be translated in elevation by applying a common  $b_z$ -correction through the operator's panel.

There is no print-out or display of residuals from absolute orientation but the operator has an excellent facility to test his results. He can initialize  $D$ -scale to the No. 1 control point and directly obtain a model distance in microns to any other control point. This value is then compared to the previously computed distances.

#### TEST OF PROCEDURE

Two adjacent models 16-17 and 17-18 each had 4 control points (for both planimetry and elevation) and about 40 additional vertical control points (see Figure 4). These were well distributed in the model to furnish data on the elevation accuracy of the system. Measuring the points in individually controlled models did not introduce any propagation of error as might occur in strip triangulation. Thus, it furnished some measure of the inherent accuracy of the system. One of the models was measured twice with different operators so as to furnish a comparison and to validate the program.

#### RESULTS OF ORIENTATION

##### TEST WITH THE AP/C

The model scale was 1:17,200. Only elevation errors were considered for this test in absolute orientation.

A comparison was made in the two sets of measurements of model 16-17. The standard deviation  $m$  from the mean, for a single measurement of these two sets, is  $\pm 3.7$  microns

(where  $m = \pm \sqrt{([dd]/2n)}$  and  $d$  is the difference between sets). The differences were random. Thus, the  $m$  of 3.7 microns indicates a reliable value for the standard error of observation for a single observation.

Another similar comparison was made for a partial set (Figure 11) with 28 points of the same model by the same operator on different days. (The partial set resulted from computer malfunction.) Here the standard deviation from the mean was  $\pm 3.1$  microns.

The models were then examined for closure to control. The model measured without any corrections entered in the AP/C, or with the partial corrections for distortions by use of the corrector plates in the printing process, showed a remarkable systematic pattern of distortions (see Figure 4). The standard deviation to control (misclosure at control points) was 16 microns. The application of further corrections in the AP/C greatly improved this pattern (see Figure 12) with the standard deviation of about 8 microns.

The juncture of adjacent models were at times rather poor, with differences as great as

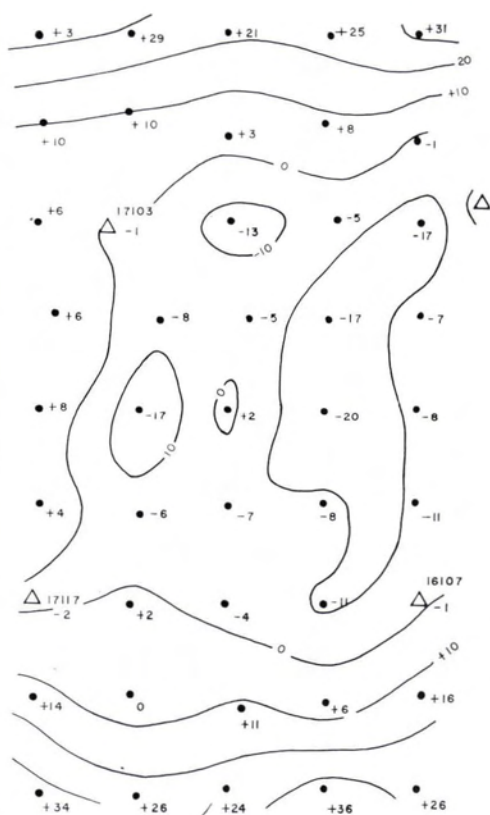


FIG. 11. Elevation Errors in Microns in Model 16-17 with Distortions Partially Corrected.



25 microns. However, if the models are superimposed by fitting the rectangular patterns of control, the correspondence of error pattern is remarkable (see Figure 13). The pattern is noticeably asymmetric. This pattern also prevails in the aerotriangulation adjustment. This indicates that only the central part of the photograph should be used if propagation effects are to be minimized.

STRIP TRIANGULATION WITH THE AP/C

Aerotriangulation procedures with the AP/C closely parallel those with a first-order analog instrument. *Base-in* and *base-out* models are achieved by a single switch located just above the operator's control panel. Junctioning of models is done by first setting the  $E_m$  for a common point by the footwheel. The measuring dot is then brought to ground level in the model by changing the  $bx$ -element by means of the incremental switch. These steps are done after dependent relative orientation is accomplished.

The adjustment program used is that of the Coast and Geodetic Survey as amended by Horsfall.<sup>11</sup> To conserve computer storage,

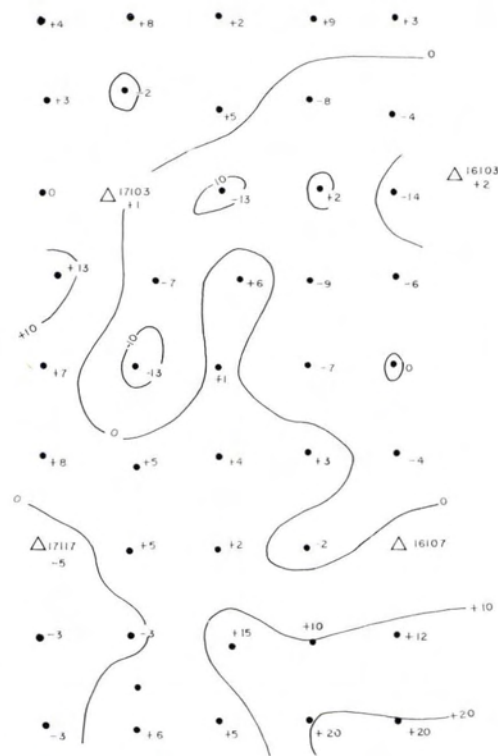


FIG. 12. Elevation Errors in Microns in Model 16-17 with Distortions Corrected.

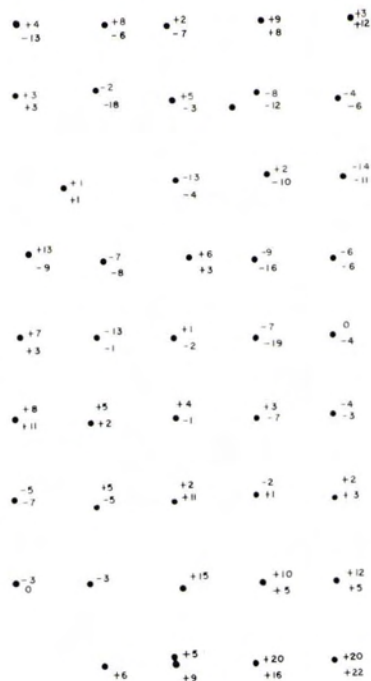


FIG. 13. Comparison of Errors in Different Models. The upper values are Model 16-17 and the lower values are from 17-18; all values are in microns in the model scale.

a ceiling is imposed in the number of control points for adjustment: five for horizontal, and seven for vertical. Thus the selection of these stations should be made with care to obtain the best solution. The factors to strive for are: (1) an even distribution along the strip and (2) one which covers the extremes of the strip. Although the first model is controlled absolutely in scale, it should not be heavily weighted in the solution as it might introduce an erroneous slope in the polynomial adjustment curve.

Formulas used in the adjustment are

$$\begin{aligned}
 x' &= x - \Delta z(2Ix + J) + Ax^3 + Bx^2 \\
 &\quad + Cx - 2Dxy - Ey + F \\
 y' &= y - \Delta z(Lx + M) + 3Ax^2y + 2Bxy \\
 &\quad + Cy - Dx^2 + Ex + G \\
 z' &= z[1 + (2Ix + J)^2 + (Lx + M)^2]^{1/2} \\
 &\quad + Ix^2 + Jx + Lxy + My + N.
 \end{aligned}$$

Here,  $x'$   $y'$   $z'$  are the newly transformed (or adjusted) coordinates.  $x$ ,  $y$ ,  $z$  are strip coordinates transformed to an axis-of-flight system.

$\Delta z$  is the increment in the  $z$ -coordinates of a point from the reference datum of the strip.

$x'$ ,  $y'$ , and  $z'$  must undergo an inverse transformation (translation, rotation and scaling)

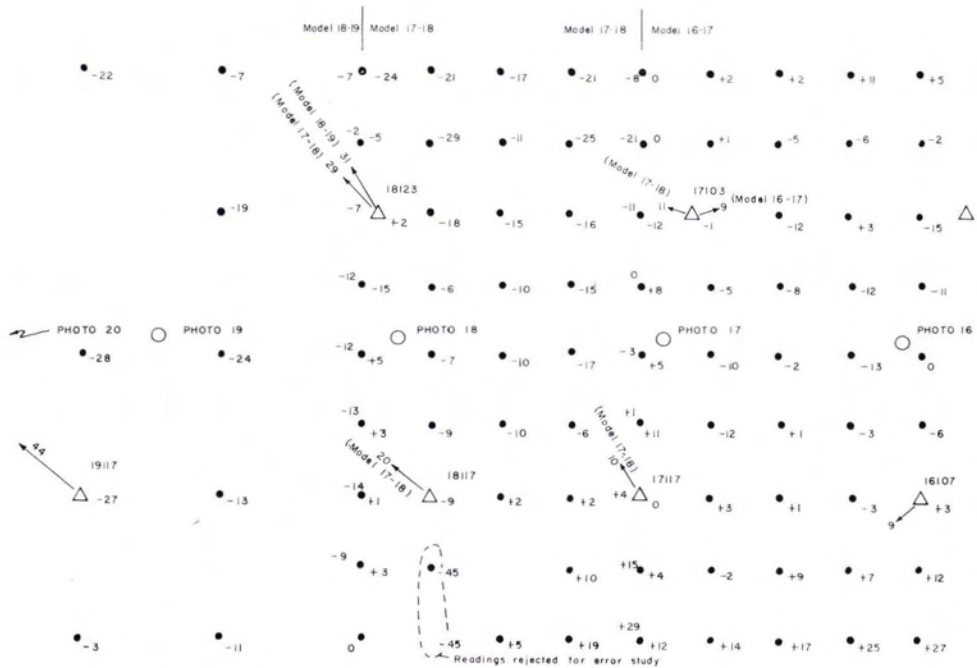


FIG. 14 Aerotriangulation—Closures to Control before Adjustment (Horizontal misclosure shown at arrowheads, others are vertical misclosures; all values are in microns at scale of the model.)

to ground coordinates. The coefficients  $A, B, C \dots N$  are the parameters of the transformation and are determined for the strip from the control stations.

The test strip comprises only four models with control as depicted in Figure 4. The strip was cantilevered by adding successive models by co-orientation (i.e., dependent relative orientation and scaling). A close fit to three transfer (tie) points generally could not be achieved because a large amount of systematic error was evident. Consequently the scale transfer was made by using the center tie point with an approximate fit to the other two points.

The closures to control before adjustment are shown in Figure 14. Very little propagation of error is evident; only 44 microns in planimetry and 29 microns in elevation. The pattern of systematic effect is evident.

The closures to control after adjustment are shown in Figure 15. The standard deviations to control at model scale of 1:17,200 were as follows:

- in  $x$ :  $\pm 4$  microns
- in  $y$ :  $\pm 6$  microns (i.e.,  $\pm 7$  microns in planimetry)
- in  $z$ :  $\pm 10$  microns.

It is evident that the northwest corner of

the strip was improperly controlled; therefore for elevation a separate adjustment using the method by Brandenberger<sup>2</sup> was performed with 9 control points. This resulted in a standard deviation of  $\pm 7$  microns in  $z$ .

#### AEROTRIANGULATION WITH THE WILD A7

For the purpose of studying the workability of SWA photography with a stereoplotter which was not equipped for this kind of photography, and of comparing the results with those obtained with the AP/C, the same test strip was triangulated at the Wild Autograph A7 in the Ohio State University, Department of Geodetic Science. The aeropolygon method of aerotriangulation was performed in this case.

The calibrated focal length 88.23 mm cannot be introduced into the projector cameras of the Wild A7 because the range of focal length column here is 98 mm to 215 mm. In this case a focal length of 105.88 mm was used. This means that an increase of 1/5 times the actual (88.23 mm) is used. The photography being vertical (i.e., the camera axis being nearly coincidental with the  $Z$ -coordinate direction) this increment in the focal length of the cameras would cause an affine deformation in the stereo models. Thus, the

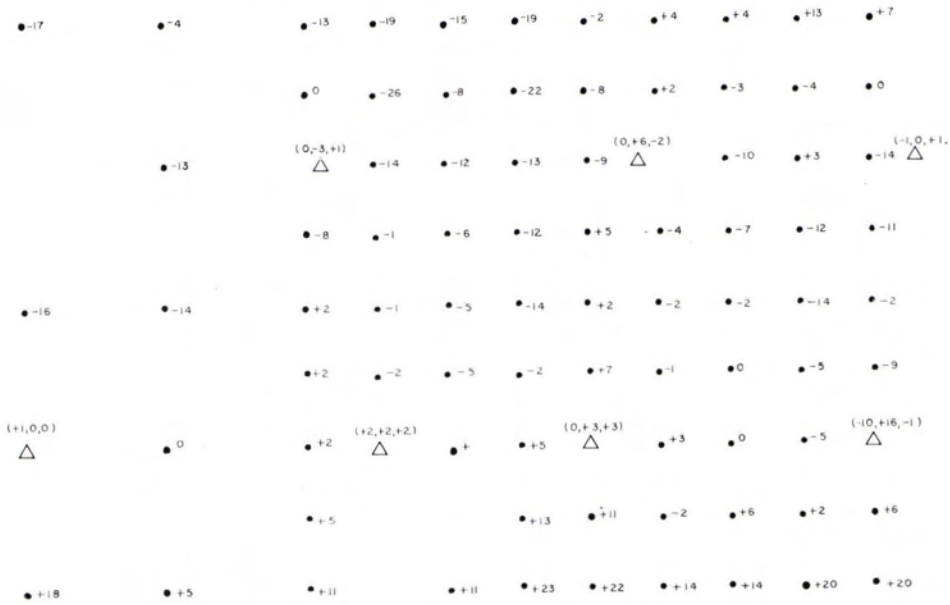
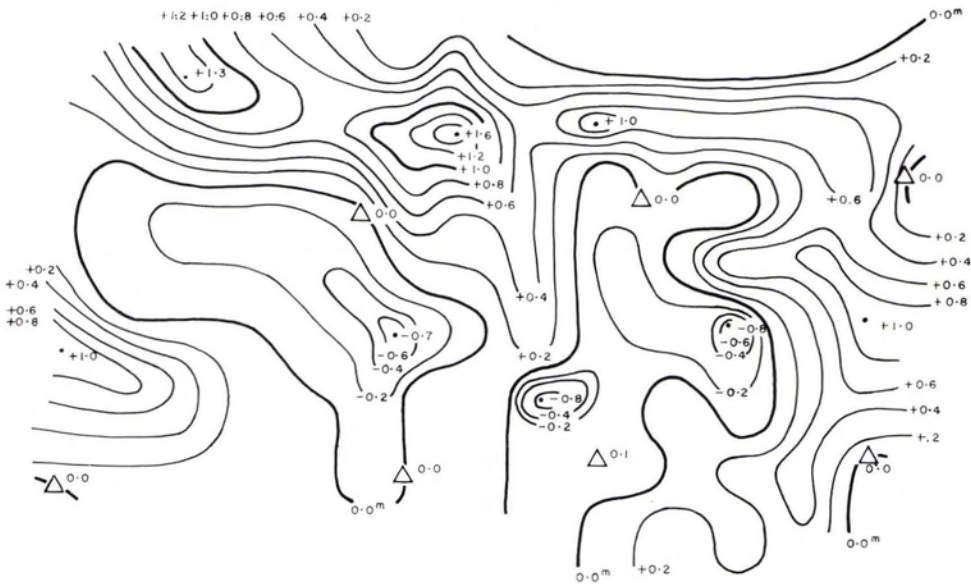


FIG. 15 Aerotriangulation—Closures to Control after Adjustment

planimetric coordinates ( $X$  and  $Y$ ) will have a scale different from that of the elevation coordinate ( $Z$ ) which, in this case is enlarged by  $1/5$  times.

The least count of the coordinates in the model was 0.01 mm. Further, each coordinate was read in two rounds of observation. Considering that the model scale, 1:6,000, was approximately three times the scale of the picture (i.e., the same as the model scale with



Interval : 0.2 m      Scale 1: 34000

FIG. 16 Elevation Differences Between A7 and AP/C in Meters

the AP/C), it may be considered that the reading accuracy of all the coordinates was about the same ( $\pm 3\mu$ ) in this test as well as with the AP/C.

The relative orientation of each model was performed by the numerical method<sup>6</sup>. After the relative orientation was performed the residual  $y$ -parallaxes were measured at eight different locations in each of the 5 models. From these 40 observations was computed the standard residual error in  $y$ -parallax which was  $\pm 0.019$  mm. This reduced to the picture scale was approximately  $\pm 0.006$  mm, i.e.,  $6\mu$ , which is reasonably good for any analogue system.

The observed coordinates ( $X$ ,  $Y$  and  $Z$ ) were next reduced to the ground system by performing linear transformation based on the ground coordinates of points 16107, 19117, 16103 and 17102 (see Figure 4). Only at this stage could the closing errors be found in the observed coordinates at the points. An analytical-graphical method of adjustment was performed for each of the coordinates.

The adjusted ground coordinates of all points in the strip were compared to those obtained from the AP/C. The standard deviation between the AP/C and A7 (adjusted) data are as follows\*:

$$\begin{aligned} X, & \pm 0.14 \text{ m} \\ Y, & \pm 0.98 \text{ m} \\ Z, & \pm 0.14 \text{ m} \end{aligned}$$

The differences in elevation are depicted in Figure 16.

It may be pointed out here that with the Wild A7 no attempt was made to compensate for the residual errors in refraction and earth curvature. It is also apparent that those effects were of little consequence in this test.

#### CONCLUSIONS

Although the test and procedures outlined above were limited in the size of data and in the control of error sources, some conclusions are evident:

★ Very high quality measurements can be obtained with the AP/C system. Measurements could be further improved by calibration of the comparators and other allied photogrammetric components of the system. In fact, the test demonstrates the value of accurate calibration data.

★ The AP/C analytical system provides a means to measure and control systematic errors

\* Since this test was completed, the procedures for relative orientation and absolute orientation have been revised. See Forrest, Robert B., "AP/C Plotter Orientation," *PHOTOGRAMMETRIC ENGINEERING*, Vol. XXX, No. 6, pages 1024-1027.

in the photograph. Thus, in this test, the standard deviation was reduced to one-half that with the corrector plate.

★ Results with the AP/C system, both in absolute orientation and in aerotriangulation, were superior to those that could normally be expected by an analogue system.

★ SWA photography can produce very high quality results with the AP/C. Thus, the standard deviation to vertical control was less than one-half foot with 1:17,000 scale photograph.

★ The test with the Wild A7 indicates that the SWA can be handled satisfactorily in analogue instruments for normal aerial stereo-photogrammetric mapping if affine deformations are properly corrected. However, this has not been validated with optical projection type instruments such as the Kelsh plotter.

★ The test was considered adequate to validate the AP/C system for SWA photography. It might be more practical to perform aerotriangulation with the AP/C to yield control for compilation instruments. Also, possibly the orientation elements for compilation instruments can be readily obtained in this procedure.<sup>15</sup> The need for more testing is indicated for these procedures.

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