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Auxiliary Data in Strip Adjustment

Horizon photo data are applied in experimental computations using ISP-Commission III fictitious photos.

(*Abstract on next page*)

SUMMARY

THE HORIZON CAMERA is one of the auxiliary instruments which provides some of the necessary information for controlling aerial triangulation by yielding the values of the orientation parameters ϕ and ω at each exposure station. A new method of deducing the orientation parameters from the horizon pictures has been established.[7] The properties of this method are:

- It allows for any number of observations along the horizon line.
- It is subject to a least-squares solution.
- It yields rigorous absolute components of tilt whenever the true horizon or a levelled apparent horizon is observed.
- Relative tilt components can be obtained from the absolute values or directly from the equations if this is required.
- It provides a fast and economical data-reduction and processing procedure.
- It is not restricted to horizon-camera pictures but can be used in association with terrestrial or high-oblique photography as well. Not only is it unrestricted to observations of the horizon line but also to definite points observable on the horizon picture, in which instance all ϕ, ω, and κ can be obtained.

Although the observation of definite points in the horizon pictures is the most promising means of deducing the three rotational elements of orientation, the present available camera and films usually are incapable of registering such points. Restricted to the rather limited accuracy of the horizon-determined orientation parameters, the incorporation of these data in aerial triangulation must be handled with care.

It is evident[2] that at present the horizon-determined orientation parameters could not be incorporated in the process of strip formation (i.e., in the relative orientation of one photograph to the preceding one) where conventional procedures yield a higher degree of accuracy. It is therefore suggested that the auxiliary data should be incorporated in the adjustment stage. In view of the above discussion it is advisable to establish an adjustment method with the following characteristics:

◐ Keeping the conventional relative orientation of each model intact;
◐ Not resorting to polynomials, especially where long strips are triangulated; and
◐ Utilizing both the auxiliary data and any available ground control point along the strip.

The suggested method in this presentation meets the above specifications using the following technique:

▲ Each model in the strip is adjusted as a rigid unit.
▲ The horizon-determined parameters of each model are viewed as known elements and are not

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subject to any change in the adjustment process.

▲ The other orientation parameters required for adjusting each model, and which are not given *a priori*, are solved simultaneously for all models in the strip.

▲ The coordinates of the ground control points are incorporated in the above simultaneous solution, thus contributing to the adjustment of the entire strip. The density and distribution of the control points will affect the accuracy of the determination of the parameters that are not deduced from the horizon camera.

▲ The orientation parameters of each individual model are deduced from the simultaneous solution, and the adjustment of each model constitutes finally the adjustment of the entire strip.

Incorporation of Auxiliary Data in the Process of Strip Formation

Model formation within the strip is defined here as a process in which the bundle of rays constituting photograph j is oriented with respect to the already oriented bundle of rays of photograph i, utilizing the horizon-determined orientation elements ϕ and ω associated with this bundle. In case the auxiliary data are not accurate enough, the above process will fail to eliminate the parallaxes within the model, resulting in model deformations as well as discrepancies between models. Thus the criterion in applying the auxiliary data to the formation of a model is the accuracy with which these data can be obtained, compared with the accuracy achieved by the conventional procedures.

Many experiments conducted by different investigators at different times[4,5,8] lead to the conclusion that the determination of ϕ and ω by utilizing the horizon pictures is, at best, within $\pm 2^c$. This accuracy seems to be quite optimistic, but even if this value is accepted, it is inferior to the accuracy obtained by the conventional procedures of relative orientation.[2] However, it is hoped that with a more sensitive emulsion capable of penetrating haze and other physical obstacles, and with a more advanced horizon camera, the accuracy of determining ϕ and ω will reach the $\pm 1^c$ level.

Incorporation of Auxiliary Data in the Adjustment Stage

The outcome of the strip formation is a set of bundles of rays oriented to each other but deviated from their corresponding absolute position in space due to the error accumulation and the effect of earth curvature. The relative orientation of each bundle to its preceding one, can be achieved with a high degree of accuracy, thus reducing to a minimum the effect of model deformation. It is, therefore, recommended that the relative orientation obtained in a conventional way be kept intact as far as possible at the adjustment stage. This can be achieved by adjusting (i.e., absolutely orienting) each individual model in the strip as a rigid unit.

ABSTRACT: *A new method for the determination of the attitude of aerial cameras (Φ and ω, and under certain conditions, also κ) by means of horizon photography has been established. After deriving the tilt components of each photograph, the absolute orientation parameters (Φ and Ω) of each model are determined and incorporated in the adjustment of the strip. The strip adjustment takes full advantage of all available auxiliary data as well as any available ground control and solves simultaneously for all the parameters of orientation of each model.*

The absolute orientation requires, among other parameters, the values of the tilt components of each model (i.e., $\Delta\Phi$ and $\Delta\Omega$). Thus, an attempt is made here to deduce these values from the individual horizon-determined tilt components of each of the photographs constituting the model. In this way the auxiliary data are indirectly incorporated in the adjustment phase.

BASIC CONCEPTS AND DEFINITIONS

A direction d in space ($O \rightarrow E$, see Figure 1), which in the case of near-vertical photography is always in the negative hemisphere ($Z < 0$), can be safely determined by the following couple of direction tangents:

$$\begin{aligned} \tan \theta_x &= \frac{X_E - X_0}{Z_E - Z_0} = t_X \\ \tan \theta_Y &= \frac{Y_E - Y_0}{Z_E - Z_0} = t_Y. \end{aligned} \tag{1}$$

The *measured direction* is defined throughout this treatment by its corresponding direction tangents as deduced from plate measurements and denoted t_X, t_Y.[3]

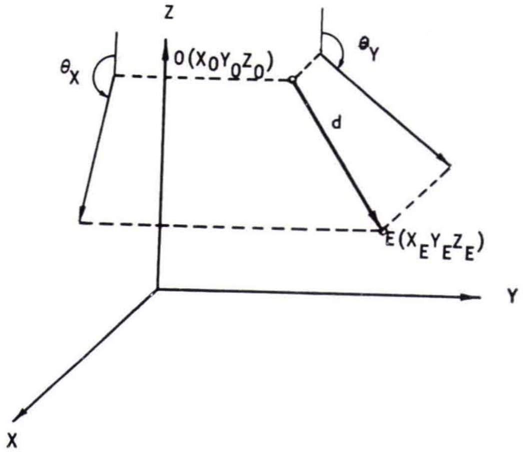

FIG. 1. A direction d in space and its projections on the YZ- and XZ-planes.

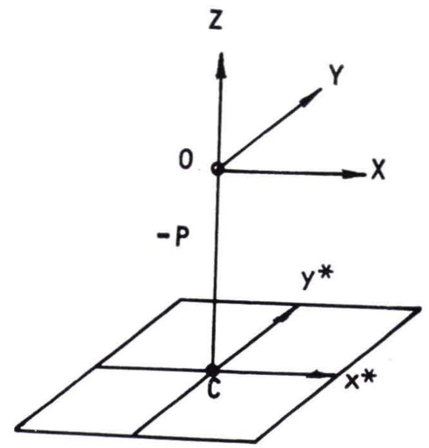

FIG. 2. An internal reference system.

If an internal reference system, having its origin at the perspective center O, is set parallel to the plate coordinate system defined by the plates' fiducial marks, and if the measured plate coordinates are reduced to the principal point of the plate and denoted x^*, y^* (see Figure 2), then the *measured* direction-tangent of point E, following Equation 1 are:

$$t_X = \frac{x_E^* - 0}{z_E^* - 0}, \quad \text{and} \quad t_Y = \frac{y_E^* - 0}{z_E^* - 0}.$$

As $z_E^* = -P$ (principal distance) in the above described internal coordinate system, it follows that:

$$t_X = -\frac{x_E^*}{P},$$

and

$$t_Y = -\frac{y_E^*}{P}. \tag{2}$$

The *transformed direction* is defined here by the values of the direction tangents which have undergone a spatial transformation and denoted t_X', t_Y'. The well-known spatial transformation formulas read, in matrix notation:

$$\boldsymbol{C}' = \boldsymbol{S} + \boldsymbol{CM} \tag{3}$$

where

$\boldsymbol{C}' = [X' \; Y' \; Z']$, the transformed coordinates,
$\boldsymbol{C} = [X \; Y \; Z]$, the original coordinates,
$\boldsymbol{S} = [S_1 \; S_2 \; S_3]$, the translations along the XYZ-axes,

$$\boldsymbol{M} = \begin{bmatrix} \cos\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ +\sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = [\boldsymbol{M}_x \boldsymbol{M}_y \boldsymbol{M}_z]. \tag{4}$$

The matrix refers to the right handed coordinate system used in the U.S.,[6] where ω is the primary rotation, ϕ the secondary and κ the tertiary rotation.

Using the above notations, the transformation formulas can be written as:

$$\begin{aligned} X_E' &= S_1 + Xm_{11} + Ym_{21} + Zm_{31}, \\ Y_E' &= S_2 + Xm_{12} + Ym_{22} + Zm_{32}, \\ Z_E' &= S_3 + Xm_{13} + Ym_{23} + Zm_{33}. \end{aligned} \tag{5}$$

Thus, Equation 1 for example can be written as:

$$\tan\theta_x' = \frac{X_E' - X_0'}{Z_E' - Z_0'} = t_x'. \tag{6}$$

Combining this equation and Equation 5, one gets:

$$t_x' = \frac{(X_E - X_0)m_{11} + (Y_E - Y_0)m_{21} + (Z_E - Z_0)m_{31}}{(X_E - X_0)m_{13} + (Y_E - Y_0)m_{23} + (Z_E - Z_0)m_{33}}.$$

Dividing the numerator and denominator by $(Z_E - Z_0)$ and using Equations 1, it follows that:

$$t_x' = \frac{t_x m_{11} + t_y m_{21} + m_{31}}{t_x m_{13} + t_y m_{23} + m_{32}} = \frac{[t_x \quad t_y \quad 1]\cdot \overline{\boldsymbol{M}}_x}{[t_x \quad t_y \quad 1]\cdot \overline{\boldsymbol{M}}_z},$$

and similarly:

$$t_y' = \frac{t_x m_{12} + t_y m_{22} + m_{32}}{t_x m_{13} + t_y m_{23} + m_{33}} = \frac{[t_x \quad t_y \quad 1]\cdot \overline{\boldsymbol{M}}_y}{[t_x \quad t_y \quad 1]\cdot \overline{\boldsymbol{M}}_z}, \tag{7}$$

where $\overline{\boldsymbol{M}}$ contains the values of the elements required to rotate a direction from its measured position to its newly transformed position.

The *oriented direction* is defined here by the values of the direction tangents satisfying certain conditions imposed upon them and denoted T_x, T_y, thus:

$$T_x = \frac{[t_x' \quad t_y' \quad 1]\cdot \overline{\overline{\boldsymbol{M}}}_x}{[t_x' \quad t_y' \quad 1]\cdot \overline{\overline{\boldsymbol{M}}}_z} \qquad T_y = \frac{[t_x' \quad t_y' \quad 1]\cdot \overline{\overline{\boldsymbol{M}}}_y}{[t_x' \quad t_y' \quad 1]\cdot \overline{\overline{\boldsymbol{M}}}_z} \tag{8}$$

where $\overline{\overline{\boldsymbol{M}}}$ contains the values of the elements required to rotate a direction from its transformed position to its oriented one. This oriented direction can also be expressed as function of the measured direction, in which case $\boldsymbol{M}$ contains the values of the elements required to rotate the direction all the way from its measured to its oriented position, that is:

$$T_x = \frac{[t_x \quad t_y \quad 1]\cdot \boldsymbol{M}_x}{[t_x \quad t_y \quad 1]\cdot \boldsymbol{M}_z} \qquad T_y = \frac{[t_x \quad t_y \quad 1]\cdot \boldsymbol{M}_y}{[t_x \quad t_y \quad 1]\cdot \boldsymbol{M}_z}. \tag{9}$$

DETERMINATION OF THE TILT-COMPONENTS $\Delta\Phi$ AND $\Delta\Omega$ OF A MODEL

The following method is based on the realization that the optical axis of the nadiral camera should coincide with that determined by the horizon camera. This condition can be mathematically expressed as:

$$T_x^m - T_x^h = 0,$$

and

$$T_y^m - T_x^h = 0, \tag{10}$$

where T_x^m, T_y^m are the oriented direction tangents of the camera axis, and T_x^h, T_y^h are the oriented direction tangents as determined by the horizon camera.

The direction tangents of the camera axis, as obtained in the process of strip formation (i.e., the unadjusted cantilever assembly), are $t_x'^m$, $t_y'^m$ whose expressions are (see Equation 7):

$$t_x'^m = \frac{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_x^m}{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_z^m}, \qquad t_y'^m = \frac{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_y^m}{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_z^m}, \tag{11}$$

where $\boldsymbol{M}_x^m$, $\boldsymbol{M}_y^m$, $\boldsymbol{M}_z^m$ are the columns of the orientation matrix containing ϕ, ω, and κ as obtained in the process of strip formation. The values of $t_x'^m$ can thus be viewed as the transformed direction tangents of the camera axis, whose initial direction was precisely nadiral, to the position obtained by the strip formation (see Figure 3). The rotation from this transformed position to the oriented one is given in the following form:

$$T_x^m = \frac{[t_x'^m \quad t_y'^m \quad 1]\cdot \overline{\boldsymbol{M}}_x}{[t_x'^m \quad t_y'^m \quad 1]\cdot \overline{\boldsymbol{M}}_z}, \qquad T_y^m = \frac{[t_x'^m \quad t_y'^m \quad 1]\cdot \overline{\boldsymbol{M}}_y}{[t_x'^m \quad t_y'^m \quad 1]\cdot \overline{\boldsymbol{M}}_z}, \tag{12}$$

where $\overline{\boldsymbol{M}}_x$, $\overline{\boldsymbol{M}}_y$, $\overline{\boldsymbol{M}}_z$ are the columns of the unknown orientation matrix.

On the other hand, the direct transformation from the initial nadiral position to the oriented direction as determined by the horizon camera is (see Figure 3) expressed by the following equations:

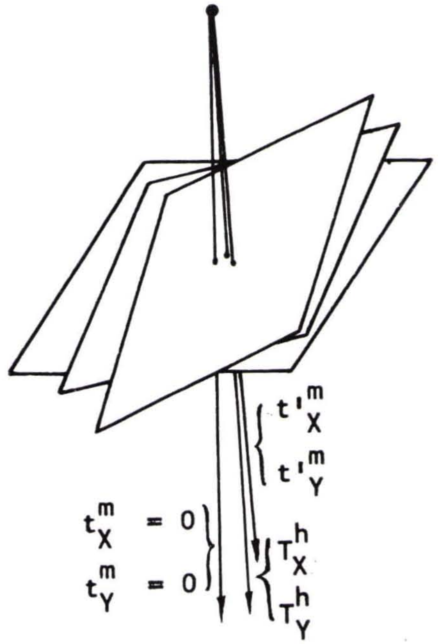

FIG. 3. Direction of the optical axis of the nadiral camera in the different phases of the triangulation.

$$T_x^h = \frac{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_x^h}{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_z^h}, \qquad T_y^h = \frac{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_y^h}{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_z^h} \tag{13}$$

where $\boldsymbol{M}_x^h$, $\boldsymbol{M}_y^h$, $\boldsymbol{M}_z^h$ are the columns of the horizon-determined orientation matrix ($\kappa = 0$ in case it cannot be determined from the horizon camera).

Substituting Equation 12 and Equation 13 in Equation 10, it follows that:

$$\frac{[t_x'^m \quad t_y'^m \quad 1]\cdot \overline{\boldsymbol{M}}_x}{[t_x'^m \quad t_y'^m \quad 1]\cdot \boldsymbol{M}_z} - \frac{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_x^h}{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_z^h} = 0,$$
$$\frac{[t_x'^m \quad t_y'^m \quad 1]\cdot \overline{\boldsymbol{M}}_y}{[t_x'^m \quad t_y'^m \quad 1]\cdot \overline{\boldsymbol{M}}_z} - \frac{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_y^h}{[0 \quad 0 \quad 1]\cdot \boldsymbol{M}_z^h} = 0. \tag{14}$$

The only unknowns in Equation 14 are the orientation elements contained in $\overline{\boldsymbol{M}}_x$, $\overline{\boldsymbol{M}}_y$, and $\overline{\boldsymbol{M}}_z$.

Expending the above functions in Taylor series and evaluating the expansion at $\phi = \omega = 0$ (see explanation below) without considering κ:

$$-(1 + t_x'^m)\Delta\Phi + t_x'^m t_y'^m \Delta\Omega + t_x'^m - T_x^h = 0$$
$$-t_x'^m t_y'^m \Delta\Phi - (1 + t_y'^m)\Delta\Omega + t_y'^m - T_y^h = 0 \tag{15}$$

The set of Equations 15 is associated with bundle i of model N. An equivalent set can be deduced for bundle j; thus four equations are available for determination of the two unknowns $\Delta\Phi$ and $\Delta\Omega$ by the least squares technique. The solution is achieved by an iterative process, starting each iteration with the approximation $\phi = \omega = \kappa = 0$ (see the conditions under which Equations 15 were obtained). At each iteration the direction tangents are transformed so that the directions assume a new position which becomes the initial position for the next iteration. This iteration process goes on until the orientation is finally achieved. From the product of all the partial orientation matrices obtained in each iteration, the final results can be deduced. The resulting values will be the amount by which the two bundles associated with the model must rotate.

Theoretically κ could be solved for here as well. Each model has two fixed directions in space, associated with the two bundles i and j (those obtained by the horizon camera). When the model undergoes a rotation to fit into these two directions, it undertakes a swing κ as well. However, practically the two fixed directions as obtained from the horizon camera are nearly parallel to each other and thus κ is insignificantly affected by the rotation of the model (Figure 3).

PARABOLIC SMOOTHING OF BREAKS BETWEEN MODELS

Because of the imperfect rectification of each model, discrepancies will remain along the line joining any two adjacent models. Most noticeable of these are the discrepancies in elevations. This is an unavoidable outcome or a *price* for controlling the error accumulation. A rather known concept can be modified here to fit the purpose of smoothing the discrepancies between the models. The suggested method is based upon the assumption that the propagation of errors in aerial triangulation can be expressed in terms of a third degree polynomial such as:

$$\Delta Z = C_0 + C_1X + C_2Y + C_3X^2 + C_4X^3 + C_5XY,$$

which, for points along the strip axis, becomes:

$$\Delta Z = C_0 + C_1X + C_3X^2 + C_4X^3.$$

Inasmuch as

$$\Delta\Phi = \frac{\partial \Delta Z}{\partial x},$$

then it follows that

$$\Delta\Phi = C_1 + 2C_3X + 3C_4X^2$$

or in a general form,

$$\Delta\Phi = A_1 + A_2X + A_3X^2. \tag{16}$$

In a similar way $\Delta\kappa$ can be expressed in a general form as:

$$\Delta\kappa = D_0 + D_1X + D_2X^2.$$

$\Delta\Omega$ is a linear function of X, but because the other two corrections are expressed as a second degree polynomial, the expression for $\Delta\Omega$ will also be raised to this degree, so that:

$$\Delta\Omega = B_1 + B_2X + B_3X^2. \tag{17}$$

As it is assumed here that no definite point can be observed in the horizon picture and thus that κ cannot be solved for, only the equations for Φ and Ω will be dealt with. Substituting Equation 16 and 17 in Equation 15 the following expressions are obtained:

$$\begin{aligned} &-(1 + t_x'^{m^2})(A_1 + A_2X + A_3X^2) \\ &\qquad - t_x'^m t_y'^m (B_1 + B_2X + B_3X^2) \\ &\qquad + t_x'^m - T_x^h = 0, \\ &-t_x'^m t_y'^{m^2}(A_1 + A_2X + A_3X^2) \\ &\qquad - (1 + t_y'^{m^2})(B_1 + B_2X + B_3X^2) \\ &\qquad + t_y'^m - T_y^h = 0. \end{aligned} \tag{18}$$

Six unknowns are involved in the above equation (A_1, A_2, A_3, B_1, B_2, B_3). For every model there are then four equations with six unknowns. In a strip of n models there will be $4n$ equations from which the six unknowns can be solved for (X can be taken as the machine coordinates of the right nadir point of each model). Once the coefficients are solved for, Equations 16 and 17 are used once again to compute Φ and Ω of each model, and these elements should be used for the rotation of each model. It should be emphasized that this suggested method does not correct discrepancies but rather smooths them in the best possible way.

ADJUSTMENT OF THE BRIDGED STRIP

Bridging can be viewed analytically as the case in which more information exists along the strip, than in the case of the cantilever strip. The additional information, in form of control points, enables an adjustment of the strip. The effectiveness of the adjustment depends on the density and distribution of the available control points. In the conventional methods, this information is utilized to establish a second or higher degree polynomial as a means of adjustment for the entire strip. The following suggested method, in contrast, is aiming at the adjustment of the individual models in the strip, taking advantage of any available type and amount of information.

Each model in the strip contains seven unknowns, i.e., three rotations, three translations and a scale factor.

The ideal situation is the case in which enough information is available to solve for the seven unknowns of each model in the strip without resorting to various assumptions and approximations. However, there is hardly a need for aerial triangulation in such a case, as the achievement of this goal requires control points at each model. If the horizon data are available, the number of unknowns is reduced to five per model, ($\Delta\Phi$ and $\Delta\Omega$ being obtained in the manner suggested in a previous section), but unless additional auxiliary data and/or ground control points are available, approximations have to be made. When ground control points are available along the strip, an adjustment for λ and κ can be applied in linear segments between the control points. Obviously, this is an approximation whose effect is lessened by the increase of the number of available control points. In this case, if $\Delta\kappa$ of each model is plotted against X (the distance between any two control points) the resulted curve is a straight line expressed mathematically as:

$$\Delta\kappa = ax + b,$$

but as

$$\Delta\kappa = \frac{d\Delta y}{dx},$$

then

$$\Delta y = \int (ax + b)dx = \frac{1}{2a}(ax + b)^2,$$

i.e., the linear change in κ has a parabolic effect on the resulting adjusted Y-coordinate. However, this approach should not be confused with the second-degree polynomial commonly used in photogrammetry as means of a direct adjustment of point coordinates throughout the entire strip. In long strips where ground control points are sparce, the underlying assumption that the error accumulation does resemble the degree of polynomial used, is quite doubtful. On the other hand, the auxiliary data will yield corrections to the errors "as they are, not as they should be."[1] The parameters which are not solved for by the horizon data will require some approximations, as already explained, but the effect of these approximations is limited to the range between the available control points. Consequently, the simultaneous adjustment offered in this method, utilizing both the auxiliary data and any amount of given ground control points, will aim at getting as close as possible to the rigorous solution.

The Adjustment Equations. Considering that all the auxiliary data has already been transformed to the local reference system, that Φ and Ω of every model have been computed from the available horizon data (as explained in the previous second), and that the strip has already been formed, the next step is the adjustment of the strip. The equations to be used for this purpose are of two types:

$$\text{I. } \boldsymbol{C}_i - (\boldsymbol{S} + \lambda \boldsymbol{C}_i^* \boldsymbol{M}) = 0 \tag{19}$$

$$\text{II. } \boldsymbol{S}_n + \lambda_n \boldsymbol{C}_n^* \boldsymbol{M}_n = \boldsymbol{S}_{n+1} + \lambda_{n+1} \boldsymbol{C}_{n+1}^* \boldsymbol{M}_{n+1} \tag{20}$$

where $\boldsymbol{C}$ represents ground coordinates of a point, $\boldsymbol{C}^*$ represents plate coordinates of a point, $\boldsymbol{S}$ are the three translations, λ is a scale factor, and $\boldsymbol{M}$ is the orientation matrix (see Equation 4).

The equation of Type I is applied wherever a ground control point is available. The equation of Type II is applied as a condition which the pass points along the line joining the two adjacent models must fulfill. Equation 19, in full, reads:

$$F_x = X - S_x - \lambda[X^* (\cos\Phi\cos\kappa) + y^* (-\cos\Phi\sin\kappa) + Z^* (\sin\Phi)] = 0,$$

$$F_y = Y - S_y - \lambda[X^*(\cos\Omega\sin\kappa + \sin\Omega\sin\Phi\cos\kappa) + y^*(\cos\Omega\cos\kappa - \sin\Omega\sin\Phi\sin\kappa) + Z^*(-\sin\Omega\cos\Phi)] = 0,$$

and

TABLE 1. THE RESULT OF DIFFERENTIATING EQUATION 21 AND EVALUATING THE FUNCTION AT $\Phi=\Omega=K=0$

$\left[\frac{\partial F_x}{\partial K}\right]_0 = \lambda Y^*,$	$\left[\frac{\partial F_y}{\partial K}\right]_0 = \lambda X^*,$	$\left[\frac{\partial F_z}{\partial K}\right]_0 = 0,$
$\left[\frac{\partial F_x}{\partial \lambda}\right]_0 = -X^*,$	$\left[\frac{\partial F_y}{\partial \lambda}\right]_0 = -y^*,$	$\left[\frac{\partial F_z}{\partial \lambda}\right]_0 = -Z^*,$
$\left[\frac{\partial F_x}{\partial S_x}\right]_0 = -1,$	$\left[\frac{\partial F_y}{\partial S_x}\right]_0 = 0,$	$\left[\frac{\partial F_z}{\partial S_x}\right]_0 = 0,$
$\left[\frac{\partial F_x}{\partial s_y}\right]_0 = 0,$	$\left[\frac{\partial F_y}{\partial S_y}\right]_0 = -1,$	$\left[\frac{\partial F_z}{\partial S_y}\right]_0 = 0,$
$\left[\frac{\partial F_x}{\partial S_z}\right]_0 = 0,$	$\left[\frac{\partial F_y}{\partial S_z}\right]_0 = 0,$	$\left[\frac{\partial F_z}{\partial S_z}\right]_0 = -1,$
$F_{x_0} = X - S_x - \lambda X^*,$	$F_{y_0} = Y - S_y - \lambda Y^*,$	$F_{z_0} = Z - S_z - \lambda Z^*.$

$$
\begin{aligned}
F_z = Z - S_z - \lambda[&X^*(\sin\Omega\sin\kappa - \cos\Omega\sin\Phi\cos\kappa)\\
&+ y^*(\sin\Omega\cos\kappa + \cos\Omega\sin\Phi\sin\kappa)\\
&+ Z^*(\cos\Omega\cos\Phi)] = 0.
\end{aligned}
\tag{21}
$$

Differentiating Equation 21 with respect to the unknowns, and evaluating the function at $\Phi=\Omega=\kappa=0$, the expressions shown in Table 1 are valid.

Thus the linearized (Taylor series) set of Equation 21 is:

$$
\begin{aligned}
\lambda Y^*\Delta\kappa - X^*\Delta\lambda - \Delta S_x + F_{x_0} &= 0,\\
-\lambda X\Delta\kappa - Y\Delta\lambda - \Delta S_y + F_{y_0} &= 0,\\
-Z^*\Delta\lambda - \Delta S_z + F_z &= 0.
\end{aligned}
\tag{22}
$$

The corresponding set of linear equations deduced from Equation 20 is:

$$
\begin{aligned}
&-\lambda Y^*\Delta\kappa + X^*\Delta\lambda + \Delta S_x)_n\\
&\quad - (-\lambda Y^*\Delta\kappa + X^*\Delta\lambda + \Delta S_x)_{n+1}\\
&\quad + [(S_x + \lambda X^*)_n - (S_x + \lambda X^*)_{n+1}] = 0,\\
&(-\lambda Y^*\Delta\kappa + X^*\Delta\lambda + \Delta S_y)_n\\
&\quad - (-\lambda Y^*\Delta\kappa + X^*\Delta\lambda + \Delta S_y)_{n+1}\\
&\quad + [(S_y + \lambda X^*)_n - (S_y + \lambda Y^*)_{n+1}] = 0,\\
&(Z^*\Delta\lambda + \Delta S_z)_n - (Z^*\Delta\lambda + \Delta S_z)_{n+1}\\
&\quad + [(S_z + \lambda Z^*)_n - (S_z + \lambda Z^*)_{n+1}] = 0.
\end{aligned}
\tag{23}
$$

The adjustment program calls for starting each iteration with the initial first approximations assigned to the unknowns. In the present case, the initial values of the unknowns are:

$$
\begin{aligned}
\kappa &= 0\\
\lambda &= 1, \text{ and}\\
S_x &= S_y = S_z = 0.
\end{aligned}
$$

Accordingly, the known terms of Equation 22 become:

$$
\begin{aligned}
F_{x_0} &= X - X^*\\
F_{y_0} &= Y - Y^*\\
F_{z_0} &= Z - Z^*,
\end{aligned}
\tag{24}
$$

and those of Equations 23 become:

$$
\begin{aligned}
G_{x_0} &= X_n^* - X_{n+1}^*\\
G_{y_0} &= Y_n^* - Y_{n+1}^*\\
G_{z_0} &= Z_n^* - Z_{n+1}^*.
\end{aligned}
\tag{25}
$$

POINTS INVOLVED IN THE ADJUSTMENT AND THEIR FUNCTION

The measured XYZ-coordinates of the pass points (u, m, l in Figure 4) in the two adjacent models enable the linking of the models to each other by imposing the condition that their coordinates in model n be the same as in model $(n+1)$. However, the Z-coordinates of pass points u and l lead to the determination of a certain value of Ω, which might disagree with the value of Ω deduced from the horizon camera. Point m, located on the X-axis about which Ω rotates, will not contribute to the above problem and hence its XYZ-coordinates can be fully utilized in the adjustment. As for the points u, l, their XY-coordinates can be utilized whereas their Z-coordinates should not. In this way, the model can freely rotate by the amount imposed by the horizon-determined Φ and Ω without causing unbearable constrains on the adjustment. For

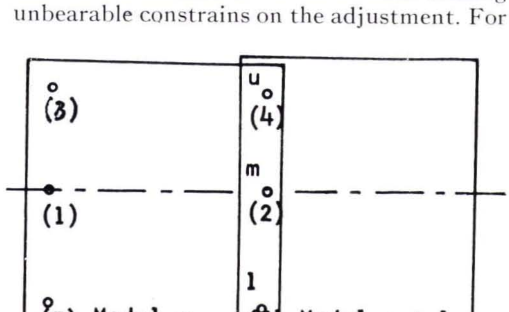

FIG. 4. Classification of points in the model.

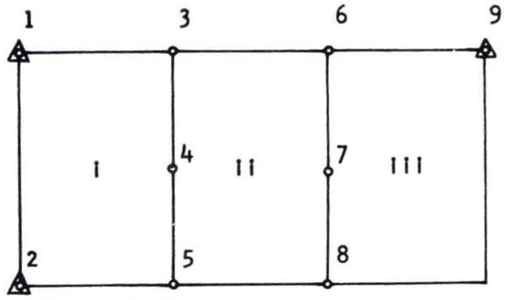

FIG. 5. A three-model strip.

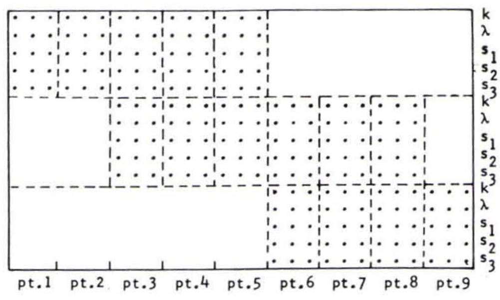

FIG. 6. The $\mathbf{A}^{\mathrm{T}}$ matrix of a three-model strip.

this reason, the third equation in the set of Equations 23 is multiplied by a numerically small factor whenever it is applied to a pass point. This factor can be looked upon as a reduced weight.

In summary, all points in the strip can be divided into four groups: (1) control points, (2) points m, (3) pass points l and u, and (4) all other points whose coordinates are required. Only groups 1, 2, and 3 are involved in the adjustment phase; groups 1 and 2 are fully involved whereas group 3 entails the exception of the Z-coordinates. The strip coordinates of points belonging to the fourth group will be transformed into the ground coordinate system using the parameters resulted from the adjustment.

THE STRIP ADJUSTMENT COMPUTER PROGRAM

The present program, as already stated, has as its purpose the adjustment of the individual models in the strip, taking advantage of any available type and amount of information. The least-squares solution is applied simultaneously to all models in the strip and the parameters of each individual model are then deduced from the simultaneous solution.

The transformed matrix of the observation equation Coefficients A^{T}, as deduced from a

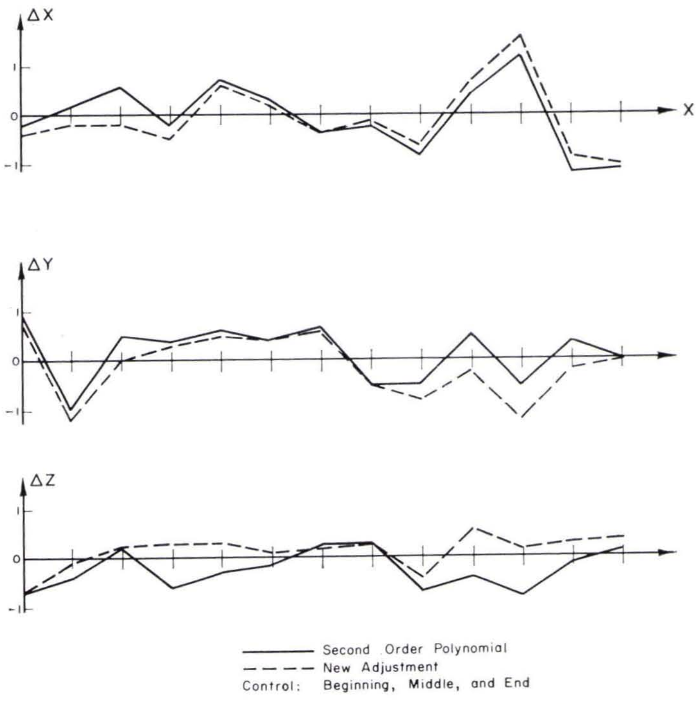

FIG. 7. Comparison of discrepancies resulting from different adjustment procedures.

three-models' strip containing three control points (shown by triangles in Figure 5) and six pass points (shown by circles), is represented in Figure 6 for clarity purposes.

The coefficients associated with control points are taken from Equation 22 and those associated with pass points, from Equation 23. It should be emphasized that in the present procedure each iteration shifts the coordinates of the points to a new position which will serve as the initial position for the next iteration. Thus every iteration starts with the initial first approximation assigned to the parameters, and ends with the resulting increments or partial values of these parameters. These values are used in the partial transformation of the coordinates from the previous to the new position. The final values of the orientation parameters, at the end of the iteration process, are deduced from the increments resulting at each iteration.

When the iteration process is terminated and the final orientation parameters of every model in the strip, are obtained, the points belonging to group number 4 of the foregoing Section are read and their coordinates are transformed to the ground system, utilizing the orientation matrix of the model in which the corresponding points are located.

EXPERIMENTS AND CONCLUSIONS

Experiments have been conducted with the ficticious data prepared by the International Society of Photogrammetry, Commission III. Photographs number 2 to 14 of strip No. 1 (12 models) were linked and adjusted using the method suggested here as well as by a second degree polynomial. The control points used in the adjustment were 5–1, 5–3, 5–5, 17–1, 17–3, 17–5, 29–1, 29–3, and 29–5, located at the beginning, middle and end of the strip. The discrepancies (adjusted-ground control values) along the center of the strip, resulted from each of the methods used, are plotted in Figure 7. The close proximity of the results is evident; however, it should be kept in mind that the strip under investigation is rather short and thus cannot demonstrate the full advantage of the suggested adjustment procedure. The effectiveness of the new program, utilizing the auxiliary data, should be more pronounced for long strips where the assumption that the error accumulation resembles any particular polynomial is doubtful 1.

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