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Automatic Data Verification

Quality control is necessary to eliminate blunders and accidental errors, and to maintain high reliability.

(*Abstract on next page*)

INTRODUCTION

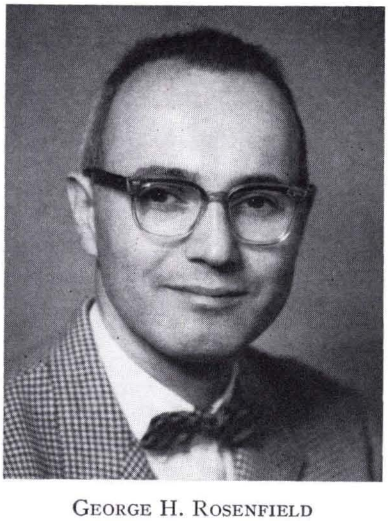
DEFINITION AND PURPOSE

DATA VERIFICATION is the detection of bad data (i.e., blunders and accidental errors, as opposed to random and systematic errors) among observations or measurements employed in analytical aerial photogrammetric data reduction operations. Blunders and accidental errors may enter the system at any stage of the data acquisition and reduction. The blunders prevent the analytical solution from converging, and accidental errors prevent the solution from converging to the most probable answer. In most cases, the blunders and accidental errors exist in the input data. Techniques of data verification (blunder elimination and data editing) will result in more nearly correct results, obtained in less time, and with greater savings.

The data verification system described in this paper was developed by Autometric/Raytheon under contract to U.S. Army Engineer Geodesy, Intelligence, Mapping Research and Development Agency (GIMRADA)†, with the purpose of editing data prior to entry into a photogrammetric data adjustment program. The data verification system is being incorporated into an automatic data reduction system which will include a triangulation data preprocessing section, a blunder elimination section, a data edit section, and the MUSAT aerial triangulation adjustment. It is presumed that this automatic system will become the operational tool for data verification and aerial triangulation at the Army Map Service. It is also presumed that all aerial triangulation data will first be processed for blunder elimination and then for data editing. The automatic svstem will soon be available for engineering test at Army Map Service.

QUALITY CONTROL

Quality control of any production operation is necessary to eliminate blunders and accidental errors, and to maintain a high reliability in a given product. Design of a quality control system must be directed to a particular operation, and requires a complete knowledge of the entire system. A comprehensive quality control system is necessary to achieve success in analytical photogrammetry operations due to the high degree of geometrical quality which can be achieved. Analytical techniques allow state-of-the-art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† More recently renamed U.S. Army Engineer Topographic Laboratories (ETL).

corrections for all known sources of error. Establishment and maintenance of an efficient quality control system for a photogrammetric operation must be based upon a complete knowledge of the fundamental problems in all phases of the data experiment, including photographic materials and processing, data acquisition, instrumentation, data translation, and data reduction. Commission II of the International Society of Photogrammetry has been studying these fundamental problems (Hallert 1961, 1964). Two domestic organizations which have already achieved success in analytical photogrammetry operations are the U.S. Coast and Geodetic Survey, and the U.S. Air Force Eastern Test Range. Aspects of quality control measures instituted at these two organizations have been published by Harris (1962) and Rosenfield (1964). The accidental errors and blunders are caused by such things as incorrect manipulation of instruments, instrument malfunctions, error in numerical computations, errors in recordings, and the like.

ABSTRACT: *This paper describes a system for verification of data prior to entry into a general analytical photogrammetric block adjustment program. The concept of verification of data consists of two parts: (1) blunder elimination, and (2) data editing. Blunder elimination refers to those accidental errors which are of such large magnitude as to prevent the solution from converging. Data editing concerns elimination of those accidental errors which do not conform to probability theory, and which prevent the solution from converging to the most probable answer.*

ERROR ANALYSIS

TYPES OF ERRORS

Fairly standard definitions have been developed for the various types of errors encountered in the measuring sciences. These definitions have been clearly explained for the fields of photogrammetry by Hallert (1960) and of geodesy by Bomford (1962). The following definitions are composites from the two references:

a. Blunders (also called large errors or lapses). Blunders are generally due to mistakes or carelessness. They may be detected by repetition or by simple external checks.

b. Systematic, constant, and periodic errors. Systematic errors are those whose occurrence, size, and direction are regulated by a certain functionally expressable law. Constant errors are the simplest case where all measurements are afflicted with errors of equal size and direction. Periodic errors are a special case of systematic error.

c. Random or accidental errors. Random or accidental errors are usually small and change their size and direction according to the law of normal distribution.

However, for specific applications it sometimes becomes necessary to redefine certain of the terms. In the considerations of applied statistics, the types of errors have been defined by Yoshitu (1959) and Mann (1963). For use with adjustment techniques, the definitions for accidental errors and blunders have been reexamined.

Random Errors. Random errors are those which can be treated by the methods of probability theory.

Accidental Errors. Accidental errors are those which cannot be treated by probability theory. Due to their gross nature, they may be detected and removed by editing of the data.

Blunders. Blunders are those particular accidental errors that are of such large magnitude as to cause the adjustment of the data to fail to converge.

Systematic Errors. Systematic errors are those whose occurence, size, and direction are regulated by a certain functionally expressable law. In general, a cause can be assigned to these errors and they can therefore be removed by calibration.

An inherent characteristic of statistical data is that an observation deviates from its true value or mathematical expectation. The measure of this deviation is the total error. In the absence of accidental errors and blunders, the total error in the observation can be partitioned into random and systematic components. Under the realistic assumption that the two parts are statistically independent, the total error variance is given by:

$$\sigma_T^2 = \sigma_R^2 + S^2,$$

where σ_T^2 stands for the total error variance, and σ_R^2 and S^2 for the contribution to the total error variance due to the random and systematic errors, respectively.

Quite often the systematic component may be described mathematically in the form of a regression equation. According to Hald (1955) formation of the mathematical-statistical model is a technological rather than a statistical task. In some instances, adequate theoretical knowledge is lacking and an empirical description must be developed. However, it must be stressed that in the long run it is the theoretical description based on professional knowledge which must be developed. Correction for systematic error must be performed by either precalibration or by error-model considerations. Location and correction of accidental errors which lie outside the probability theory of the random errors is based on the system being free of systematic error.

EXISTING DATA EDITING TECHNIQUES

It is the purpose of data editing to detect and find the magnitude of the *accidental* error, locate the erroneous observation, and make the necessary correction, thus leaving the good statistical data. The work of the U.S. Coast and Geodetic Survey in the field of analytical photogrammetry has been comprehensively documented (see Bibliography). All of the articles stress aspects of both quality control and of data editing. The present data editing technique of the C&GS has been developed about the concept of segmented data reduction. The first phase is a three-photo relative orientation conducted in a moving arc graduation process. At the third rejection in a given triplet, the strip is automatically stopped, and the system moves to the next strip. The second phase is the separate strip adjustment.

The method for editing missile data at the Eastern Test Range was developed by Yoshitsu (1959), and is based on the underlying mathematical model of the missile flight trajectory. The method considers the data as a nonstationary time series and utilizes finite differences and the variate difference technique over given spans of data. The rejection criterion is a multiple of the computed standard deviation. Erroneous data are automatically located, rejected, replaced, and flagged, within the computer.

BASIC ASSUMPTIONS

It is evident from the preceding section on Existing Data Editing Techniques *that accidental errors and blunders are expected to occur only very infrequently*. The Coast Survey, for example, will stop the computer run for a given strip upon determination of the third accidental error in a given triplet. Yoshitsu at the Eastern Test Range has made the statement that his data editing procedure works well if the accidental errors are reasonably scattered.

Operational use of the technique has indicated this to be true. It follows that when a large number of accidental errors exist in a system, that they are no longer accidental—but that the data is of poor quality. This causes the variance to be inflated, and further editing will result in a raw data smoothing rather than editing. *It is also evident that the relatively few accidental errors are considered to occur in a large quantity of data.* The C&GS does not expect to locate more than two bad points in a triplet which may contain at least 12 data points. Flight data editing at the ETR will locate up to 5 consecutive bad points in a span of 61 data points. Data editing will not take place for less than 13 and preferably less than 25 data points in a span. In addition, it is considered to be operationally poor data if more than 6 points are edited in a given flight coverage. Finally, *an underlying mathematical model must be considered for adequate data editing.* The C&GS model is the stereo triplet, for which simultaneous relative orientation is performed. The ETR mathematical model expresses the trend of the missile powered flight trajectory as a random series at the fourth finite difference level.

BLUNDER ELIMINATION

Blunders, which may be defined as those accidental errors which cause the adjustment to fail to converge, may be detected and eliminated by relatively simple external checks. Blunders may occur at any stage of the data acquisition and reduction process; however, they may be kept to a minimum by adequate quality control techniques throughout the data experiment. Final investigation for determining and eliminating blunders must take place within the internal structure of the computer and must consider all aspects of the data problem. Those areas of interest in which blunders may occur are: the air station position and attitude parameters; the identification and measurement of the image points; and the ground control data. As indicated above, blunders may be detected by analysis of the data with respect to the underlying trend of the mathematical model. The underlying model selected is that for a system of analytical aerial triangulation.

AIR STATION PARAMETERS

The air station position and attitude parameters are input into the computer in terms of geographic position and local attitude angles together with estimates of their standard errors:

$$\phi, \quad \lambda, \quad h, \quad T_X, \quad T_Y, \quad H,$$

$$\sigma_\phi, \quad \sigma_\lambda, \quad \sigma_h, \quad \sigma_{T_X}, \quad \sigma_{T_Y}, \quad \sigma_H.$$

The basic consideration for blunder elimination from the air station parameters is that the values of the parameters represent a time series of data obtained along a single strip of aerial triangulation. Each flight strip is a separate entity, the blunder elimination procedure being reinitiated for each strip. Extensive breaks in the strip cannot be accomodated, but must be considered as if a new strip were being investigated.

The following analysis, a modification of the technique developed by Grubbs (1950), will eliminate blunders in the time series of the air station position and attitude data: the time series of air station position values in the geographic coordinate system, and of the respective attitude values in the local coordinate system are input to the computer. Each individual set of time series data for the length of a particular strip are collected in series. A linear polynomial in the form:

$$\mu = a_0 + a_1 t$$

is fit by a moving arc, (shift one), technique over six points to the time series data by a standard least squares adjustment technique. The discrepancies from the fit are calculated for the *i*th data point by:

$$\delta_i = \mu_i - x_i,$$

in which u_i is the computed value, and x_i is the input value. The mean of the discrepancies, and the sum of the squares of the deviation from the mean of the discrepancies are computed. The largest discrepancy of the group is then determined. A new polynomial fit is made to the remaining data points after removing that with the largest discrepancy. A new set of discrepancies is determined, and the mean and sum of the squares of this second set are computed. The following statistic is then computed:

$$\frac{S_n^2}{S^2} = \frac{\sum_{i=1}^{n-1} (\delta_i - \bar{\delta}_n)^2}{\sum_{i=1}^{n} (\delta_i - \bar{\delta})^2},$$

in which

$$\bar{\delta}_n = \frac{1}{n-1} \sum_{i=1}^{n-1} \delta_i, \qquad \bar{\delta} = \frac{1}{n} \sum_{i=1}^{n} \delta_i,$$

where S^2 represents the set of readings, and $S_n{}^2$ represents the set of readings less the value to be tested. The statistic $S_n{}^2/S^2$ is compared with the value 0.05. The tested data point is accepted if:

$$S_n^2/S^2 \geq 0.05,$$

and is rejected if:

$$S_n^2/S^2 < 0.05.$$

The test value 0.05 represents the fact that more than 99 percent of the error represented by the discrepancies is caused by the rejected data point.

If the data point has been rejected, a replacement data point is computed to fit on the polynomial determined from the five remaining data points.

If two consecutive bad points are discovered, the analysis assumes that a discontinuity in the time series has occurred. The two points are accepted and a new strip is initiated at the first of these points. Upon initiation of this new strip, if three consecutive bad points are discovered, the evaluation of the entire original strip for that coordinate is terminated and an error message is printed. The program then continues with the next following strip.

PASS POINT IMAGES

All data points imaged on the plates are first considered to be pass points, regardless of whether or not they might also be images of control points. As the points must, of course, be imaged on the plate, the first inspection is for this source of error. The coordinate of any image which exceeds the plate limits is to be rejected. Remaining pass points are to be tested for blunder elimination by the manner outlined below:

Two types of equations are required for consideration of pass points. These are the pass point equations and the differential scale restraint equations of the MUSAT program (GIMRADA 1965, Elassal 1966).

The *k*th pass point equation from the conjugate images of the ith pass point from the *j*th and $(j+1)$th camera stations has the linear form:

$$q_{ik} = \bar{B}_j \cdot (\bar{A}_{j+1}{}^0 \times \bar{A}_j{}^0),$$

and the *l*th differential scale restraint equation from the conjugate image of the *i*th pass point from the *j*th, $(j+1)$th, $(j+2)$th camera stations has the linear form:

$$\Delta d a_{il}^{j+1} = -(\bar{B}_j \cdot \bar{Q}_j) + (\bar{B}_{j+1} \cdot \bar{Q}_{j+1}),$$

in which

$\bar{A}^0{}_j$ represents the vector from the *j*th air station to the target,

$\bar{B}_j$ represents the vector from the $(j+1)$th air station to the *j*th air station,

$\overline{Q}_j$ represents the normalized vector at the target, perpendicular to the air station-target plane.

For each *j*th air station and *i*th pass point, the following data are given—values for the air station position and attitude parameters in the coordinate system for the adjustment, together with estimates of their standard errors:

$$X_j,\ Y_j,\ Z_j,\ T_{X_j},\ T_{Y\cdot},\ H_j,\ \sigma_{x\cdot},\ \sigma_{y_j},\ \sigma_{z_j},\ \sigma T_{X_j},\ \sigma T_{Y_j},\ \sigma_{H_j};$$

and values for the plate coordinates of the pass point images together with estimates of their standard errors:

$$x_i,\ y_i, \qquad \sigma_{x\cdot},\ \sigma_{y_i}.$$

From the *k*th pass point equation and the *l*th differential scale restraint equation the following constants are computed:

$$\epsilon_{ik} = q_{ik},$$
$$\epsilon_{il} = \Delta d\overset{i+1}{a_{il}},$$

together with estimates of their variances:

$$\overset{2}{s}_{\epsilon_{ik}} = A_{ik}\ \overset{0}{\Sigma}_{ik}\ \overset{T}{A}_{ik},$$
$$\overset{2}{s}_{\epsilon_{il}} = A_{il}\ \overset{0}{\Sigma}_{il}\ \overset{T}{A}_{il},$$

in which Σ^0_{ik}, Σ^0_{il} represent the covariance matrices of the image coordinates, and air station position and attitude coordinates, for the two images of the pass point equations, and for the three images of the scale restraint equations, respectively, and A_{ik}, A_{il} represent the matrices of partial derivatives of the pass point and scale restraint error equations, respectively, with respect to the image coordinate observations, and to the air station position and attitude parameters.

The constant representing a pair of conjugate pass point images is compared with the value of its respective propagated standard error. If the value of the constant exceeds a given multiple of the standard error, the pair of image points is flagged as being in possible error. Additional pass point equations of different conjugate pairs of the same pass point are also inspected and, if necessary, flagged. The same is done for triplets of pass point images using the constants from the scale restraint equations. Erroneous pass point images are identified by inspection of the flagged data, and are eliminated from the problem. The multiplier value for the rejection criteria is under operator control.

GROUND CONTROL POINTS

Each set of ground control point coordinates is examined with respect to each corresponding image point. That is, a ground control point which is imaged on a plate will be tested against that plate image. Since the image points are already consistent within themselves, the ground point will be rejected if it does not favorably compare with any image point. Logically, it will be rejected at the first image point. For each *i*th ground point, the following data are given—values of the ground point coordinates in the system for the data reduction, together with estimates of their standard errors:

$$X_i,\quad Y_i,\quad Z_i,\quad \sigma X_i,\quad \sigma Y_i,\quad \sigma Z\cdot.$$

The equation for consideration of the ground control points is the complete ground control point equation of the MUSAT program (GIMRADA 1965). Inspection of the ground control point data is performed in a manner similar to that for the pass point data. The constants from a pair of complete ground control point equations for the *i*th ground control point of the *j*th camera station have the linear form:

$$\epsilon_{ij} = \overline{D}\cdot\overline{A}^{0}_{02}, \qquad \tilde{\epsilon}_{ij} = \overline{D}\cdot\overline{A}^{0}_{01}$$

in which D represents the vector from the air station to the ground control point, $\bar{A}^o{}_{01}$, $\bar{A}^o{}_{02}$ represent synthetic vectors emanating from the ground control point. Their estimated variances are:

$$\overset{2}{s}_{\epsilon ij} = A_{ij}\ \overset{0}{\Sigma}_{ij}\ \overset{T}{A}_{ij},$$
$$\overset{2}{s}_{\tilde{\epsilon}_{ij}} = \tilde{A}_{ij}\ \overset{0}{\Sigma}_{ij}\ \overset{T}{\tilde{A}}_{ij},$$

in which Σ^0_{ij} represents the covariance matrix of the image coordinates, ground control point coordinates, and air station position and attitude coordinates for the *i*th image of the *j*th camera, A_{ij}, $\tilde{A}_{ij}$ represent the matrices of partial derivatives of the pair of ground control point error equations respectively, with respect to the image coordinates, ground control point coordinates, and air station position and attitude coordinates.

The constants representing the pair of ground control point images are compared with their respective propagated standard errors. If the constants exceed a given multiple of the standard error, the ground control point data for that particular image is eliminated from the problem. The multiplier value for the rejection criterion is under operator control.

DATA EDITING

An early decision in the plans for data editing selected the Creusen sequential algorithm (Creusen, 1966) in order to determine its application to analytical photogrammetry problems. The decision was also made to use portions of the already available MUSAT program (GIMRADA, 1965) which is based on the coplanarity equations, in order to save time in development of the data edit program. The MUSAT program simultaneously applies the pass point equation, the differential scale restraint equation, and the ground control point equation to the data adjustment problem.

CREUSEN SEQUENTIAL ALGORITHM

The Creusen sequential algorithm, (Creusen, 1966), operates sequentially on one equation at a time. Each equation is brought into the system without the necessity of repeating the preceding adjustment in order to incorporate the new information. Within the concept, the ground position coordinates are considered as parameters in the pass point equations, and as observations in the ground control point equations. Within the algorithm, a discrepancy for each equation is computed. This discrepancy represents failure to meet the condition represented by the equation. For the coplanarity equation, the discrepancy represents failure of the conjugate images to lie on a plane with the pass point.

The data editing procedure prepared by Autometric utilizes the Creusen sequential algorithm for solution of the condition equations. Operation of the algorithm in the Data Edit Program may be summarized as follows.

The linearized observation equation has the matrix form:

$$BX = 0,$$

in which

$$\underset{(1\times n+1)}{B} = [b^1b^2 \cdots b^n-1],$$

$$\underset{(n+1\times 1)}{X} = [x^1x^2 \cdots x^nd]^T,$$

where d represents the observation equation evaluated at the approximations, and

$$b = \frac{\partial d}{\partial p}, \qquad p = \text{parameters}.$$

Improved values for approximations to the vector of parameters X and to the covariance matrix of the parameters Q_{XX} are given by:

$$X = X^0 - \delta X,$$

$$Q_{XX} = Q^0{}_{XX} - \delta Q_{XX},$$

in which X^o, $Q^o{}_{XX}$ represent the approximations, and where

$$\delta X = Q^0{}_{XX}B^T(BQ^0{}_{XX}B^T)^{-1}BX^0,$$

$$\delta Q_{XX} = Q^0{}_{XX}B^T(BQ^0{}_{XX}B^T)^{-1}(Q^0{}_{XX}B^T)^T$$

where

$$\underset{(n+1\times n+1)}{Q_{XX}} = \begin{bmatrix} \underset{(1\times1)}{\sigma^2{}_{p_1}} & & & & \\ & \sigma^2{}_{p_2} & & & \\ & & \ddots & & \\ & & & \sigma^2{}_{p_n} & \\ & & & & \underset{(1\times1)}{Q_{LL}} \end{bmatrix},$$

(note: Q_{XX} may or may not be diagonal), in which

Q_{LL} represents the transformed covariance matrix of the observations, performed in the manner:

$$\underset{(1\times1)}{Q_{LL}} \quad \underset{(1\times u)}{A} \quad \underset{(u\times u)}{\Sigma} \quad \underset{(u\times1)}{A^T}$$

in which Σ represents the covariance matrix of the observations, and

$$A = \frac{\partial d}{\partial u}, \qquad u = \text{observations}.$$

W represents a discrepancy from the condition:

$$W = BX^0.$$

DATA EDIT PROGRAM

The data edit program inspects each pass point image on a given frame to determine if it appears on at least one preceding frame. If it does, the pass point equation is generated and entered into the solution. After all images on that frame have been evaluated for the pass point condition, each is inspected again to determine whether it appears on at least two previous frames. If it does, the scale restraint equation is generated and entered into the solution. Each pass point and scale restraint equation is applied independently to the conjugate images of the pass point. A single pass point equation is formed for each pair of images, and a single scale restraint equation is formed for each triplet of images. These single equations are entered one at a time into the solution.

After processing of all pass points on the frames which are present in the computer memory, the strip is inspected for ground control. The conjugate images of a tested pass point are used with only the first two frames on which they appear in order to generate the ground control point equation. These equations are individually entered into the solution. Three ground control points (not on a straight line) are required for the frames within the computer memory in order to orient the strip absolutely. If three ground control points are not found, editing of all ground control data cannot be performed. For this case of sparse ground control, the standard edit procedure in the data reduction adjustment system (based on the weighted residuals) will easily edit the ground control data.

The program will accomodate the data of only seven photographs at a time within the computer memory. Upon processing of these seven photographs, the first photograph and its associated data (including ground control) will be removed from core, and the eighth photograph and its associated data (including ground control) will be read in. This system will continue until all frames and their data have been processed.

Data are rejected within the basis of the discrepancy which is computed for each equation. If a weighted discrepancy is found which exceeds the rejection criterion, the data leading to the equation are rejected. The rejection criterion is based on the standard deviation of all weighted discrepancies determined up to that point in the solution. In this manner, the solution is allowed to find its own statistical limits. All of the discrepancies are computed using the same set of approximations to the parameters and covariance matrix. Thus all discrepancies belong to the same statistical population.

DATA REJECTION CRITERION

The data rejection criterion is selected to edit data in such a manner that there is γ percent confidence that at least α percent of the individual values are within acceptable tolerance limits. Any data lying outside of these limits are to be rejected. The limits of rejection are herein referred to as the data rejection criterion.

If the data to be edited are assumed to be composed of random variables from the same statistical population with estimate of the mean as zero, and estimate of the standard error as σ_o, the rejection criterion c then has the value:

$$c = p\sigma_0,$$

where σ_o represents the *aposteriori* value of the standard error of unit weight estimated from the least squares adjustment procedure, and p represents a constant multiplier. When the mean μ and the standard deviation σ of the population is absolutely known, it is correct to say that 99.73 percent of the individual values will lie within $\pm 3\sigma$. However, the p-factor in the program is under operator control for a given test run, and any value desired may be chosen.

The Creusen sequential algorithm used in the Data Edit program operates individually on single equations. Each equation expresses a physical condition relating the data from which it is formed. The Creusen algorithm directly computes a discrepancy for each equation which represents the failure of that condition to be fulfilled. The data to be inspected by the edit program are represented by the set of discrepancies from all equations used in the adjustment to that point.

Assuming that the discrepancies are unbiased and uncorrelated, the standard error of unit weight for all the discrepancies in the sample is estimated by the equation:

$$(\sigma_0)_u = [E_u/f_u]^{1/2},$$

in which

$$E = \sum_{i=1}^{n} [(W^2{}_u)_i/(Q_{WW})_i],$$

where W_{ui} represents the discrepancy computed in turn from the pass point, scale restraint, and ground control point equations for the ith equation, in the manner:

$$\boldsymbol{W} = \boldsymbol{B}\boldsymbol{X}^0,$$

and $(Q_{WW})_i$ represents the propagated estimate of the variance for that discrepancy computed in the manner:

$$\boldsymbol{Q}_{WW} = \boldsymbol{B}\boldsymbol{Q}_{XX}\boldsymbol{B}^T,$$

in which $\boldsymbol{B}$ represents the vector of partial derivatives of the error equations with respect to the parameters, $\boldsymbol{Q}_{XX}$ represents the covariance matrix of the parameters.

f_u represents the degrees of freedom for the separate systems of pass point, scale restraint, or ground control point equations, and is computed in the following manner:

For the pass point equations,
$f = k(n-5)$
in which n is the number of conjugate image pairs per model, $\boldsymbol{k}$ is the number of models, $\boldsymbol{k} = j\text{-}1$, and j is the number of photographs.

For the scale restraint equations
$f = n - k + 1$
in which n is the number of conjugate triplets, k is the number of models.

For the ground control point equations:

$$f = j(2n) - 6$$

in which n is the number of ground control point images per photograph, j is the number of photographs.

These equations for determining degrees of freedom are specialized for the case where all models contain the same number of image points. However, they can be applied to the general case by the following modifications: For each model, let the value of k be the number 1. Furthermore, let the number of degrees of freedom for each model be added to the total accumulation of degrees of freedom for all the preceding models. In this manner, the cumulated value will be correct. For the ground control, let the value of j be the number 1 and proceed as above.

The weighted discrepancy representing a given equation is determined in the manner:

$$w_{W_u} = [(W_u)_i/Q_{WW})_i]^{1/2},$$

and is compared with the rejection criterion established up to that point in the editing process. If the value of the weighted discrepancy exceeds the rejection criterion, editing is performed. Editing of the data in the sequential manner does not take place until adequate degrees of freedom are available to establish confidence in the validity of the rejection. Adequate degrees of freedom are presently considered to be available upon completed processing of:

The first model for the pass point equations.
The first triplet for the scale restraint equations.
The first seven frames for the ground control equations.

However, the first cumulation of data before reaching adequate degrees of freedom must also be edited. Upon first reaching the desired number of degrees of freedom, all of the weighted discrepancies for the first accumulation of data are compared with the rejection criterion. If a data point is rejected in this operation, the next data point is brought in to reestablish the desired number of degrees of freedom, and the test is repeated for the entire initial set of weighted discrepancies.

In addition, the program will also allow editing to be performed for the minimum degrees of freedom being equal to one. For example a complete model with only 6 pass points yields one degree of freedom; and editing will be performed for this model. However, the analyst is cautioned on the amount of confidence to be placed on this type of editing.

EPILOG

Knowledge gained during development and test of this data verification system has indicated that a more elegant mathematical model and a different solution algorithm might result in a more efficient data reduction operation, although the system developed is both valid and adequate for its purpose. The revised analysis would use a sequential algorithm other than the Creusen sequential algorithm and a mathematical model based on relative orientation for the pass points, and on the collinearity condition equations for the ground control points, instead of on the coplanarity condition equations of the MUSAT program.

REFERENCES

1. Bomford G., *Geodesy*, Oxford, Clarendon Press, 2nd Edition 1962, pg. 507.
2. Creusen, M. J., A Sequential Procedure to Solve Linear Systems, paper presented to annual meeting of American Society of Photogrammetry, March 1966, Washington, D.C.
3. Elassal, A. A., Simultaneous Multiple Station Analytical Triangulation Program, *Photogrammetria*, Volume 21, Number 3, June 1966, 83–95.
4. GIMRADA, Multiple Camera Analytical Triangulation Program, U. S. Army Engineer Geodesy, Intelligence and Mapping Research and Development Agency, Fort Belvoir, Va. December 1965.
5. Grubbs, F. E., Sample Criteria for Testing outlying Observations, *Annals of Mathematical Statistics*, Vol. 21, 1950, 27–58.
6. Hald, A., *Statistical Theory with Engineering Applications*, John Wiley and Sons, New York 1955, pg. 758.
7. Hallert, B. P., The Working Group of Fundamental Problems, *Photogrammetria*, 18/1, 1961–62, 1–5.
8. Hallert, B. P., Fundamental Problems in Photogrammetry, *International Archives of Photgrammetry*, Commission II, 1964.
9. Hallert, B. P., *Photogrammetry*, McGraw Hill Book Co., Inc., 1960, pg. 261.
10. Harris, W. D. et al, Analytical Aerotriangulation in the Coast and Geodetic Survey, PHOTOGRAMMETRIC ENGINEERING, 28/1, 1962, 44–69.
11. Mann, H. P., The Accuracy of AMR Instrumentation, Technical Documentary Report No. MTC-TDR-64-1, Patrick Air Force Base, Florida, 13 December 1963.
12. Rosenfield, G. H., Panel—The Ballistic Camera Accuracy Review Project, PHOTOGRAMMETRIC ENGINEERING, 30/2, 1964, 307–311.
13. Yoshitsu, M., The Editing of Flight Data, *RCA Data Processing Technical Report* No. 50, 3 August 1959.

BIBLIOGRAPHY

1. C&GS Satellite Triangulation in the C&GS, *C&GS Technical Bulletin* No. 24, February 1965.
2. Davis, R. G., Automatic Editing of Data in Large Simultaneous Photogrammetric Adjustments, *Proceedings 1967 Symposium on Computational Photogrammetry*, December 1967, Paper for Wednesday, December 6.
3. Doyle, F. J., *Analytical Photogrammetry*, Chapter X for Manual of Photogrammetry, 3rd Edition.
4. Elassal, A. A., The Use of Modular Computer Programs to Edit Data for Very Large Simultaneous Photogrammetric Solutions, paper presented at October 1967 ASP meeting.
5. Harris, Wm. D., Tewinkel, G. C., Whitten, A., Analytic Aerotriangulation in the Coast and Geodetic Survey, *Photogrammetric Engineering*, 28/1, March 1962, pp. 44–69.
6. Harris, W. D., Report on Analytic Aerotriangulation, *Photogrammetric Engineering*, 28/3, July 1962, pp. 403–410.
7. Harris, W. D. et al Analytical Triangulation, *C&GS Technical Bulletin* No. 21, July 1962.
8. Keller, M., Tewinkel, G. C., Aerotriangulation Strip Adjustment, *C&GS Technical Bulletin* No. 23, August 1964.
9. Keller, M., Documented Computer Program, *Photogrammetric Engineering*, 31/5, September 1965, 892–896.
10. Keller, M., Tewinkel, G. C., Block Analytic Aerotriangulation, *ESSA Technical Report*, C&GS 35, November 1967.
11. Keller, M., C&GS Block Method of Analytic Aerotriangulation, *Proceedings 1967 Symposium on Computational Photogrammetry*, December 1967, Paper for Wednesday, December 6.
12. McKenzie, M. L., Eller, R. C., Computational

Methods in the USGS, *Photogrammetric Engineering*, 31/5, September 1965, pp. 884–892.

13. Mikhail, E. M., Modification of a Least Squares Solution due to Data Rejection, *Proceedings 1967 Symposium on Computational Photogrammetry*, December 1967, Paper for Friday, December 8
14. Rosenfield, G. H., Autometric Data Verification, *Proceedings 1967 Symposium on Computational Photogrammetry*, December 1967 Paper for Friday, December 8.
15. Tewinkel, G. C., Panel Discussion on the Future of Analytical Aerial Triangulation, *Photogrammetric Engineering*, 24/1 March 1958, 87–89.
16. ——Analytic Photogrammetry, *Revista Cartografica*, Argentinia 8, No. 8, 1959, 139–157.
17. —— *Analytic Aerotriangulation in the C&GS*, Symposium, Geodesy in the Space Age, OSU, February 1961.
18. ——Analytic Aerotriangulation with Moderate Priced Equipment, *Boll di Geodisia*, 20/3, September 1961, 504–516.
19. —— *Analytic Relative Orientation in Photogrammetry*, C&GS, April 1961.
20. —— Analytic Absolute Orientation in Photogrammetry, *C&GS Technical Bulletin* No. 19, March 1962.
21. —— *Operation of an Aerotriangulation System at the USC and GS*, Paper presented at October 1962 ASP meeting.
22. —— Block Analytic Aerotriangulation, *Photogrammetric Engineering*, 32/6, November 1966, 1056–1061.
23. —— The Analytic Photogrammetry System of the Coast and Geodetic Survey, *Proceedings 1967 Symposium on Computational Photogrammetry*, December 1967, Paper for Wednesday, December 6.
24. Wilke, T. A., Useful Alternatives to Chauvenet's Rule for Rejection of Measurement Data, *Preceedings 1967 Symposium on Computational Photogrammetry*, December 1967, Paper for Wednesday, December 6.
25. Woodcock, L. F., Lampton, B. F., Measurement of Crustal Movements by Photogrammetric Methods, *Photogrammetric Engineering*, 30/6 November 1964, 912–916.

Articles for Next Month

James P. Scherz, Donald R. Graff, William C. Boyle, Photographic characteristics of water pollution.

Harold T. Rib and *Robert D. Miles*, Automatic interpretation of terrain features.

Richard S. Williams, Jr., Degredation of infrared caused by condensation.

Arch C. Gerlach, Geographic applications program of the USGS.

H. Gyde Lund, Factors for computing photo coverage.

G. H. Schut, Photogrammetric refraction.

Simha Weissman, Auxiliary data in strip adjustment.

Afifi H. Soliman, Standard error in strip adjustment.

Articles in Other Photogrammetric Journals

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Rudolf Schuller, Electronic data processing for the Bavarian land consolidation.

V. Kratky, Photogrammetric solution for exterior orientation in satellite geodesy.

W. Brindöpke, Accuracy of photogrammetric elevations for the production of German 1:5,000 maps in flat terrain.

R. Reiser, Testing two-meter invar bars.

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