GERALDINE M. WUOLLET*
HERBERT E. SKIBITZKE
U. S. Geological Survey
Phoenix, Arizona 85025

Plotting DME Arcs for Aerial Photography

Accurate circular flight paths based on existing navigation aids apply day or night and with or without maps, such as for infrared sensing.

(*Abstract on page 190*)

INTRODUCTION

AERIAL PHOTOGRAPHY and other techniques of remote sensing are being used with increasing frequency by hydrologists and geologists in the U. S. Geological Survey, whose requests are most often for special-purpose large-scale black-and-white photography, color and false-color photography, and infrared imagery. Last year approximately 15,000 aerial photographs were made by the Phoenix Research Office primarily for use by Water Resources Division and the Geologic Division personnel throughout the United States. Infrared imagery was also obtained for various special-project sites throughout the country. In general, the special sites consisted of small areas requiring medium- to low-altitude reconnaissance.

For those projects requiring infrared imagery the usual problem of effectively covering a small area at a low altitude, without the aid of elaborate and costly navigational devices, has been complicated by the need for collecting the data at night. To circumvent these problems several members of the Geological Survey have developed a system of pilotage which can be used in flying parallel flight lines solely by reference to the aircraft radio-navigational equipment. Such flight lines need only be arcs of concentric circles, centered upon a suitable radio-navigation station. They can be plotted on maps in advance of the flight by using Table 1 prepared by the authors and their staff.

The system discussed here for positioning aircraft on parallel circular flight lines is based on the use of present-day radio-navigational aids in the airways network established by the Federal Aviation Agency. A circular arrangement is often more suitable than any other simple geometric pattern for collecting hydrologic data from the air or on the ground. Other vehicles and navigational facilities or tracking radar may also be used to develop circular paths of movement, which are plotted almost as easily as straight lines through use of Table 1.

RADIO NAVIGATION

Direction (azimuth) and distance from a known ground point are the two items of

GERALDINE M. WUOLLET

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information necessary to establish position in air navigation. Very-high frequency—Omnidirectional Range (VOR)—radio stations constitute the primary source of directional information. They are maintained by the Federal Aviation Agency and operate in the very-high frequency (VHF) band. Tactical Air Navigation (TACAN) stations, originally developed by the military air service, provide both direction and distance information. TACAN stations operate in the ultra-high frequency (UHF) band.

VORTAC stations, as the label implies, incorporate in one unified facility the azimuth information from a VOR and the distance information from a TACAN. Many VORTAC stations are now in operation throughout the United States, serving as the standard navigational aids most commonly used by the modern pilot in instrument navigation.

An aircraft Distance-Measuring-Equipment (DME) receiver continuously measures the time between an *interrogating* signal transmitted by the airplane equipment and a *reply* transmitted by the ground station. The time interval is converted into distance and the information presented to the pilot by the position of a needle on a graduated scale or by actual numbers on a digital-type counter. The distance indicated is really the line-of-sight or slant distance in nautical miles from the aircraft receiver to the VORTAC station. In the practical use described here, however, the aircraft distance from the station is great enough (usually tens of miles) and the aircraft altitude low enough (usually less than 10,000 feet) so that no intolerable error is made by considering the indicated slant distance equal to the desired horizontal or radial distance from the TACAN station.

Digital DME receivers are capable of indicating distance accurately to tenths or hundredths of a nautical mile. Although such a high precision of measurement may or may not be warranted, the inaccuracies inherent in the receiver would probably be constant at a uniform distance from the station. This, coupled with the fairly large distances that usually separate a project area from the nearest TACAN station, ensures that the slight changes in radial distance, dictated by the desired spacing of the circular flight lines, can be measured (DME) with a high degree of accuracy. It is, in fact, possible to fly nearly parallel (concentric) circular flight paths by use of the DME technique without visual reference to the ground and without elaborate guidance systems.

ABSTRACT: *Navigation of aircraft may be accomplished precisely by reference to modern distance measuring equipment. A set of tables has been prepared for use in designing flight lines on a map which the pilot can follow accurately without visual reference to the ground. The method can be particularly useful when making aerial surveys at night or over water or wherever suitably detailed maps are not available. When circular flight paths are acceptable, the method is more accurate than any known method of aircraft placement.*

COMPUTATION OF DISTANCE AND AZIMUTH FROM VORTAC STATION

In order to plot circular flight lines (i.e., lines of constant DME readings) on a map it is necessary first to know, within the area to be

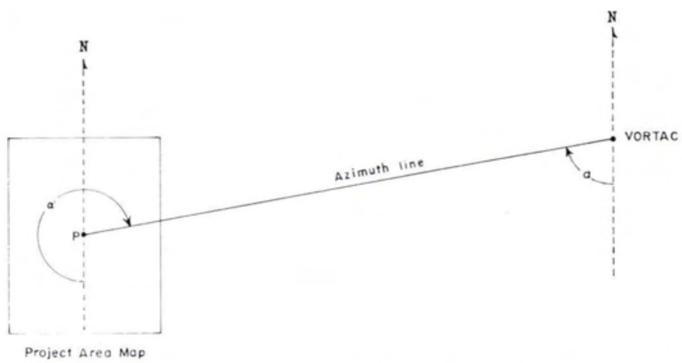

FIG. 1. Diagram showing orientation of a typical project area and a VORTAC station.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 662
Rev. Sept. 1942

INVERSE POSITION COMPUTATION

$$s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{\Delta\lambda_1 \cos\phi_m}{A_m}$$

$$s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{-\Delta\phi_1 \cos\frac{\Delta\lambda}{2}}{B_m}$$

$$-\Delta\alpha=\Delta\lambda \sin\phi_m \sec\frac{\Delta\phi}{2}+F(\Delta\lambda)^3$$

in which $\log \Delta\lambda_1=\log(\lambda'-\lambda)$ − correction for arc to sin*; $\log \Delta\phi_1=\log(\phi'-\phi)$ − correction for arc to sin*; and $\log s=\log s_1+$ correction for arc to sin*.

	° ′ ″	NAME OF STATION		° ′ ″
1. ϕ	33 25 53	VORTAC	λ	111 53 17
2. ϕ'	33 20 54	Reference Pt. P	λ'	112 41 36
$\Delta\phi\ (=\phi'-\phi)$	− 4 59	$\Delta\lambda\ (=\lambda'-\lambda)$		48 19
$\frac{\Delta\phi}{2}$	− 2 29.5	$\frac{\Delta\lambda}{2}$		24 9.5
$\phi_m\left(=\phi+\frac{\Delta\phi}{2}\right)$	33 23 23.5			
$\Delta\phi$ (secs.)	−299″	$\Delta\lambda$ (secs.)		2899″

log $\Delta\phi$	2.4756712			**log $\Delta\lambda$**	3.4622482
cor. arc − sin	−			cor. arc − sin	36
log $\Delta\phi_1$	2.4756712			log $\Delta\lambda_1$	3.4622446
log cos $\frac{\Delta\lambda}{2}$	9.9999893			**log cos ϕ_m**	9.9216580
colog B_m	1.4886841			**colog A_m**	1.4907284
log $\left\lvert s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\rvert$	3.9643446	(opposite in sign to $\Delta\phi$)		**log $\left\lvert s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\rvert$**	4.8746310
				log $\left\lvert s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\rvert$	3.9643446
log $\Delta\lambda$	3.4622482	3 log $\Delta\lambda$	10.3867	**log tan $\left(\alpha+\frac{\Delta\alpha}{2}\right)$**	.9102864
log sin ϕ_m	9.7406272	log F	7.8759	$\alpha+\frac{\Delta\alpha}{2}$	82° 59′ 27″
log sec $\frac{\Delta\phi}{2}$	0.0000001	log b	8.2626	**log sin $\left(\alpha+\frac{\Delta\alpha}{2}\right)$**	9.9967422
log a	3.2028755			**log cos $\left(\alpha+\frac{\Delta\alpha}{2}\right)$**	9.0864600
a	1595.422			**log s_1**	4.8778888
b	0.018			cor. arc − sin	+ 25
−$\Delta\alpha$ (secs.)	1595.440″			log s	4.8778913
−$\frac{\Delta\alpha}{2}$	797.720″ = 13′ 18″				
$\alpha+\frac{\Delta\alpha}{2}$	82 59 27				
α (1 to 2)	83 12 45				
$\Delta\alpha$	−26 36				
	180				
α' (2 to 1)	262 46 09				

* Use the table on the back of this form for correction of arc to sin.

s = 75,490 m
= 40.761 Nautical miles

NOTE.—For log s up to 4.0 and for $\Delta\phi$ or $\Delta\lambda$ (or both) up to 3′, omit all terms below the heavy line except those printed (in whole or in part) in heavy type or those underscored, if using logarithms to 7 decimal places.

FIG. 2. Inverse Position Computation (Form 662, U. S. Coast and Geodetic Survey, Rev. Sept. 1942).

flown, the distance and azimuth of at least one point from the VORTAC station to be used. Figure 1 shows the reference point P in a project area and its relation to a VORTAC. Accurate determination of both the distance and azimuth values α and α' is possible by using the *Inverse Position Computation* described in the U. S. Department of Commerce, Coast and Geodetic Survey Special Publication No. 200 (Lambert, W. D., and Swick, C. H., 1935). The latitude and longitude of the VORTAC station and of a point within or near the area to be surveyed are used to determine the direction and distance of the point P in the area from the VORTAC station. The latitude and longitude of VORTAC stations can be obtained from the Federal Aviation Agency.

Figure 2 shows the computation procedure for determining the distance and azimuth

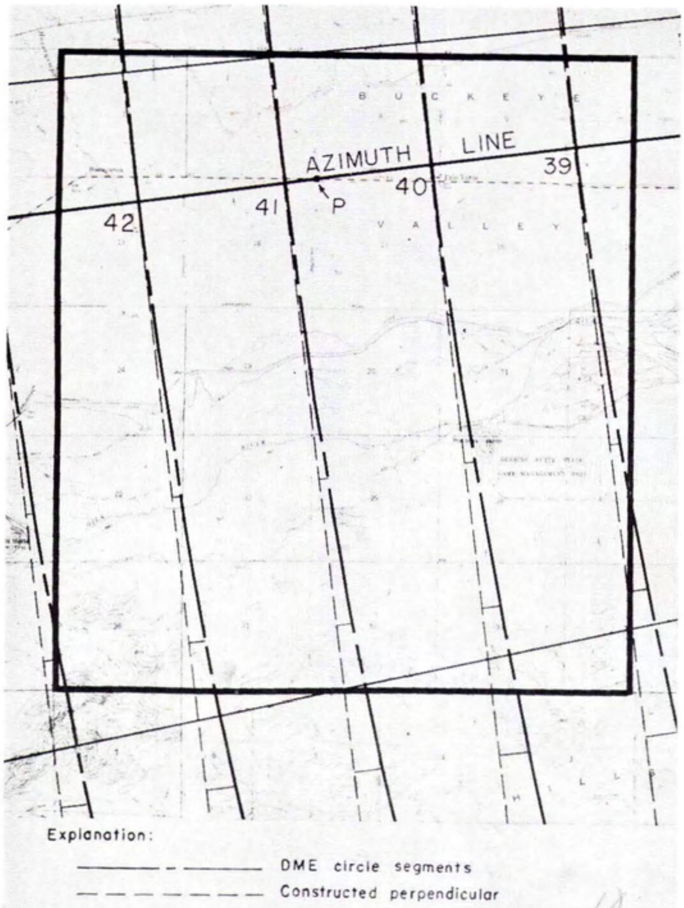

FIG. 3. Map of a project area west of Phoenix, Ariz., showing construction of DME circle segments.

from the VORTAC station at Phoenix, Arizona, to point P on the map (Figure 3) of an area west of the city. Once the azimuth line is determined as shown on Figure 3, construction lines can be drawn perpendicular to it at the interval representing the desired spacing of the flight lines, and offsets plotted therefrom to describe the circular arcs. Because the sample project area shown on the map in Figure 3 was to be photographed at a scale of 1:10,000, a circle segment was plotted for each whole-mile (nautical) DME radial distance falling inside the area. Thus arcs are shown for the 39.0-, 40.0-, 41.0-, 42.0-, and 43.0-mile radial distances, and these choices allow sufficient side lap of the photographs. Perpendiculars were constructed at these distances on the azimuth line, and offsets (from Table 1) plotted as shown on the map. The flight lines shown were drawn through the resulting points. The ground area actually covered by each photograph made on the subsequent test flight is outlined on the map in Figure 4. Although all photographs are not perfectly centered on the flight lines, probably due to the inherent inaccuracies of the equipment, the actual flight paths are more nearly parallel than normally can be flown by visual reference to the ground using a good map.

OFFSETS FOR PLOTTING CIRCULAR FLIGHT PATHS

Determining the offset distance x (Table 1) is a problem in basic geometry where x is simply a function of r and a, r being the radius of a circle with the VORTAC at its center, and a being a perpendicular distance from the azimuth line to the offset. Figure 5 describes the geometry of the problem. From Figure 5 it can be demonstrated that

$$x = r\,(1 - \cos\theta) \tag{1}$$

but

$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

and

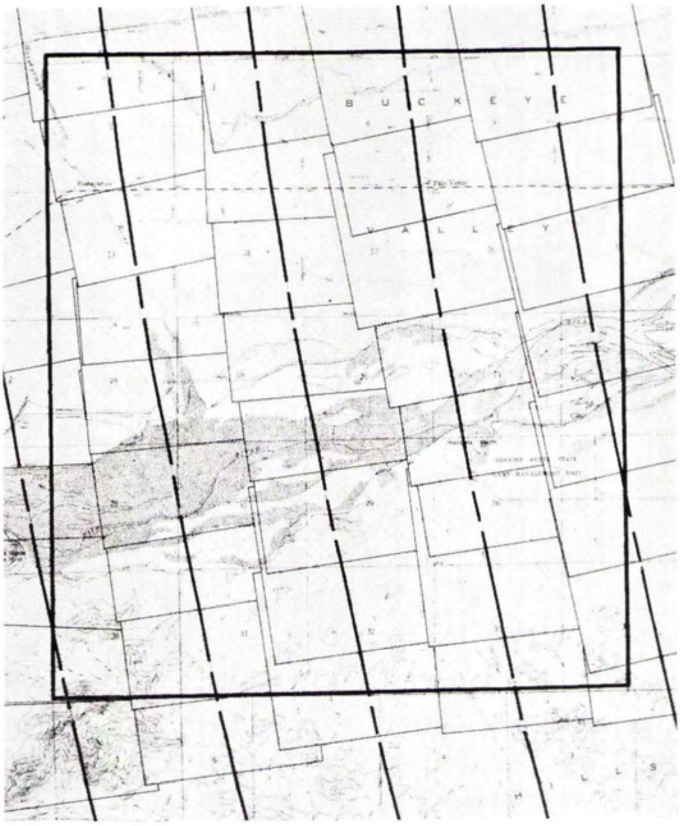

FIG. 4. Map of the project area west of Phoenix, Ariz., showing coverage of aerial photographs.

$$\sin \theta = \frac{a}{r}$$

Substituting the last two relations in Equation 1 yields

$$x = r - r\sqrt{1 - \left(\frac{a}{r}\right)^2}$$

or

$$x = r - \sqrt{r^2 - a^2} \qquad (2)$$

Equation 2 was used to compute values for the offset distance x given in Table 1. The table spans a range in r values, in unit increments, from 20 to 150, and a range in a values, also in unit increments, from 1 to 20. Although the tables were computed for using any desired unit of measure as long as everything is regarded as in that unit, DME receivers indicate the radial distance r in nautical miles; therefore, all distances in the tables may be regarded as in nautical miles. Should it be desired to use a statute miles scale for laying out the offsets on a map the values to be measured may be converted to statute miles by multiplying by 1.15157. The DME radials should be labeled in nautical miles for the pilot's benefit.

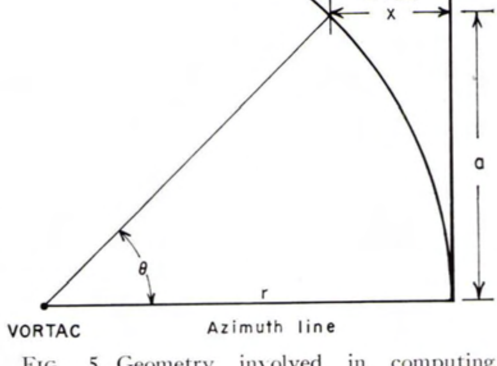

FIG. 5. Geometry involved in computing values shown in Table 1.

Table 1. Offset Distances for Plotting DME Circles

a*	*Radius (Any desired linear measure)* 20	21	22	23	24	25	26	27	28	29
1	.025	.024	.023	.022	.021	.020	.019	.018	.018	.017
2	.100	.096	.091	.087	.084	.080	.077	.074	.072	.069
3	.226	.215	.206	.196	.188	.181	.174	.167	.161	.156
4	.404	.384	.367	.351	.336	.322	.310	.298	.287	.277
5	.635	.604	.576	.550	.527	.505	.485	.467	.450	.434
6	.921	.875	.834	.796	.762	.731	.702	.675	.650	.628
7	1.265	1.201	1.143	1.091	1.044	1.000	.960	.923	.889	.858
8	1.670	1.584	1.506	1.436	1.373	1.315	1.261	1.212	1.167	1.125
9	2.145	2.026	1.925	1.834	1.751	1.676	1.607	1.544	1.486	1.432
10	2.680	2.534	2.404	2.288	2.183	2.087	2.000	1.920	1.845	1.779
11	3.297	3.112	2.947	2.801	2.669	2.550	2.442	2.342	2.251	2.167
12	4.000	3.766	3.561	3.379	3.215	3.068	2.935	2.813	2.702	2.599
13	4.801	4.508	4.252	4.026	3.826	3.646	3.483	3.336	3.201	3.077
14	5.717	5.348	5.029	4.752	4.506	4.288	4.091	3.913	3.751	3.603
15	6.771	6.303	5.906	5.564	5.265	5.000	4.763	4.550	4.357	4.181
16	8.000	7.398	6.900	6.477	6.112	5.791	5.506	5.251	5.022	4.813
17	9.464	8.671	8.036	7.508	7.059	6.670	6.328	6.024	5.751	5.505
18	11.282	10.183	9.351	8.682	8.126	7.651	7.238	6.875	6.552	6.262
19	13.755	12.056	10.910	10.038	9.337	8.752	8.252	7.817	7.433	7.091
20	20.000	14.597	12.835	11.642	10.734	10.000	9.387	8.862	8.404	8.000

a*	30	31	32	33	34	35	36	37	38	39
1	.017	.016	.016	.015	.015	.014	.014	.014	.013	.013
2	.067	.065	.063	.061	.059	.057	.056	.054	.053	.051
3	.150	.146	.141	.137	.133	.129	.125	.122	.119	.116
4	.268	.259	.251	.243	.236	.229	.223	.217	.211	.206
5	.420	.406	.393	.381	.370	.359	.349	.339	.330	.322
6	.606	.586	.568	.550	.534	.518	.504	.490	.477	.464
7	.828	.801	.775	.751	.728	.707	.687	.668	.650	.633
8	1.086	1.050	1.016	.984	.955	.927	.900	.875	.852	.829
9	1.382	1.335	1.292	1.251	1.213	1.177	1.143	1.111	1.081	1.053
10	1.716	1.657	1.603	1.552	1.504	1.459	1.417	1.377	1.339	1.304
11	2.089	2.017	1.950	1.887	1.829	1.774	1.722	1.673	1.627	1.583
12	2.504	2.417	2.335	2.259	2.188	2.121	2.059	2.000	1.944	1.892
13	2.963	2.858	2.760	2.668	2.583	2.504	2.429	2.359	2.293	2.230
14	3.467	3.341	3.225	3.117	3.016	2.922	2.834	2.751	2.673	2.599
15	4.019	3.871	3.737	3.606	3.488	3.377	3.274	3.177	3.086	3.000
16	4.623	4.448	4.287	4.138	4.000	3.871	3.751	3.638	3.533	3.433
17	5.282	5.077	4.889	4.716	4.555	4.406	4.267	4.137	4.015	3.900
18	6.000	5.761	5.542	5.341	5.156	4.983	4.823	4.674	4.534	4.402
19	6.784	6.505	6.251	6.018	5.804	5.606	5.422	5.251	5.091	4.941
20	7.639	7.315	7.020	6.751	6.504	6.277	6.067	5.871	5.689	5.519

a*	40	41	42	43	44	45	46	47	48	49
1	.013	.012	.012	.012	.011	.011	.011	.011	.010	.010
2	.050	.049	.048	.047	.045	.044	.043	.043	.042	.041
3	.113	.110	.107	.105	.102	.100	.098	.096	.094	.092
4	.201	.196	.191	.186	.182	.178	.174	.171	.167	.164
5	.314	.306	.299	.292	.285	.279	.273	.267	.263	.256
6	.453	.441	.431	.421	.411	.402	.393	.385	.376	.369
7	.617	.602	.587	.574	.560	.548	.536	.524	.513	.503
8	.808	.788	.769	.751	.733	.717	.701	.686	.671	.657
9	1.026	1.000	.976	.952	.930	.909	.889	.870	.851	.834
10	1.270	1.238	1.208	1.179	1.151	1.125	1.100	1.076	1.053	1.031
11	1.542	1.503	1.466	1.431	1.397	1.365	1.335	1.305	1.277	1.251
12	1.842	1.795	1.751	1.708	1.668	1.630	1.593	1.558	1.524	1.492
13	2.171	2.116	2.063	2.012	1.964	1.919	1.875	1.834	1.794	1.756
14	2.530	2.464	2.402	2.343	2.287	2.233	2.182	2.134	2.087	2.043
15	2.919	2.842	2.770	2.701	2.636	2.574	2.514	2.458	2.404	2.352
16	3.339	3.251	3.167	3.088	3.012	2.941	2.872	2.807	2.745	2.686
17	3.792	3.690	3.594	3.503	3.417	3.335	3.257	3.182	3.111	3.043
18	4.279	4.163	4.053	3.949	3.850	3.757	3.668	3.583	3.503	3.426
19	4.801	4.668	4.543	4.425	4.314	4.208	4.107	4.012	3.921	3.834
20	5.359	5.209	5.068	4.934	4.808	4.689	4.575	4.468	4.365	4.267

* Same unit of linear measure as that chosen for "radius."

TABLE 1—*Continued*

a*	*Radius (Any desired linear measure)* 50	51	52	53	54	55	56	57	58	59
1	.010	.010	.010	.009	.009	.009	.009	.009	.009	.008
2	.040	.039	.038	.038	.037	.036	.036	.035	.034	.034
3	.090	.088	.087	.085	.083	.082	.080	.079	.078	.076
4	.160	.157	.154	.151	.148	.146	.143	.141	.138	.136
5	.251	.246	.241	.236	.232	.228	.224	.220	.216	.212
6	.361	.354	.347	.341	.334	.328	.323	.317	.311	.306
7	.492	.483	.473	.464	.456	.447	.439	.431	.424	.417
8	.644	.631	.619	.607	.596	.585	.574	.564	.554	.545
9	.817	.800	.785	.770	.755	.741	.728	.715	.703	.690
10	1.010	.990	.971	.952	.934	.917	.900	.884	.869	.854
11	1.225	1.200	1.177	1.154	1.132	1.111	1.091	1.071	1.053	1.034
12	1.461	1.432	1.404	1.376	1.350	1.325	1.301	1.277	1.255	1.233
13	1.720	1.685	1.651	1.619	1.588	1.558	1.530	1.502	1.476	1.450
14	2.000	1.959	1.920	1.882	1.846	1.812	1.778	1.746	1.715	1.685
15	2.303	2.256	2.210	2.167	2.125	2.085	2.046	2.009	1.973	1.939
16	2.629	2.575	2.523	2.473	2.425	2.379	2.334	2.292	2.251	2.211
17	2.979	2.917	2.857	2.800	2.746	2.693	2.643	2.594	2.547	2.502
18	3.352	3.282	3.215	3.150	3.088	3.029	2.972	2.917	2.864	2.813
19	3.751	3.671	3.595	3.523	3.453	3.386	3.322	3.260	3.200	3.143
20	4.174	4.085	4.000	3.918	3.840	3.765	3.693	3.624	3.557	3.493

a*	60	61	62	63	64	65	66	67	68	69
1	.008	.008	.008	.008	.008	.008	.008	.008	.007	.007
2	.033	.033	.032	.032	.031	.031	.030	.030	.030	.029
3	.075	.074	.073	.072	.070	.069	.068	.067	.066	.065
4	.133	.131	.129	.127	.125	.123	.121	.120	.118	.116
5	.209	.205	.202	.199	.196	.193	.190	.187	.184	.181
6	.301	.296	.291	.286	.282	.279	.273	.269	.265	.261
7	.410	.403	.396	.390	.384	.378	.372	.367	.361	.356
8	.536	.527	.518	.510	.502	.494	.487	.479	.472	.465
9	.679	.668	.657	.646	.636	.626	.617	.607	.606	.590
10	.839	.825	.812	.799	.786	.774	.762	.750	.739	.728
11	1.017	1.000	.984	.968	.952	.938	.923	.909	.896	.882
12	1.212	1.192	1.172	1.154	1.135	1.117	1.100	1.083	1.067	1.052
13	1.425	1.401	1.378	1.356	1.334	1.313	1.293	1.273	1.254	1.236
14	1.656	1.628	1.601	1.575	1.550	1.526	1.502	1.479	1.457	1.435
15	1.905	1.873	1.842	1.812	1.783	1.754	1.727	1.701	1.675	1.650
16	2.173	2.136	2.100	2.066	2.032	2.000	1.969	1.938	1.909	1.881
17	2.459	2.417	2.376	2.337	2.299	2.262	2.227	2.193	2.159	2.127
18	2.764	2.716	2.670	2.626	2.583	2.542	2.502	2.463	2.426	2.389
19	3.088	3.034	2.983	2.933	2.885	2.839	2.794	2.750	2.708	2.668
20	3.431	3.372	3.314	3.259	3.205	3.152	3.103	3.055	3.008	2.962

a*	70	71	72	73	74	75	76	77	78	79
1	.007	.007	.007	.007	.007	.007	.007	.007	.007	.006
2	.029	.028	.028	.028	.027	.027	.026	.026	.026	.025
3	.064	.064	.063	.062	.061	.060	.059	.058	.058	.057
4	.114	.113	.111	.110	.108	.107	.105	.104	.103	.101
5	.179	.176	.174	.172	.169	.167	.835	.163	.160	.158
6	.258	.254	.251	.247	.244	.240	.237	.234	.231	.228
7	.351	.346	.341	.336	.332	.327	.323	.319	.315	.311
8	.459	.452	.446	.440	.434	.428	.422	.417	.411	.406
9	.582	.573	.565	.557	.549	.542	.465	.528	.521	.514
10	.718	.708	.698	.688	.679	.670	.661	.652	.644	.636
11	.870	.857	.845	.834	.822	.831	.800	.790	.780	.770
12	1.036	1.022	1.007	.994	.980	.966	.953	.941	.929	.917
13	1.218	1.200	1.183	1.167	1.151	1.135	1.120	1.105	1.091	1.077
14	1.414	1.394	1.374	1.355	1.336	1.318	1.301	1.284	1.267	1.250
15	1.626	1.603	1.580	1.558	1.536	1.515	1.495	1.475	1.456	1.437
16	1.853	1.826	1.800	1.775	1.751	1.728	1.703	1.681	1.659	1.637
17	2.096	2.065	2.036	2.007	1.979	1.952	1.926	1.900	1.875	1.851
18	2.354	2.320	2.286	2.254	2.223	2.192	2.162	2.134	2.105	2.078
19	2.628	2.590	2.552	2.516	2.481	2.447	2.413	2.381	2.350	2.319
20	2.918	2.875	2.834	2.795	2.754	2.716	2.679	2.643	2.608	2.574

* Same unit of linear measure as that chosen for "radius."

TABLE 1—*Continued*

a*	*Radius (Any desired linear measure)* 80	81	82	83	84	85	86	87	88	89
1	.006	.006	.006	.006	.006	.006	.006	.006	.006	.006
2	.025	.025	.024	.024	.024	.024	.023	.023	.023	.023
3	.056	.056	.055	.054	.054	.053	.052	.052	.051	.051
4	.100	.099	.098	.096	.095	.094	.093	.092	.091	.090
5	.156	.154	.153	.151	.149	.147	.146	.144	.142	.141
6	.225	.223	.220	.217	.215	.213	.210	.207	.205	.202
7	.307	.303	.299	.296	.292	.289	.285	.282	.279	.276
8	.401	.396	.391	.386	.382	.377	.373	.369	.364	.360
9	.508	.502	.495	.489	.484	.478	.472	.467	.462	.456
10	.628	.620	.612	.605	.597	.590	.583	.577	.570	.564
11	.761	.750	.741	.732	.723	.715	.706	.698	.690	.682
12	.905	.894	.883	.872	.862	.851	.841	.832	.822	.813
13	1.063	1.050	1.037	1.024	1.012	1.000	.988	.977	.966	.955
14	1.235	1.219	1.204	1.189	1.175	1.161	1.147	1.134	1.121	1.108
15	1.419	1.401	1.384	1.367	1.350	1.334	1.318	1.303	1.288	1.273
16	1.616	1.596	1.576	1.557	1.538	1.521	1.502	1.484	1.467	1.450
17	1.827	1.804	1.782	1.760	1.738	1.717	1.697	1.677	1.658	1.639
18	2.051	2.025	2.000	1.975	1.951	1.928	1.905	1.882	1.861	1.839
19	2.289	2.260	2.232	2.204	2.177	2.151	2.125	2.100	2.076	2.052
20	2.540	2.508	2.476	2.446	2.416	2.387	2.358	2.330	2.290	2.276

a*	*90*	*91*	*92*	*93*	*94*	*95*	*96*	*97*	*98*	*99*
1	.006	.006	.006	.005	.005	.005	.005	.005	.005	.005
2	.022	.022	.022	.022	.021	.021	.021	.021	.020	.020
3	.050	.050	.049	.048	.048	.047	.047	.046	.046	.046
4	.089	.088	.087	.086	.085	.084	.083	.083	.082	.081
5	.139	.138	.136	.135	.133	.132	.130	.129	.128	.126
6	.200	.198	.196	.194	.192	.190	.188	.186	.184	.182
7	.278	.270	.267	.264	.261	.258	.256	.253	.250	.248
8	.356	.352	.348	.345	.341	.338	.334	.330	.327	.324
9	.451	.446	.441	.437	.432	.427	.423	.418	.414	.410
10	.557	.551	.545	.539	.534	.528	.522	.518	.512	.506
11	.675	.667	.660	.653	.646	.639	.632	.626	.619	.613
12	.804	.795	.786	.778	.769	.761	.753	.745	.738	.730
13	.944	.933	.923	.913	.903	.894	.884	.875	.866	.857
14	1.096	1.083	1.072	1.060	1.048	1.037	1.026	1.016	1.005	.995
15	1.259	1.245	1.231	1.218	1.205	1.192	1.179	1.167	1.155	1.143
16	1.434	1.418	1.402	1.387	1.372	1.357	1.343	1.329	1.315	1.302
17	1.620	1.602	1.584	1.567	1.550	1.534	1.517	1.501	1.486	1.471
18	1.818	1.798	1.778	1.759	1.740	1.721	1.703	1.685	1.667	1.650
19	2.028	2.006	1.983	1.962	1.946	1.919	1.899	1.879	1.860	1.840
20	2.250	2.225	2.200	2.176	2.152	2.129	2.106	1.084	2.063	2.041

a*	*100*	*101*	*102*	*103*	*104*	*105*	*106*	*107*	*108*	*109*
1	.005	.005	.005	.005	.005	.005	.005	.005	.005	.005
2	.020	.020	.020	.020	.019	.019	.019	.019	.019	.018
3	.045	.045	.044	.044	.043	.043	.043	.042	.042	.041
4	.080	.079	.079	.078	.077	.076	.076	.075	.074	.074
5	.125	.124	.123	.122	.120	.119	.118	.117	.116	.115
6	.180	.178	.177	.175	.173	.173	.170	.168	.167	.165
7	.245	.243	.241	.238	.236	.234	.231	.229	.227	.225
8	.321	.317	.314	.311	.308	.305	.302	.300	.297	.294
9	.406	.402	.398	.394	.390	.387	.383	.379	.376	.372
10	.501	.496	.491	.487	.482	.477	.473	.468	.464	.460
11	.607	.601	.595	.589	.583	.578	.572	.567	.562	.557
12	.723	.716	.708	.702	.695	.688	.682	.675	.669	.663
13	.849	.840	.832	.824	.816	.808	.800	.793	.785	.778
14	.985	.975	.965	.956	.947	.938	.929	.920	.911	.903
15	1.132	1.120	1.109	1.098	1.088	1.077	1.067	1.057	1.047	1.037
16	1.288	1.275	1.263	1.250	1.238	1.226	1.215	1.203	1.192	1.181
17	1.456	1.441	1.427	1.413	1.399	1.385	1.372	1.359	1.346	1.334
18	1.633	1.617	1.601	1.585	1.570	1.554	1.540	1.525	1.511	1.497
19	1.822	1.803	1.785	1.768	1.750	1.733	1.717	1.701	1.685	1.669
20	2.021	2.000	1.980	1.961	1.941	1.922	1.904	1.886	1.868	1.851

* Same unit of linear measure as that chosen for "radius."

TABLE 1—*Continued*

a*	Radius (Any desired linear measure)									
	110	*111*	*112*	*113*	*114*	*115*	*116*	*117*	*118*	*119*
1	.005	.005	.005	.005	.004	.004	.004	.004	.004	.004
2	.018	.018	.018	.018	.018	.017	.017	.017	.017	.017
3	.041	.041	.040	.040	.040	.039	.039	.039	.038	.038
4	.073	.072	.072	.071	.070	.070	.069	.068	.068	.067
5	.114	.113	.112	.111	.110	.109	.108	.107	.106	.105
6	.164	.162	.161	.160	.158	.157	.155	.154	.153	.151
7	.223	.221	.219	.217	.215	.213	.211	.210	.208	.206
8	.291	.289	.286	.284	.281	.279	.276	.274	.272	.269
9	.369	.366	.362	.359	.356	.353	.350	.347	.344	.341
10	.456	.451	.447	.444	.440	.436	.432	.428	.425	.421
11	.551	.546	.542	.537	.532	.527	.523	.518	.514	.510
12	.657	.651	.645	.639	.633	.628	.622	.617	.612	.607
13	.771	.764	.757	.750	.744	.737	.731	.725	.718	.712
14	.895	.887	.879	.871	.863	.855	.848	.841	.834	.826
15	1.028	1.018	1.009	1.000	.991	.983	.974	.966	.957	.949
16	1.170	1.159	1.149	1.139	1.128	1.119	1.109	1.099	1.090	1.081
17	1.322	1.310	1.298	1.286	1.275	1.264	1.253	1.242	1.231	1.221
18	1.483	1.469	1.456	1.443	1.430	1.418	1.405	1.393	1.381	1.369
19	1.653	1.638	1.623	1.609	1.595	1.581	1.567	1.553	1.540	1.527
20	1.834	1.817	1.800	1.784	1.768	1.753	1.737	1.722	1.707	1.693

a*	*120*	*122*	*124*	*126*	*128*	*130*	*132*	*134*	*136*	*138*
1	.004	.004	.004	.004	.004	.004	.004	.004	.004	.004
2	.017	.016	.016	.016	.016	.015	.015	.015	.015	.014
3	.038	.037	.036	.036	.035	.035	.034	.034	.033	.033
4	.067	.066	.065	.064	.063	.062	.061	.060	.059	.058
5	.104	.103	.101	.099	.098	.096	.095	.093	.092	.091
6	.150	.148	.145	.143	.141	.138	.136	.134	.132	.130
7	.204	.201	.198	.195	.192	.189	.186	.183	.180	.178
8	.267	.263	.258	.254	.250	.246	.243	.239	.236	.232
9	.338	.332	.327	.322	.317	.312	.307	.303	.298	.294
10	.417	.411	.404	.398	.391	.385	.379	.374	.368	.363
11	.506	.497	.489	.481	.474	.466	.459	.452	.446	.439
12	.602	.592	.582	.573	.564	.555	.547	.538	.530	.523
13	.706	.695	.683	.672	.662	.652	.642	.632	.623	.614
14	.820	.806	.793	.780	.768	.756	.744	.733	.722	.712
15	.941	.926	.911	.896	.883	.868	.855	.842	.830	.818
16	1.072	1.054	1.037	1.020	1.004	.988	.973	.959	.944	.931
17	1.210	1.190	1.171	1.152	1.134	1.116	1.099	1.083	1.067	1.051
18	1.358	1.335	1.314	1.292	1.272	1.252	1.233	1.215	1.196	1.179
19	1.514	1.489	1.464	1.441	1.418	1.396	1.375	1.354	1.334	1.314
20	1.679	1.651	1.624	1.598	1.572	1.548	1.524	1.501	1.479	1.457

a*	*140*	*142*	*144*	*146*	*148*	*150*
1	.004	.004	.004	.003	.003	.003
2	.014	.014	.014	.014	.014	.013
3	.032	.032	.031	.031	.030	.030
4	.057	.056	.056	.055	.054	.053
5	.089	.088	.087	.086	.084	.083
6	.129	.127	.125	.123	.122	.120
7	.175	.173	.174	.168	.166	.163
8	.229	.226	.222	.219	.216	.214
9	.290	.286	.282	.278	.274	.271
10	.358	.353	.348	.343	.338	.334
11	.433	.427	.421	.415	.409	.404
12	.515	.508	.501	.494	.487	.481
13	.605	.596	.588	.580	.572	.564
14	.702	.692	.682	.673	.664	.655
15	.806	.795	.783	.773	.762	.752
16	.917	.904	.892	.879	.867	.856
17	1.036	1.021	1.007	.993	.980	.966
18	1.162	1.146	1.129	1.114	1.099	1.084
19	1.295	1.277	1.259	1.242	1.225	1.208
20	1.436	1.412	1.396	1.376	1.358	1.339

* Same unit of linear measure as that chosen for "radius."