DONOVAN N. HUBER*
*Aeronautical Chart and*
*Information Center*
*St. Louis, Mo. 63118*

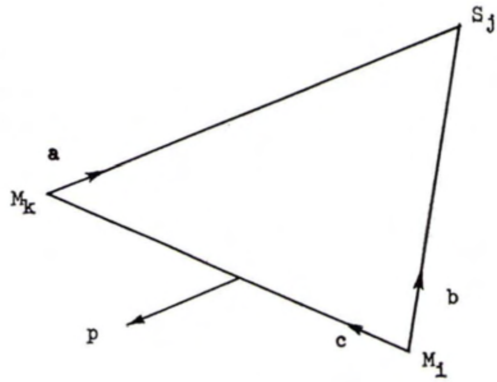


FIG. 1. Camera stations $M_i$ and $M_k$ simultaneously record the satellite flash $S_j$.

# Geodetic Positioning with the PC-1000 Camera

**Results indicate that the error propagation of the system is about 15 feet every 1000 miles.**

(*Abstract on next page*)

## INTRODUCTION

AIR FORCE PC-1000 cameras have been engaged in satellite observation for geometric geodetic position determinations since September 1963. First results with ANNA 1-B data were published by the Air Force Cambridge Research Laboratories in 1964 and again early in 1965. The findings presented here were obtained with satellite observational data from two Air Force projects observing ANNA 1-B and GEOS-A.

The latest analytical photogrammetric methods are used in the ACIC photographic plate reduction method which is described in a forthcoming technical report. The satellite triangulation adjustment procedures programmed at ACIC are defined here along with the solutions of simple test triangles and the more complex positioning of the Island of Trinidad with respect to the North American Datum in southeastern United States.

## METHOD

In Figure 1, $M_i$ and $M_k$ are camera stations from which the satellite flash $S_j$ was observed simultaneously. The unit vectors $a$ and $b$ represent the observed directions from $M_k$ to $S_j$ and $M_i$ to $S_j$ respectively. The unit vector $c$ points in the direction of the chord from station $M_i$ to $M_k$ and is a function of the station coordinates.

The exact WWV (UT-2) time for each flash event was furnished by the Applied Physics Laboratory. For the star background time, the clock at the site was cali-

* Presented at the 1966 Semi-Annual Convention of the American Society of Photogrammetry, Los Angeles, Calif., Sept., 1966. The views expressed here are those of the author and do not necessarily reflect the views of the U. S. Air Force nor the Department of Defense.

brated against WWV before and after each event. The time signals were corrected for emission and propagation before the shutter times were computed.

The apparent topocentric right ascensions $\alpha$ and declinations $\delta$ are the observed directions used by ACIC. The WWV (UT-2) observation time is converted to apparent Greenwich sidereal time $\gamma$. All observations are referred to the Greenwich meridian through the angle $(\alpha-\gamma)=G$.

ABSTRACT: *The Air Force PC-1000 camera system has proven to be an extremely accurate measuring tool for geodetic positioning. ACIC has performed many geodetic adjustments using data from observations of approximately 300 passes of the ANNA 1-B and the GEOS-A flashing light satellites. Cameras have been located mainly in the eastern half of the United States plus five additional cameras on islands forming a chain of triangles from Florida to Trinidad. Trinidad was related to Florida with an error less than 30 feet. Errors in relative positioning of Antigua, Curaçao, Grand Turk, and Swan Island with respect to Florida were: 24, 20, 11, and 10 feet, respectively. Specific recommendations for the future utilizations of the PC-1000 camera system include the densification of the PAGEOS World Net and the extension of the Trinidad tie downrange to Ascension Island.*

## BASIC CONDITION AND EQUATION

Development of the mathematical formulation for the method of intersecting planes starts with the basic condition that vectors $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are coplanar. In vector notation,

$$\boldsymbol{a} \times \boldsymbol{b} \cdot \boldsymbol{c} = 0 \tag{1}$$

where the components of $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are

| $\boldsymbol{a}$ | $\boldsymbol{b}$ | $\boldsymbol{c}$ |
|---|---|---|
| $\cos\delta_k \cos G_k$ | $\cos\delta_i \cos G_i$ | $X_k - X_i$ |
| $\cos\delta_k \sin G_k$ | $\cos\delta_i \sin G_i$ | $Y_k - Y_i$ |
| $\sin\delta_k$ | $\sin\delta_i$ | $Z_k - Z_i$. |

If $\boldsymbol{a}\times\boldsymbol{b}=\boldsymbol{p}$, the components of the $\boldsymbol{p}$ vector are

$$\cos\delta_k \sin G_k \sin\delta_i - \cos\delta_i \sin G_i \sin\delta_k$$
$$\cos\delta_i \cos G_i \sin\delta_k - \cos\delta_k \cos G_k \sin\delta_i$$
$$\cos\delta_k \cos G_k \cos\delta_i \sin G_i - \cos\delta_i \cos G_i \cos_k \sin G_k$$

which, when simplified, become

$$\cos\delta_i \cos\delta_k (\sin G_k \tan\delta_i - \sin G_i \tan\delta_k)$$
$$\cos\delta_i \cos\delta_k (\cos G_i \tan\delta_k - \cos G_k \tan\delta_i)$$
$$\cos\delta_i \cos\delta_k (\sin G_i \cos G_k - \cos G_i \sin G_k).$$

The dot product of vectors $\boldsymbol{p}$ and $\boldsymbol{c}$ results in the desired basic equation to express the coplanar condition:

$$(X_k - X_i)(\tan\delta_i \sin G_k - \tan\delta_k \sin G_i) + (Y_k - Y_i)(\tan\delta_k \cos G_i - \tan\delta_i \cos G_k) + (Z_k - Z_i)\sin(G_i - G_k) = 0. \tag{2}$$

The coordinates and trigonometric functions refer to a right handed system with the Greenwich meridian in the $XZ$-plane.

OBSERVATION EQUATIONS

For the actual least squares adjustment, the basic equation is expanded into two first-order series expressions so that two different observational situations or cases can be utilized.

*Case 1.* Simultaneous observation of a flash from two unknown stations, $i$ and $k$.

$$A\Delta X_i + B\Delta Y_i + C\Delta Z_i - A\Delta X_k - B\Delta Y_k - C\Delta Z_k + \ell^1 = 0 \tag{3}$$

where:

$$\begin{aligned} A &= \tan \delta_i \sin G_k - \tan \delta_k \sin G_i \\ B &= \tan \delta_k \cos G_i - \tan \delta_i \cos G_k \\ C &= \sin (G_i - G_k) = \sin (\alpha_i - \alpha_k) \end{aligned}$$

and

$$\ell^1 = A(X_i - X_k) + B(Y_i - Y_k) + C(Z_i - Z_k)$$

where:

$$\begin{aligned} X_i,\ Y_i,\ Z_i \quad \text{and} \quad X_k,\ Y_k,\ Z_k &= \text{approximate values} \\ i &= 1, 2, 3, 4 \\ k &= 2, 3, 4, 5 \\ i &< k. \end{aligned}$$

*Case 2.* Simultaneous observations from one unknown $i$ and one known $k$ station.

$$A\Delta X_i + B\Delta Y_i + C\Delta Z_i + \ell^2 = 0 \tag{4}$$

where:

$$i = 1, 2, 3, 4, 5$$

and

$$\ell^2 = A(X_i - X_k) + B(Y_i - Y_k) + C(Z_i - Z_k)$$

where:

$$\begin{aligned} X_i,\ Y_i,\ Z_i &= \text{approximate values} \\ X_k,\ Y_k,\ Z_k &= \text{exact values} \\ i &= 1, 2, 3, 4, 5 \\ k &= 6, 7, 8. \end{aligned}$$

The adjustment, as initially programmed for the IBM 1620, will handle a maximum of five unknowns and three knowns.* (The program has since been modified to accept 10 knowns.) There is no limit, however, to the number of observations which can be included in a solution because the flash positions are not computed and, therefore, the matrix remains at the 15 by 15 size.

* This explains the numbering just given for $i$ and $k$.

TABLE 1. COORDINATES FOR CAMERA SITES

| *Station Name* | | *NASA No.* | *Latitude North* | *Longitude West* | *Height* | |
|---|---|---|---|---|---|---|
| | | | | | *msl* | *Geoid* |
| Semmes, Ala. | 1 | 3402 | 30° 46′ 49.″159N | 88° 15′ 06.″927W | 72.7 | −2.5 |
| Hunter, Ga. | 1 | 3648 | 32 00 05.868 | 81 09 13.641 | 12.7 | 0.0 |
| Hunter, Ga. | 2 | 3648 | 32 00 06.0104 | 81 09 13.7885 | 12.7 | −5.0 |
| Homestead, Fla. | 1 | 3861 | 25 30 24.686 | 80 23 17.312 | 2.4 | 10.0 |
| Homestead, Fla. | 2 | 3861 | 25 30 24.7791 | 80 23 17.3354 | 2.4 | 5.6 |
| Swan Is. | 3 | 3404 | 17 24 16.57 | 83 56 30.13 | 40.4 | |
| Grand Turk | 3 | 3405 | 21 25 47.05 | 71 08 45.97 | 2.2 | |
| Curaçao | 3 | 3406 | 12 05 22.11 | 68 50 16.24 | 7.0 | |
| Antigua | 3 | 3106 | 17 08 51.68 | 61 47 22.59 | 1.8 | |
| Trinidad | 3 | 3407 | 10 44 32.78 | 61 36 36.33 | 249.5 | |

1 NAD 27.
2 Cape Canaveral Datum (CCD).
3 Approximate NAD 27.

## FEATURES OF THE ACIC METHOD

*Preliminary Editing.* It is always desirable to eliminate invalid data before it enters into the adjustment. For this reason, a pre-edit procedure was incorporated into the ACIC program.†

The program computes $\boldsymbol{a}\times\boldsymbol{b}=\boldsymbol{p}$ and the angle between the vectors $\boldsymbol{p}$ and $\boldsymbol{c}$ (Figure 1). The angle would equal 90° if there were no errors in the observations, station coordinates, or orientation.‡ Since this is rarely, if ever, the case, the difference between the computed value and 90° is considered a residual $s$. The residuals for the observations from each pair of stations are summed and a mean value computed. Those flashes with residuals varying from the mean by more than three seconds are eliminated. The three-second criteria was determined by extensive testing and seems suitable for the PC-1000 system at this stage.

*Preliminary Adjustment.* Multiple station events—three or more cameras observing a flash simultaneously—are very desirable, especially when two or more of the cameras are known (fixed) sites. In the latter circumstance, the observed directions from the fixed sites can be adjusted to agree with the geodetic system on which they are positioned to add control to the main least squares program.

The method of computing the *control* directions—adjustment by conditions—uses Lagrange's multipliers or correlates in the solution. For example, a condition equation for known stations $i$ and $k$ observing flash $j$ would be

$$m_{(ik)j}\Delta\alpha_{ij} + n_{(ik)j}\Delta\delta_{ij} + p_{(ik)j}\Delta\alpha_{kj} + q_{(ik)j}\Delta\delta_{kj} + \mathfrak{L}_{(ik)j} = 0 \qquad (5)$$

† Since each equation in the adjustment is formed from a simultaneous observation of a single flash from two stations, the pre-edit evaluates only this type event.

‡ The error in station coordinates is no problem because the effect is systematic for any particular pair of stations. The mean value is considered reliable for preliminary editing if it is derived from thirty or more flashes and six or more different passes of the satellite.

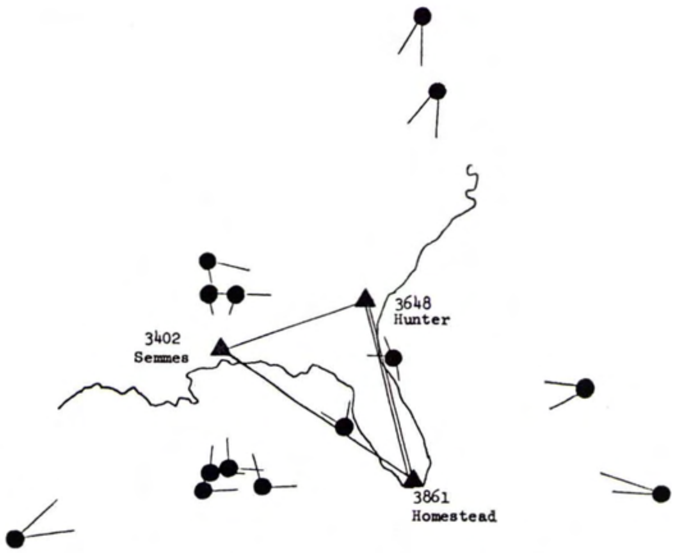


FIG. 2. Station 3402 determined from 3648 and 3861.

where:

$$m_{(ik)j} = \tan \psi_{ik} \cos (\alpha_{ij} - \alpha_{kj}) - \tan \delta_{kj} \cos (G_{ij} - A_{ik})$$

$$n_{(ik)j} = \sec^2 \delta_{ij} \sin (G_{kj} - A_{ik})$$

$$p_{(ik)j} = -\tan \psi_{ik} \cos (\alpha_{ij} - \alpha_{kj}) + \tan \delta_{ij} \cos (G_{kj} - A_{ik})$$

$$q_{(ik)j} = -\sec^2 \delta_{kj} \sin (G_{ij} - A_{ik})$$

$$l_{(ik)j} = \tan \psi_{ik} \sin (\alpha_{ij} - \alpha_{kj}) + \tan \delta_{ij} \sin (G_{kj} - A_{ik}) - \tan \delta_{kj} \sin (G_{ij} - A_{ik})$$

$$\tan \psi_{ik} = \frac{Z_k - Z_j}{r}, \sin A_{ik} \frac{Y_k - Y_i}{r},$$

$$\cos A_{ik} = \frac{X_k - X_i}{r} \qquad r = [(X_k - X_i)^2 + (Y_k - Y_i)^2].$$

After the preliminary adjustment, an auxiliary program reads out the amount of change in the directions to force them to agree with the position and orientation of the fixed sites. The consistency of the individual adjustments to the observed directions $\alpha$, $\delta$ for a specific satellite pass and the amount of the adjustment is noted.

*Constraints.* Although provision is made for differential weighting of known stations, it has not been done to date because the fixed sites have always been first-order surveyed positions. Unknown stations are allowed to adjust freely unless two or more are eccentric to the same triangulation station. In the latter case, the surveyed distance or spatial orientation between them is entered as a condition.

*Output.* Each observation equation formed for the principal adjustment represents a plane with three points—two camera positions and a flash position. The magnitude of the residual read out for each equation indicates the degree to which the coplanar condition (Equation 1) was satisfied. The standard error (sigma) computed for the solution represents the overall internal accuracy of the data. Observations creating residuals larger than three sigma are edited out and the solution is rerun starting with the formation of new normal equations—actually, those data cards with the

TABLE 2. GEODETIC POSITION DETERMINATION OF STATION 3648 FROM STATIONS 3402 AND 3861

| | $\phi$ | $\lambda$ | $H$ |
|---|---|---|---|
| CCD | 32° 00′ 06″.010N | 81° 09′ 13″.789W | 7.7m |
| PC-1000 | 32 00 05.991 | 81 09 13.889 | 8.2 |
| $\Delta$ | 0.019 | −0.100 | −0.5 |
| $\sigma$ | ±0.073 | ±0.019 | ±2.6 |

| *Agreement with CCD Spatial Position* | | *Internal Consistency of Data* | |
|---|---|---|---|
| $\Delta X$ | 2.5 meters | $\sigma_X$ | 0.7 meters |
| $\Delta Y$ | 1.2 meters | $\sigma_R$ | 1.9 meters |
| $\Delta Z$ | 0.3 meters | $\sigma_Z$ | 2.8 meters |
| $\Delta R$ | 2.8 meters | $\sigma_R$ | 1.8 meters |

| *Line* | *Nets* | *Flashes* |
|---|---|---|
| Hunter-Semmes | 4 | 13 |
| Hunter-Homestead | 14 | 37 |
| Hunter-Semmes-Homestead | 1 | 4 |

$\Delta R = (\Delta X^2 + \Delta Y^2 + \Delta Z^2)^{1/2}$
$\sigma_R = 0.3333\ (\sigma_X + \sigma_y + \sigma_Z)$ when $\sigma_{min}/\sigma_{max} \geq 0.35$

*A*, *B*, *C* coefficients of the observation equations (Equations 3 and 4). New observational data applicable to the adjustment in work can also be added at this point.

The corrected (adjusted) station coordinates ($X$, $Y$, $Z$) are the most important output along with the covariance matrix containing the variances $\sigma X^2$, $\sigma Y^2$, $\sigma Z^2$ along the diagonal. There is an additional transformation-propagation program which produces the ellipsoidal coordinates $\phi$, $\lambda$, and $H$ with associated error figures.

## Camera Sites

Since January 1965, PC-1000 cameras have occupied 18 locations in the Western Hemisphere. The coordinates for eight sites occupied from 12 to 18 months are listed in Table 1. The first three stations—Semmes, Hunter, and Homestead—were used for testing and then as fixed stations for positioning the five islands—Swan, Grand Turk, Curaçao, Antigua, and Trinidad.

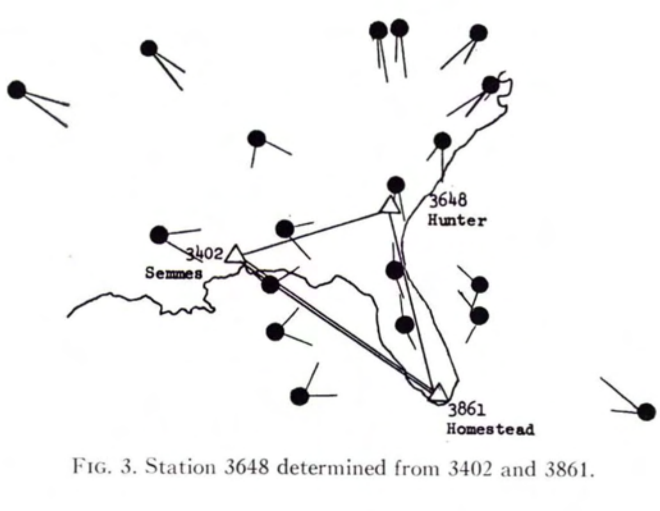


FIG. 3. Station 3648 determined from 3402 and 3861.

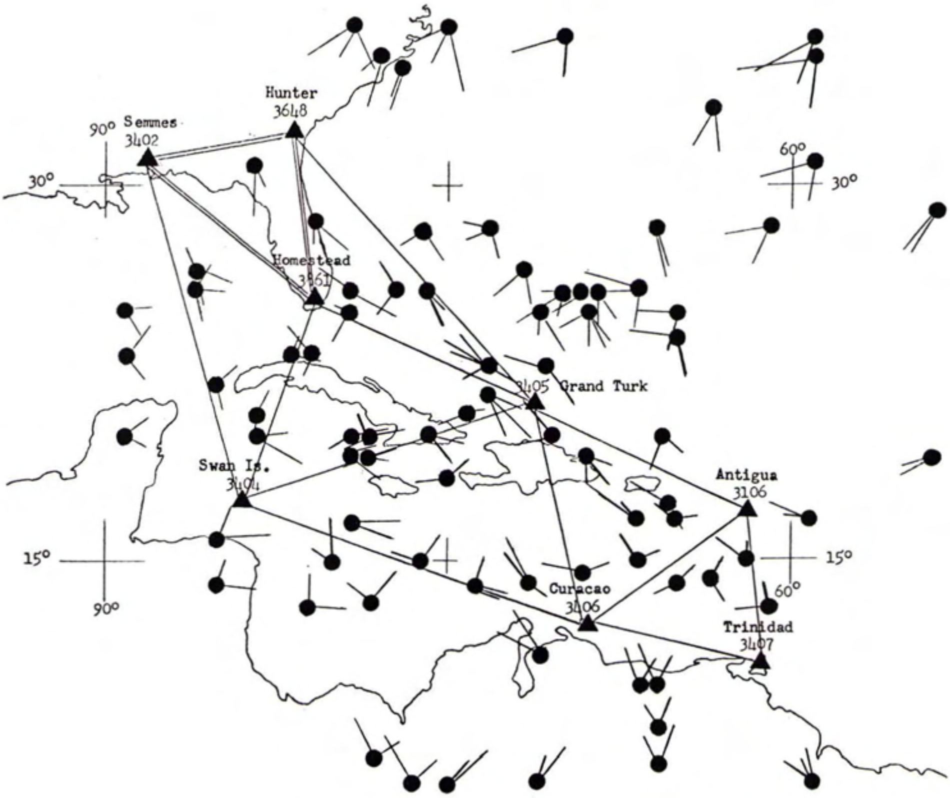


FIG. 4. Trinidad adjustment.

## RESULTS

The Semmes-Hunter-Homestead triangle was one of several test triangles. Testing was done in two operations using Cape Canaveral Datum (CCD) coordinates for Hunter and Homestead.

In the first operation, CCD coordinates were computed for Semmes holding Hunter and Homestead fixed. Fourteen nets were selected* (Figure 2). There were four nets consisting of ten flashes for the Hunter-Semmes line, nine nets with 24 flashes for the Homestead-Semmes line, and one net with all three stations. The computed coordinates for Semmes and the corresponding standard errors were as follows:

| | | | |
|---|---|---|---|
| Latitude: | 30°46′ | 49″.336N | ±0″.042 |
| Longitude: | 88°15′ | 07″.064W | ±0″.084 |
| Height: | 66.8 meters | | ±0.9m |

where the height used is the distance along the normal above the ellipsoid.

In the second test, CCD coordinates were computed for Hunter holding Semmes and Homestead fixed. The coordinates from the first operation were used for Semmes but entirely different observations were employed. The results are tabulated in Table 2 and Figure 3 shows the geometry.

The position computed for Hunter differed by 2.8 meters from the published USC&GS super-survey position. The computed position for Hunter produced only

* A net is defined as a flash sequence simultaneously observed from two or more stations.

TABLE 3.—ADJUSTED NAD-27 COORDINATES

| *Station* | $\phi$ | $\lambda$ | $H$* |
|---|---|---|---|
| Swan Is. | 17° 24′ 17″.066N | 83° 56′ 29″.765W | 50.0m |
| Grand Turk | 21 25 47.132 | 71 08 46.305 | 4.2 |
| Curacao | 12 05 23.216 | 68 50 17.795 | 30.7 |
| Antigua | 17 08 53.222 | 61 47 22.897 | 13.5 |
| Trinidad | 10 44 32.991 | 61 36 38.958 | 286.1 |

* Above ellipsoid.

a 0.4 meter difference in the chord distance between Hunter and Homestead. Expressed in proportional parts the spherical standard error—1.8 meters—is equivalent to 1/393,000.

## POSITIONING OF TRINIDAD

In November 1964, the Defense Intelligence Agency validated certain Air Force Systems Command geodetic requirements for the Eastern Test Range and directed the Air Force to accomplish a tie from Cape Kennedy to Trinidad. The decision was then made to accomplish the task through utilization of PC-1000 Geodetic Stellar Cameras and the ANNA 1-B satellite. (The GEOS-A satellite was used later to complete the project.) Homestead, Fla., Semmes, Ala., and Hunter AFB, Ga., were the stations used as base stations with Grand Turk, Swan Island, Curaçao, Antigua, and Trinidad as the downrange stations.

Observational data for the period May 1965 to June 1966 were used for the solution. More than 300 flashes from approximately 100 nets formed more than 320 observation equations for the least-squares adjustment. There were 17 nets with three or more stations two of which had five stations. Swan Island was in 27 nets with one or more of the three base stations, Grand Turk was in 16, Curaçao in 15, Antigua in 10 and Trinidad in 3. There were approximately 48 nets involving just the island stations. Figure 4 depicts the relative geometry between the nets and stations.

The coordinates for Semmes, listed in Table 1, were computed with PC-1000 data holding Hunter and Homestead fixed. Eight nets were used for the Semmes-Hunter line, nine nets for the Semmes-Homestead line, and there were two three-station nets. The standard errors for the coordinates were as follows: Latitude: ±0″.037, Longitude: ±0″.080, Height: ±1.0m.

Table 3 shows the adjusted NAD-27 coordinates for the five island stations. Table 4 presents the corresponding accuracy figures (standard error). The proportional accuracy figure is the ratio between $\sigma_R$ and the average distance to the three base stations from each island station. The $\sigma_R$'s represent only the errors in the extension. To these values must be added the error in the NAD positions of the base stations relative to Meades Ranch. This error is approximately 18 feet (one sigma).

TABLE 4. ACCURACY FIGURES

| *Station* | $\sigma\phi$ | $\sigma\lambda$ | $\sigma_H$ | $\sigma_R$ | *Proportional Accuracy* |
|---|---|---|---|---|---|
| Swan Is. | ±0″.148 | ±0″.058 | ±2.9m | 3.1m | 1/446000 |
| Grand Turk | 0.111 | 0.160 | 2.7 | 3.4 | 1/448000 |
| Curacao | 0.266 | 0.222 | 3.4 | 6.0 | 1/405000 |
| Antigua | 0.201 | 0.353 | 4.7 | 7.2 | 1/357000 |
| Trinidad | 0.366 | 0.352 | 4.9 | 8.8 | 1/345000 |

## Conclusions

The PC-1000 camera system has proven to be an accurate measurement device for geodetic positioning and should be used to satisfy future requirements for geodetic ties and strengthening geodetic control. The results from testing and the Trinidad adjustment indicate that, with the PC-1000 camera, the error propagation through a chain of triangles is about 15 feet every 1000 miles.

However, this accuracy was achieved with an adequate amount of observational data. For example, three three-station nets for each triangle and four or five two-station nets for each line. If only one three-station net is available for a triangle, then about eight two-station nets are needed for each line. This criteria assumes good geometry between station and satellite positions and observations accurate to one second of arc or better.

## References

1. Brown, D. C. *A Treatment of Analytical Photogrammetry with Emphasis on Ballistic Camera Applications*, RCA Data Reduction Technical Report No. 39, August 1957.
2. Greenwalt, C. R. and M. E. Shultz. *Principles of Error Theory and Cartographic Applications*, ACIC Technical Report 96, February 1962.
3. Hadgigeorge, G., O. W. Williams and P. H. Dishong. *Numerical Results from Geodetic Satellite (ANNA) Optical Data*, presented at the Second International Symposium, The Use of Artificial Satellites foı Geodesy, Athens, Greece, 27 April–1 May 1965.
4. Mancini, A. and L. A. Gambino. *Results of Space Triangulation Adjustments from Satellite Data*, GIMRADA Research Note No. 13.
5. Sheldon, L. L. and D. H. Eckhardt. *Geodetic Positioning from Simultaneous Optical Observation of the ANNA 1-B Satellite*, AFCRL Environmental Research Papers No. 35, July 1964.
6. Veis, G. *Precise Aspects of Terrestrial and Celestial Reference Frames*, Smithsonian Institution Astrophysical Observatory, Research in Space Science Special Report No. 123, April 1963.
7. Zhongolovich, I. D. "Earth Satellites and Geodesy," *Soviet Astronomy*, Vol. 8, No. 1, July and August 1964.