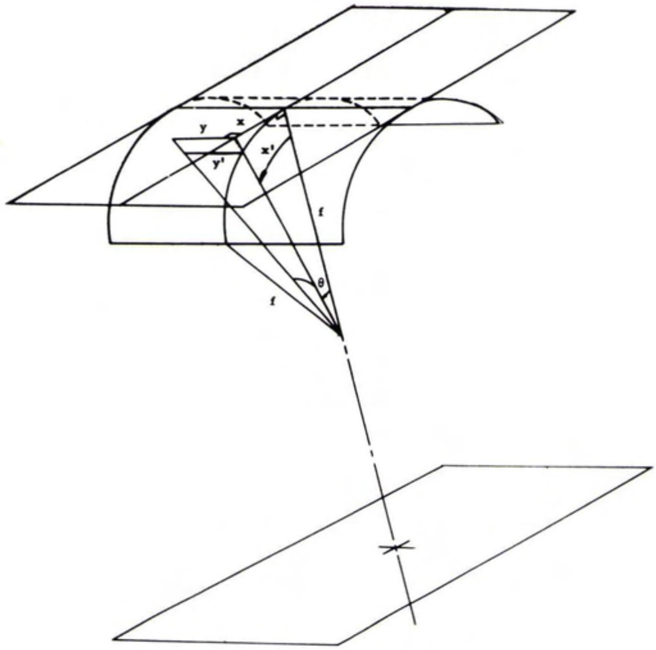

FRONTISPIECE. Panoramic projective geometry.

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Resection without Camera or Station Parameters

Computer programs are based on control points, line lengths, and projective geometry.

(Abstract on next page)

INTRODUCTION

FOR THE PAST SEVERAL years Technology Incorporated has been under contract to develop digital computer methods to resect photographs with no camera or station parameters known. Many techniques have been developed for various classes of input data and many types of cameras. The only input data required are a few control points or line endpoints within the photograph with known ground positions or lengths. Methods are presented here for flat image plane and panoramic cameras. This paper describes one computer program which can accept any of these input

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types for either type of camera. The program evaluates input data and rejects any point or line which has significant errors.

Projective Technique for Resection

The instantaneous exposure of film in a flat image plane camera creates a perspective relationship between the target area on the ground and the image upon the film. Figure 1 illustrates the geometry of film exposure. Projectivity exists between the film plane and the ground plane.

ABSTRACT: *Techniques are described which have been used to calculate resection coefficients with only control points or line lengths or both as input data. Camera parameters and aircraft location are not used. Computer programs have been written to resect both flat image plane and panoramic photographs. The mathematical techniques, based on projective geometry, are described for the basic methods of calculation. Practical innovations are also described which were developed to stabilize the solution for wide ranges of input data. A minimum mean-square-error criterion is used to measure the quality of the iteratively calculated coefficients. Practical problems encountered with this technique are discussed with some examples.*

Geometrical projective methods can be applied to the photogrammetric problem of mapping the film into the ground. If we assume that the film and ground are both flat planes then eight coefficients will uniquely map the film plane into the ground plane. The projective equations are:

$$X_i = \frac{a_{11}x_i + a_{12}y_i + a_{13}}{a_{31}x_i + a_{32}y_i + 1} \quad (1)$$

$$Y_i = \frac{a_{21}x_i + a_{22}y_i + a_{23}}{a_{31}x_i + a_{32}y_i + 1} \quad (2)$$

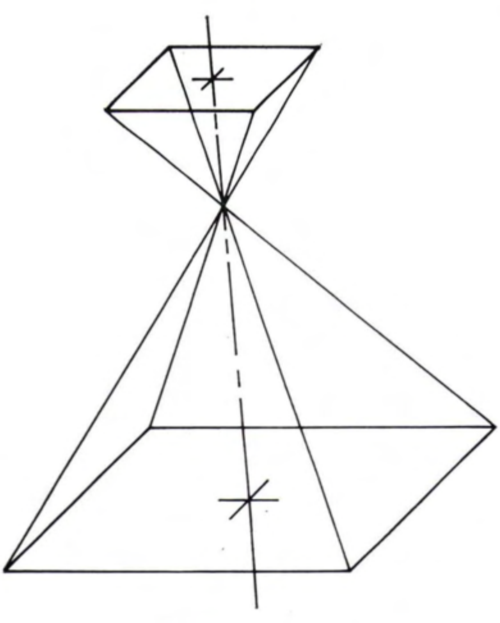

FIG. 1. Geometry of film exposure.

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where X_i and Y_i are ground plane coordinates for film plane coordinates x_i and y_i respectively of the i-th point.

The projective equations describe the mathematical relationship between points on the film plane and corresponding points on the ground plane. If we assume that the ground area photographed is a perfectly flat plane then these equations can be solved by inserting the values of x_i, y_i, X_i and Y_i for at least four known points, called control points. Four points are required because eight equations are needed to specify uniquely the eight a-coefficients of (1) and (2). If more are available, a least-squares solution can be calculated.

A significant aspect of this technique is that no knowledge is required of the interior or exterior camera parameters, as is required for collinearity resection techniques. Also this type of projective resection automatically compensates for many film errors such as nonuniform shrinkage in different directions. This basic technique projects a plane onto a plane, and additional calculations would be required to extract the third dimension. Techniques are being developed to extract the third dimension from multiple photographs of the same object.

Projective Resection Equations

The basic projective equations are stated above. These equations project points of the film plane onto the ground plane. The basic equations are adequate for flat-image-plane, frame exposure cameras.

Panoramic cameras can be modeled for this method by mapping the curved film plane onto a flat plane and then projecting that flat plane onto the ground plane with Equations 1 and 2. The Frontispiece depicts the geometry involved for panoramic photography. The flat plane is tangent to the cylindrical panoramic film at the center of the swing. Let the panoramic film coordinates for point i be x_i', y_i'. If the effective focal length of the camera is f, then the angle θ_i in the plane of the x-axis is

$$\theta_i = \frac{x_i'}{f} \cdot \tag{3}$$

Coordinates in the tangent plane for point i are

$$x_i = f \tan \theta_i \tag{4}$$

$$y_i = (y_i' + D(\sin \theta_i - \theta_i \cos \theta_i)) \sec \theta_i \tag{5}$$

where D is a parameter of the image motion compensation. Points in the tangent plane are projected to the ground using the basic Equations 1 and 2.

In many instances, photography to be resected will not contain four or more well-defined and accurately measured points to be used for control. Four points are needed to calculate the eight resection coefficients, and f and D require one additional point for panoramic photographs. Line lengths can be used for control purposes to alleviate this situation. Line lengths are often known more accurately than the exact positions of many juxtapositioned points in a specific coordinate system.

Resection with line-length control starts with the measurement of the end points of each line on the film. For line i these are designated x_{iL}, y_{iL} for the left end and x_{iR}, y_{iR} for the right end. These points are projected to the ground and the projected length is determined as

$$L_i^* = \sqrt{(X_{iL}^* - X_{iR}^*)^2 + (Y_{iL}^* - Y_{iR}^*)^2} \tag{6}$$

where the * indicates a quantity projected to the ground plane.

If no control points are used for a photograph, the following equations project X^* and Y^*:

$$X_i^* = \frac{a_{11}x_i}{a_{31}x_i + a_{32}y_i + 1} \tag{7}$$

$$Y_i^* = \frac{a_{21}x_i + a_{22}y_i}{a_{31}x_i + a_{32}y_i + 1}\,. \tag{8}$$

Comparison with Equations 1 and 2 will reveal that some terms are missing. Terms a_{13} and a_{23} cannot be specified without at least one control point to locate the photograph with respect to a ground coordinate system. At least two control points are needed to specify the rotation of the photograph with respect to the ground, therefore, a_{21} is deleted unless two points are available.

Both flat-image-plane and panoramic photographs can be resected using line length control data. The tangent plane coordinates are calculated for the panoramic case. Each end point for a line on the film is projected to the tangent plane using Equations 3, 4, and 5. Next Equations 6, 7, and 8 are used to calculate the projected ground length for each line.

Calculation Techniques

Calculation of resection coefficients follows the same pattern regardless of the type of camera or control data. See Figure 2 for a logic diagram of the calculation technique. An estimate of the resection parameters is calculated after reading the input control data. The resection parameters to be calculated vary with the camera type and whether control data consists of points, lines, or both in combination. This completes preparations to enter the iterative loop which corrects errors in the parameter estimates to drive them to a minimum mean squared error projection.

The iterative loop begins with a projection to generate ground coordinates X_i^*, Y_i^*, and/or L_i^* using the resection parameter estimates. Assume that the control data is correct and that projected coordinates are possibly in error due to in correct parameter estimates. The following expressions include terms for errors in the ground plane:

$$X_i^* + dX_i^* = X_i \tag{9}$$

$$Y_i^* + dY_i^* = Y_i \tag{10}$$

$$L_i^* + dL_i^* = L_i \tag{11}$$

or in general

$$G_i^* + dG_i^* = G_i \tag{12}$$

where G indicates ground plane data.

Differential elements dG_i^* are a total differential which can be expanded as a function of the resection parameters.

$$dG_i^* = \frac{\partial G_i^*}{\partial a_{11}}\,da_{11} + \frac{\partial G_i^*}{\partial a_{12}}\,da_{12} + \cdots + \frac{\partial G_i^*}{\partial D}\,dD. \tag{13}$$

The differential elements of the parameters da_{ij} can be linearly approximated by elements Δa_{ij}, which can be added to the previous value of each parameter to improve the estimate. A test variable is evaluated to determine if a minimum mean square error projection has been obtained. The iterative loop repeats until it is obtained.

Calculations proceed as follows:

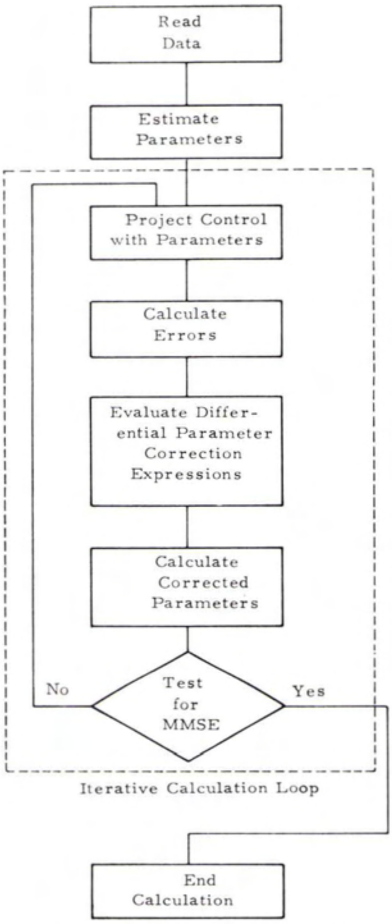

FIG. 2. Calculation technique.

TABLE 1. MODELS IN GENERALIZED PROGRAM

Camera Type	*Data Type*
Flat Image Plane	Control Points
Flat Image Plane	Line Lengths
Flat Image Plane	Both Control Points and Line Lengths
Panoramic	Control Points
Panoramic	Line Lengths
Panoramic	Both Control Points and Line Lengths

1. Estimate parameters, p^0, where 0 identifies estimates.
2. Project control data to ground, G^{*0}.
3. Calculate errors in projection as $\Delta G^{*0} = G - G^{*0}$.
4. Evaluate partial differential expressions, $\partial G^{*0}/\partial p^0$.
5. Solve linearized form of (13) as

$$[\Delta G^{*0}] = \left[\frac{\partial G^{*0}}{\partial p^0}\right][\Delta p^0]$$

$$\left[\frac{\partial G^{*0}}{\partial p^0}\right]^T[\Delta G^{*0}] = \left[\frac{\partial G^{*0}}{\partial p^0}\right]^T\left[\frac{\partial G^{*0}}{\partial p^0}\right][\Delta p^0]$$

$$\left[\left[\frac{\partial G^{*0}}{\partial p^0}\right]^T\left[\frac{\partial G^{*0}}{\partial p^0}\right]\right]^{-1}\left[\frac{\partial G^{*0}}{\partial p^0}\right]^T[\Delta G^{*0}] = [\Delta p^0]$$

6. Calculate improved estimates p^1 as

$$p^1 = p^0 + \Delta p^0$$

7. Check $\left[\frac{\partial G^{*0}}{\partial p^0}\right]^T[\Delta G^{*0}]$ for zero elements for minimum mean square error projection

8. Repeat 2 through 7 until minimum mean square error projection is obtained or until the number of iterations has reached a specified value without obtaining a minimum mean square error projection.

Detailed Calculation Techniques

The basic calculation technique is indicated in Figure 2. In the actual computer program more complexity is required to logically accomplish the necessary data checks, make the necessary decisions, and give the program the greatest *a priori* ability to calculate a satisfactory resection coefficient.

This computer program contains six calculation models which will generate resection coefficients for two types of cameras and three types of input data. Table 1 itemizes the combinations.

The number of parameters calculated to resect a photograph varies with the type of camera, the type of input data, and the quantity of input data. Table 2 describes the parameters calculated for each combination of the variables.

Figure 3 contains the logic diagram in more detail. Exact calculation methods are presented later. The general philosophy of the program is that it be self-correcting for the expected types of problems that can occur. Input data is processed into amplitude ranges which allow more stable calculations than raw data. The most likely camera model is initially selected for resection. A least squares fit will be calculated or the specified number of iterations will be exceeded. The data points or lines are tested at this stage and all bad data is rejected. Remaining data is used to recalculate a second set of resection coefficients. If no resection is obtained for the camera initially assumed, the other camera model will be used to attempt a resection.

Table 2. Parameters Calculated for Various Input Data

Case	*Camera Type*	*No. Points*	*No. Lines*	*Coefficients Calculated*										
				a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}	a_{31}	a_{32}	a_{33}	f	D
1	Flat	0	≧5	×	No (=0)	No (=0)	×	×	No (=0)	×	×	No (=1)	No	No
2	Flat	1	≧5	×	No (=0)	×	×	×	×	×	×	No (=1)	No	No
3	Flat	2	≧4	×	×	×	×	×	×	×	×	No (=1)	No	No
4	Flat	3	≧2	×	×	×	×	×	×	×	×	No (=1)	No	No
5	Flat	≧4	≧0	×	×	×	×	×	×	×	×	No (=1)	No	No
6	Pan	0	≧7	×	No (=0)	No (=0)	×	×	No (=0)	×	×	No (=1)	×	×
7	Pan	1	≧7	×	No (=0)	×	×	×	×	×	×	No (=1)	×	×
8	Pan	2	≧6	×	×	×	×	×	×	×	×	No (=1)	×	×
9	Pan	3	≧4	×	×	×	×	×	×	×	×	No (=1)	×	×
10	Pan	4	≧2	×	×	×	×	×	×	×	×	No (=1)	×	×
11	Pan	≧5	≧0	×	×	×	×	×	×	×	×	No (=1)	×	×

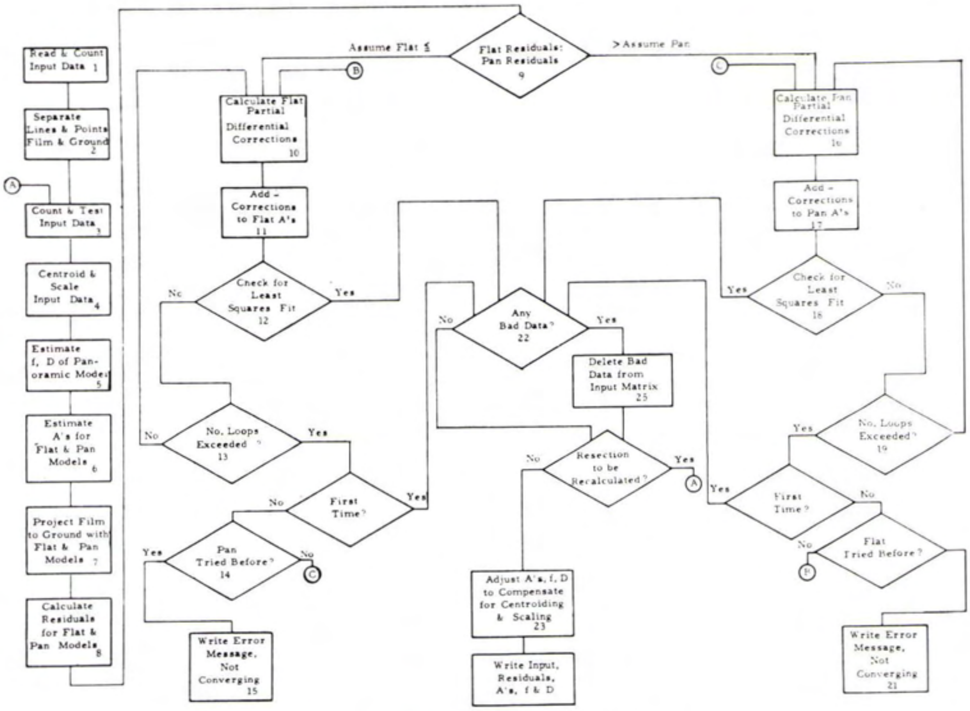

FIG. 3. Logic diagram of generalized program.

ADJUSTMENT OF RAW DATA

Numerical values of raw input data may vary widely. Computation problems can be caused if certain undesirable ranges of amplitudes occur in the data. Significant information may differ by many orders of magnitude less than the data, such as 123456789 compared with 123456790. The film and ground data may vary by many orders of magnitude. Estimates required for initial values in many of the techniques must be reasonably accurate for any input units which might be used during the lifetime of the computer program. The mathematical iteration routine must be stable for all input data units; it must converge to a set of rectification coefficients. The effect of internal computer roundoff and truncation could become significant for some input data units.

A general solution has been developed which permits the computer program to compensate for variations in the input. This solution consists of centering the actual data on a local coordinate system about (0, 0) and scaling so that the maximum amplitude of any element is unity.

The centroiding (centering at local 0, 0) and scaling cannot be applied to all types of data. Specific input processing is presented in Table 3.

It is impossible to centroid panoramic data because the center of the image is the center of the panoramic sweep. This center must be preserved to calculate the proper relationship between the film and its tangent plane. Consequently, the panoramic input data must be presented to the program in frame coordinates, with the center of the panoramic image being at $x=0$, $y=0$.

As centroiding only subtracts a bias from the input, the line length data is not centroided. If both line lengths and control points for the same photograph are used, the control points can be centroided in the ground plane.

TABLE 3. SPECIFIC INPUT DATA PROCESSING

Camera Type	*Data Type*	*Centroid Film*	*Centroid Ground*	*Scale Film*	*Scale Ground*
Flat	Points	Yes	Yes	Yes	Yes
Flat	Lines	Yes	No	Yes	Yes
Flat	Both	Yes	Yes	Yes	Yes
Pan	Points	No	Yes	Yes	Yes
Pan	Lines	No	No	Yes	Yes
Pan	Both	No	Yes	Yes	Yes

INITIAL ESTIMATES OF RESECTION PARAMETERS

Resection of flat image plane photographs with four or more control points and any number of line lengths requires eight a elements. A ninth element, a_{33}, is fixed at unity. Each point yields two equations; therefore, $2N$ independent linear equations can be written in terms of N ground coordinates X, Y and the image plane point coordinates x, y. Line lengths are not used for initial estimates since four points are sufficient to calculate the eight a's.

$$X_i = \frac{x_i a_{11} + y_i a_{12} + a_{13}}{x_i a_{31} + y_i a_{32} + 1}$$

$$Y_i = \frac{x_i a_{21} + y_i a_{22} + a_{23}}{x_i a_{31} + y_i a_{32} + 1} \cdot \qquad i = 1, 2, \cdots, N \tag{14}$$

Rewritten in matrix form, Equation 14 becomes

$$\begin{bmatrix} X_1 \\ \cdot \\ \cdot \\ \cdot \\ X_N \\ Y_1 \\ \cdot \\ \cdot \\ \cdot \\ Y_N \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1X_1 & -y_1X_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_NX_N & -y_NX_N \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1Y_1 & -y_1Y_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_NY_N & -y_NY_N \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{31} \\ a_{32} \end{bmatrix}. \tag{15}$$

Written in matrix notation, (15) becomes

$$(Z) = (A)(a). \tag{16}$$

Solving this simultaneous set requires eight equations instead of $2N$ equations ($2N \geqq 8$) because there are only eight unknowns. Equation 16 is solved in the following manner:

$$(A)^T(Z) = (A)^T(A)(a) \tag{17}$$

and

$$[(A)^T(A)]^{-1}(A)^T(Z) = (a). \tag{18}$$

The solution for the flat image plane photograph with no control points and at least five line lengths yields five of the a coefficient elements. Coefficients a_{11} and a_{22} are estimated as the average of the ratios between the line lengths in the ground and film images. The equation to estimate these coefficients is

$$\text{ratio} = \frac{1}{N}\sum_{i=1}^{N} \frac{L_i}{\sqrt{(x_{i\,\text{left}} - x_{i\,\text{right}})^2 + (y_{i\,\text{left}} - y_{i\,\text{right}})^2}} .$$

Coefficients a_{21}, a_{31}, and a_{32} are initially estimated to be 0. Coefficients a_{12}, a_{13}, and a_{23} are assigned the value 0 because the data is insufficient to calculate these parameters.

With one control point its coordinates become the center for the system, a_{13} and a_{23}. However, in centroided data this is (0, 0); hence, the above line length estimates are adequate.

With two or three control points, initial estimates are calculated from both line and point data. The points are centroided and scaled. The ratio of Equation 19 is assigned to a_{11} and a_{22}, whereas 0 is assigned to a_{13} and a_{23}. Point data is used to estimate the other parameters by solving:

$$\begin{bmatrix} X_1 - a_{11}x_1 \\ X_2 - a_{11}x_2 \\ Y_1 - a_{22}y_1 \\ Y_2 - a_{22}y_2 \end{bmatrix} = \begin{bmatrix} y_1 & 0 & -x_1X_1 & -y_1X_1 \\ y_2 & 0 & -x_2X_2 & -y_2X_2 \\ 0 & x_1 & -x_1Y_1 & -y_1Y_1 \\ 0 & x_2 & -x_2Y_2 & -y_2Y_2 \end{bmatrix} \begin{bmatrix} a_{12} \\ a_{21} \\ a_{31} \\ a_{32} \end{bmatrix}. \tag{20}$$

With three points at least squares fit must be calculated.

Panoramic resection coefficient estimates are calculated in exactly the same manner as stated above for flat image plane estimates, after the film is projected onto a flat tangent plane. Panoramic parameters f and D are estimated at 1.3 and 0 respectively for scaled input data. Observing the geometry of the Frontispiece calculate

$$\theta_i = \frac{x_i'}{f} \tag{21}$$

where the prime mark denotes a film coordinate.

Tangent plane coordinates are

$$x_i = f \tan \theta_i \tag{22}$$

$$y_i = (y_i' + D(\sin \theta_i - \theta_i \cos \theta_i)) \sec \theta_i. \tag{23}$$

Matrix Equation 15 calculates panoramic estimates if four or more data points exist. Equation 19 is used if no or one point exists. Equation 20 is used if two or three points are present in the input data.

(The complete general partial differential equation for any case is expressed as a matrix, and also the terms are completely expanded in the manuscript form of this article. In the interest of brevity, however, these details are not included here. The reader may obtain these items by writing the author.—*Editor*)

Rejection of Bad Data Points

Many sources of potential control data errors occur. Photo interpreters can mislocate coordinate reader crosshairs, or correctly locate them over an adjacent similar object. If manual data copying occurs, it is possible to transpose adjacent digits, or copy the wrong number, misread an instrument, etc. These types of errors are normally much larger in amplitude than typical reading errors or uncorrected systemic errors.

Typical reading errors occur if no mistakes are made. These reading errors are due to such causes as parallax in the positioning of crosshairs, backlash in geartrains of the coordinate reader, roundoff in the digitization of the least significant digit to be keypunched, etc. Errors of this type are caused by small flaws in the design or performance of the photo interpreter and each machine which processes the data prior to its entering the resection program input format.

Systemic errors are caused by the processes which generate the control data. Ground points could be measured with inaccurate instruments, such as an inaccurate transit angle or a biased length measurement. Atmospheric refraction causes a somewhat predictable bending of the light rays which expose the film. Lens distortion is a typical systemic camera error, as well as displacement of the principal point from the exact center of the film format. Uncorrected systemic errors are those for which no compensation is made in the control data prior to its entering the computer for resection.

Reading errors and uncorrected systemic errors are considered independent of any bad control data readings. Bad data is usually confined to one or two points within a set of ten to twenty. These bad data points have errors which are significantly larger than those of the good points.

A rejection technique has been developed which has proven quite adequate. This technique is utilized after a series of iterations has been performed on a set of input data. First the amplitude of the distance errors for each point is calculated using:

$$\Delta L_i^* = \sqrt{\Delta X_i^{*2} + \Delta Y_i^{*2}}. \tag{24}$$

For line lengths this is calculated during the iterative loop. A rejection criteria C is calculated where

$$C = 2 \cdot \frac{1}{N} \sum_{i=1}^{N} \Delta L_i^*. \tag{25}$$

Any point is rejected if its ΔL_i^* exceeds the value C.

$$\Delta L_i^* > C \rightarrow \text{data}_i \text{ is bad} \tag{26}$$

Adjustment of Resection Parameters to Input Units

The data units used to calculate resection coefficients have been centroided and scaled. An adjustment is required to convert these internal computer program units to be compatible with the actual input and output units. Typical actual units are reader counts for film data and feet for ground data.

The actual units are projected by matrix A, where

$$[A] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & 1 \end{bmatrix} \tag{27}$$

using Equations 1 and 2. This will project either the flat image plane or the panoramic tangent plane to the ground.

Panoramic parameters f and D are adjusted to be:

$$f_{\text{out}} = f \cdot SFF \tag{28}$$

$$D_{\text{out}} = D \cdot SFF \tag{29}$$

Equations 3, 4, and 5 are used to project the panoramic film data to a tangent plane which is properly scaled.

Matrix A of (27) has the following general elements, where Table 4 describes the parameter values to be used for particular types of input control data.

$$
[A] = \begin{bmatrix}
\dfrac{a_{11}SFG + X_c a_{31}}{\bar{a}_{23}} & \dfrac{a_{12}SFG + X_c a_{32}}{\bar{a}_{33}} & \dfrac{\bar{a}_{13}SFG + X_c \bar{a}_{32}}{\bar{a}_{33}} \\
\dfrac{a_{21}SFG + Y_c a_{31}}{\bar{a}_{33}} & \dfrac{a_{22}SFG + Y_c a_{32}}{\bar{a}_{33}} & \dfrac{\bar{a}_{23}SFG + Y_c \bar{a}_{33}}{\bar{a}_{33}} \\
\dfrac{a_{31}}{\bar{a}_{33}} & \dfrac{a_{32}}{\bar{a}_{33}} & \dfrac{\bar{a}_{33}}{\bar{a}_{33}}
\end{bmatrix}
$$

where

$$\bar{a}_{13} = a_{13}SFF - a_{11}x_c - a_{12}y_c$$

$$\bar{a}_{23} = a_{23}SFF - a_{21}x_c - a_{22}y_c$$

$$\bar{a}_{33} = SFF - a_{31}x_c - a_{32}y_c$$

Applications and Extensions

The obvious advantage of this technique is that only control data is used to calculate resection coefficients. No internal camera or external exposure station parameters are used as input data. The ability to use line lengths as control data, especially for panoramic photography, enhances the value of this technique.

This method appears to be as accurate as the collinearity technique for resection of any ground data which appears in a reasonable approximation to a flat plane, which can be tilted. Research is nearly completed to apply this method for extraction of the third dimension, and thus make it perfectly general. Three-dimensional mapping processes will differ from those using collinearity models. This will effectively remove any planar restrictions of this technique.

Strip photography can often be resected quite accurately using the flat image plane equations. Other more exact strip camera resection models have been developed using this same general method.

Hopefully, this method will become one of the standard techniques of analytical photogrammetry.

Practical Problems Encountered

These techniques have undergone considerable testing in many computer programs utilizing several coding languages. Fortran IV seems to perform better than matrix manipulation macro languages. The computer programs seem quite stable and normally a resection is obtained for the typical quality of aerial photography.

Input data must be examined to insure that three points do not lie on a line. Likewise, the data should be well distributed across the area to be resected, to avoid excessive errors due to extrapolation outside of the periphery established by the control coordinates.

Table 4. Elements of Matrix A

Camera Type	*No. Points*	x_c	y_c	X_c	Y_c	a_{12}	a_{13}	a_{23}
Flat	$\geqq 2$	x_c	y_c	X_c	Y_c	a_{12}	a_{13}	a_{23}
Flat	1	x_c	y_c	X_c	Y_c	0	a_{13}	a_{23}
Flat	0	x_c	y_c	0	0	0	0	0
Pan	$\geqq 2$	0	0	X_c	Y_c	a_{12}	a_{13}	a_{23}
Pan	1	0	0	X_c	Y_c	0	a_{13}	a_{23}
Pan	0	0	0	0	0	0	0	0

Comparison with the Collinearity Technique

Identical control point data was used with this projective model and the collinearity model. The collinearity model used has the focal length given and eight parameters are iteratively calculated: three aircraft station coordinates, three aircraft orientation angles, and two interior camera parameters, the coordinates of the principal point on the film. The collinearity fit was calculated in the film plane. The flat image plane projective model was used and eight a_{ij} parameters were iteratively calculated.

Control point data was calculated with noise inserted under controlled conditions. The ground data was in a flat plane to force the collinearity model to solve the same physical problem that the projective model solves. Thus two different mathematical models were used with identical input data.

The accuracy of the two methods is identical. Residual errors for each program are shown in Figure 4, and are identical for each control point. This indicates that no accuracy is lost in using the projective resection model. It is anticipated that the third dimension will be added to the projective technique, and at that point the two methods will probably be of equal accuracy for all types of resections. Then, and even now for flat ground target areas, the projective technique will have an advantage over the collinearity technique, because projective methods require no estimates for the interior and exterior parameters of the camera.

Control Data

X	Y	x	y
-3000.0000	1500.0000	-19577.0CC0	6989.0000
-1500.0000	15C0.0000	-9801.0C00	7080.0000
0.	1500.0000	33.0CC0	7018.0000
1500.0000	1500.0000	9747.0CC0	7094.0000
3000.0000	15C0.0000	19641.0000	6915.0000
-3000.0000	0.	-21172.00C0	47.0000
-1500.0000	0.	-10596.0000	-19.0000
0.	0.	-50.0CC0	191.0000
1500.0000	0.	10657.0C00	8.0000
3000.0000	0.	21075.0CC0	-74.0000
-3000.0000	-15C0.0000	-22992.0C00	-8038.0000
-1500.0000	-15C0.0000	-11579.0C00	-7997.0000
0.	-1500.0000	85.0C00	-8091.0000
1500.0000	-15C0.0000	11343.0C00	-8167.0000
3000.0000	-1500.0000	22943.0000	-8155.0000

Note: Z = 0 for all points

Residuals for Projective Resection		Residuals for Collinearity Resection	
18.30962	-65.41144	18.31000	-65.41193
-4.57687	48.29749	-4.57692	48.29697
28.07806	9.01212	28.07789	9.01178
-61.72650	107.73255	-61.72575	107.73204
26.00840	-48.54137	26.00868	-48.54187
-12.49002	-53.43657	-12.48996	-53.43676
-5.91291	-94.C0053	-5.91318	-94.00077
-32.19635	141.44240	-32.19633	141.44212
99.65869	-16.10778	99.65871	-16.10808
-60.34882	-72.65107	-60.34861	-72.65151
-1.25261	2.70971	-1.25180	2.70955
-59.73653	72.51275	-59.73618	72.51248
129.41006	7.32442	129.41030	7.32414
-90.81463	-39.85561	-90.81448	-39.85606
27.58831	0.97276	27.58820	0.97230

FIG. 4. Comparison between residuals calculated by projective and collinearity resection techniques for flat ground plane control data.

THIS IS THE PROJECTIVE TRANSFORMATION MATRIX

5.3853317E-01	1.0873094E-02	-1.2603916E C4
-5.4347647E-03	5.4414369E-01	-2.5198177E 03
3.2159701E-08	5.646061CE-07	1.0000000E 00

INPUT DATA				RESIDUALS (GROUND UNITS)	
(FILM)		(GROUND)			
X	Y	X	Y	X	Y
23991.CCCCO	16382.CCCCO	482.24000	6206.73999	7.01358	-4.92911
24614.CCCOO	14089.00CCO	792.34000	4973.21C02	5.41308	-3.82241
27421.CCCOO	23022.CCCCO	2370.26001	9710.26C01	10.22071	13.20787
24434.CCCCO	11114.00CCO	684.5000C	3373.57999	-13.78855	-2.38111
27420.CCCOO	20533.0000O	2348.42999	8394.46997	8.09331	4.81342
3C198.CCCOO	23072.CCCCC	3848.07999	9735.15595	7.52284	-0.91158
30232.CCCOO	19893.COCCO	3848.07999	8029.40997	-1.70380	12.96996
3C329.CCCCO	16901.0CCCO	3865.89001	6432.28998	6.40467	11.85569
30311.CCCOO	13825.COCCO	3840.38000	4773.23599	-4.18023	22.88343
30355.0C000	1C836.CCCCO	3842.67999	3196.32599	-8.79937	-7.40204
33226.CCCOC	23162.00CCO	5470.91998	9744.44995	-6.98290	20.47888
33242.CCCCO	2C063.CCCCO	5456.95001	8088.78C03	-8.34453	27.28191
33312.CCCOO	16939.COCCO	5464.44000	6426.58C02	-2.64684	21.23571
33345.CCCOO	1C931.0COCO	5439.57001	3209.78C00	-6.60054	13.86245
33447.CCCCC	4967.CCCCO	5443.23999	11.02000	-1.94290	-9.85688
36291.CCCCO	23228.00CCO	7097.27002	9742.21997	-5.99461	40.38583
36364.CCCOO	19282.CCCCO	7108.39001	8061.58002	-5.06984	-379.46592
36368.CCCOO	17053.00CCO	7091.94000	6458.88C00	-1.62278	32.83672
36395.CCCOO	14C17.CCCCO	7083.44000	4843.85999	0.61147	21.58580
36422.0CCOO	11C00.COCCO	7078.00000	3240.20999	-0.10586	3.66172
36425.CCCOO	7977.00CCO	7055.50000	1617.92000	3.32850	-4.22289
393C0.0CCOO	23266.COCCO	8691.00000	9740.18005	-2.70058	45.54866
39321.0CCOO	20155.00CCO	8688.57996	8088.23999	-7.45179	42.64953
39333.CCOOO	17122.C0000	8666.27002	6474.53003	3.33106	37.52430
39352.CCCOO	14127.00000	8653.27002	4878.21997	8.72354	29.85223
39639.CCCOO	1C912.00000	8780.25000	3163.13000	15.99067	15.68275
39455.CCCCO	8049.00000	8679.68005	1641.38C00	1.28185	-5.32459

THE FOLLOWING POINTS REJECTED, RESIDUALS LARGER THAN ALLOWED

36364.CCCCC	19282.CCCCO	7108.39C01	8061.58C02

FIG. 5. Typical computer output with one bad data point identified.

CALCULATION EXAMPLES

An example of typical computer printout is shown in Figure 5. Data points from an aerial photograph were used as control, and one was accidentally misread. The program calculated a set of resection coefficients using the flat image plane math model. The bad point (17th from top) was detected and rejected from the set of control points, and a second set of resection coefficients was calculated, see Figure 6.

THIS IS THE PROJECTIVE TRANSFORMATICN MATRIX

5.4258177E-01	8.7633G48E-03	-1.2674074E C4
-5.9281625E-03	5.4295397E-01	-2.4762621E 03
3.492109CE-07	2.106011CE-07	1.CCCCCCCE CC

INPUT DATA				RESIDUALS (GROUND UNITS)	
(FILM)		(GROUND)			
X	Y	X	Y	X	Y
23991.CCC00	16382.00CC0	482.24CCC	6206.73599	-1.36256	-3.91968
24614.CCC00	14089.00CC0	792.34CC0	4973.21C02	2.96359	-3.17618
27421.CCC00	23022.CCCC0	2370.26001	9710.26C01	1.34062	10.59262
24434.CCCC0	11114.0C000	684.5C0C0	3373.57595	-11.05866	2.98538
2742C.CCC00	2C533.0CCC0	2348.42999	8394.46997	2.34964	-1.46824
30198.CCC00	23072.C0CC0	3848.07999	9735.15595	5.55361	-13.20885
30232.0C000	19893.CCCC0	3848.07959	8029.40997	-1.22296	-2.28378
30329.0C000	16901.00CC0	3865.89CC1	6432.28598	9.27079	-2.86254
30311.0CC00	13825.000C0	384C.38CCC	4773.23995	1.04839	12.55648
30355.C0CC0	1C836.0CCCC	3842.67599	3196.32555	-1.21291	-10.13793
33226.CCC00	23162.0CCC0	5470.91598	9744.44595	-4.29146	-2.33951
33242.0CCC0	2C063.0CCC0	5456.95C01	8088.78003	-5.C2756	3.05513
33312.CCC00	16539.CCCC0	5464.44CCC	6426.58C02	1.32978	-0.89608
33345.CCC0C	1C931.C0CC0	5439.570C1	3209.78CCC	-1.30852	6.45786
33447.C0C00	4967.0CCC0	5443.23999	11.020CC	4.61528	11.01061
36291.CCC00	23228.0CCC0	7097.27CC2	9742.21597	-1.59144	6.87041
36368.CCCCC	17053.00CC0	7091.94CC0	6458.88CCC	0.49366	2.98311
36395.CCCC0	14017.C0CC0	7083.44C00	4843.85595	1.62137	-1.13627
36422.CCC00	11C0C.0CCC0	7078.0C000	3240.20599	-0.18390	-8.48481
36425.CCC00	7977.0CCC0	7055.5CCCC	1617.92CCC	2.24011	-2.23748
393C0.CCC00	23266.CCCC0	8691.CCC0C	9740.180C5	0.40881	1.52024
39321.CCC0C	20155.CCCC0	8688.57996	8088.23999	-7.22811	0.23550
39333.0CC00	17122.0GC00	8666.27CC2	6474.53C03	0.77077	0.21201
39352.CCC0C	14127.0CCC0	8653.27002	4878.21597	3.41608	0.97652
35639.CCC00	10912.CCCC0	8780.25C00	3163.130CC	6.87136	-0.70775
39455.0CC00	8049.CCCC0	8679.68CC5	1641.3800C	-9.80600	-6.59671

FIG. 6. Typical computer output after bad data has been deleted and resection coefficients are recomputed.

ACKNOWLEDGMENTS

Technology Incorporated is entering its fourth year of research in developing this technique. The basic projective resection equations for flat image plane cameras and control points were developed about five years ago by Dr. Paul Pepper. During the past research this technique was examined, developed and extended in logical steps, to the point where this type of general program is now available.

Dr. Pepper is preparing a paper which will contain the mathematical details and derivations of the basic projective process. Mr. Michael Hord plans to report on several panoramic techniques which he has developed. The author is grateful to Mr. Paul Whittemore who has been analyzing collinearity methods and provided comparison results for presentation here.

XI CONGRESS ISP, LAUSANNE, SWITZERLAND

Here are brief samples of the package tours that are completely described in a brochure available on request from the American Society of Photogrammetry, 105 N. Virginia Ave., Falls Church, Va. 22046.

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