JESSE R. CHAVES\* *Bureau of Public Roads Washington, D.C. 20591*

# **Two Methods of Analytic Triangulation for Highways**

**The accuracies of the Canadian National Research Council and the U. S. Coast & Geodetic Survey systems are compared.**

*(Abstract on page 698)*

## **INTRODUCTION**

THE RESULTS OF AN investigation are reported in which two methods of analytic aerial triangulation are compared. The methods selected for evaluation are those developed by the National Research Council of Canada  $(NRC)^{1,2}$  and the U.S. Coast and Geodetic Survey (CGS).3,4 Two scales of photographs were used to evaluate the accuracy of computed ground coordinates. Two previous investigations<sup>5,6</sup> have been conducted to determine the feasibility of using analytic aerial triangulation in highway engineering. This included evaluating computed ground coordinates for supplemental control for large-scale, small contour interval mapping employed in highway location and design. In the work previously reported, coordinate measurements were made with monocular comparators, while in this investigation a stereocomparator was utilized.

#### AERIAL PHOTOGRAPHY

The aerial photographs used for this evaluation were made available by the Region 15 office of the Bureau of Public Roads. The photographs were used in a mapping project for an extension of Colonial Parkway near Williamsburg, Virginia. Two photographic flight strips which were used for the comparative evaluation experiment were taken with a Wild RC-8 mapping camera equipped with a six-inch focal length Aviogon lens. The first flight strip which was about 12,000 feet long consisted of ten photographs at scale of 1 :4,800. The second flight strip which was

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about 16,000 feet long, consisted of six photographs at a scale of 1 :9,600. The larger scale photographs were used for map compilation as well as for bridging. The smaller scale photographs were used exclusively for analytic bridging. Diapositive plates were printed emulsion-to-emulsion from the aerial negative film using an automatic dodging printer.

### GROUND CONTROL SURVEY AND PHOTO-GRAPHIC TARGETS

Basic horizontal control was surveyed to better than second-order accuracy using a Tellurometer and Wild T-2 theodolite. The vertical control was surveyed to second-order accuracy using a Zeiss N-2 automatic level. The ground positions of 39 points and elevations of 42 points were surveyed on the ground. The ground coordinates of these points were available for controlling the triangulated strips as well as for testing the accuracy of the analytically computed coordinates.

All surveyed points except four were premarked using photographic targets. The other four points were natural objects that could be readily identified in the aerial photographs. Three types of target designs were used as markers of surveyed ground control. Nine targets of type *A* shown in Figure 1 were placed throughout the photographed area by the mapping contractor. The legs of target type *A* were made of white muslin. Seven targets of design *B* and twenty-three of design C were placed throughout the project by personnel of the Bureau of Public Roads. The center of target design *B* consisted of alternating colored cloth wedges. The alternating colors of these wedges were either blue and black or brown and black. The centers of

target design C were solid block squares. \\'henever targets were placed in wooded areas the legs were extended somewhat to make them easier to find on the aerial photographs.

#### PHOTOGRAMMETRIC INSTRUMENTS AND MEASURING PROCEDURE

Photographic *x* and *y* plate coordinates were measured with a Wild STK 1 Stereoused to drill six pass points along the center of each photographic plate and perpendicular to the flight axis. Two holes were drilled in the vicinity of the customary pass point locations. These were located in areas of relatively flat topography wherever feasible. Removal of *x* and *y* parallaxes at the time of coordinate measurement was more accurately and readily accomplished in areas of flat topography.

ABSTRACT: *The resnlts of a comparison of two methods of analytic aerial triangulation are given. The evaluation was performed with* 1: *4,800- and* 1: *9,600 scale photographs. The strip coordinate computations and strip adjustments for the Canadian National Research Council and U.* S. *Coast and Geodetic Survey methods were tested 'Using the same measured plate coordinates and ground control. Second degree polynomials in both methods gave better overall results.* A1 *though the error propagalion within each strip computation was undoubtedly dig'erent, the resulting computed ground coordinates are not significantly affected.*

comparator. The comparator was equipped with a Wild EK 4 Electric Coordinate Printer which recorded coordinate measurements to the nearest even micron. Comparator output was recorded by means of a typewriter and card punch. Punched cards were in the proper format for computer program input. Measurements were made in a temperature controlled room (70°F  $\pm$  1°) using 11  $\times$  magnification and a 40-micron diameter measuring mark.

A Wild Pug 3 Point Transfer Device was

Although a three-dimensional view was available for selection of pass point locations, all holes were actually drilled monocularly using the same drill.

The orientation of each plate during measurement was with its emulsion side down and the photographic x-axis nearly parallel to the com parator x-axis. The *x* and *y* coordinates were measured on the left-hand stage while the parallaxes, *px* and *py,* were recorded from the right-hand stage.

Each of the image points was measured



FIG. 1. Photographic target designs.

iour times. Because the fiducial marks in the camera had open centers, it was necessary to measure each of the four legs and then mathematically intersect for the center of the fiducial. Five measurements were made on each leg.

#### ANALYTIC SYSTEMS

It is beyond the scope of this paper to present the mathematical basis of the analytic aerial triangulation systems used in this investigation. The reader who is interested in pursuing the theoretical basis of the subject should consult the References I, 2, 3, 4, 7, 8, 9, and 10. References 1, 2, 3, 4, and 9 also contain documented Fortran programs.

#### **COMPUTERS**

Four computers were used in this investigation, namely, the IBM 7030 (STRETCH), IBM 7010 (60K), IBM 7090, and IBM 360 Model 50. These four computers were used because of their availability at the time. In addition, no computer program conversions were attempted in order to avoid unnecessary delay. At the present time, however, all programs are operational on the IBM 360.

The STRETCH computer was made available through the courtesy of the U. S. Coast and Geodetic Survey for computing the strip coordinates using the triplet method. The NRC strip computations were performed on the IBM 7010 system using an 18-digit mantissa. The CGS strip adjustments were computed on the IBM 7090 and the NRC strip adjustments with the IBM 360.

# COMPUTER PROGRAM FEATURES

There are a number of features regarding the strip computation and strip adjustment program that are worthy of note.

For the CGS strip computation, the photographic *x* and *y* coordinates of each image point occur on a separate card. From one to ten measurements can be made for each point, but all of the cards must be together. For multiple observations, coordinates which deviate more than 25 microns from the mean are rejected. If two such rejections occur in a given set, the computation is stopped and a new triangulation is started. The coordinates of pass points for each photograph of the triplet must be in a strict sequence. A card sort is performed to insure the proper order. In the triplet solution a given pass point image is rejected if its residual parallax exceeds the limit set by the user. The companion pass point in the same area is then substituted for it. If two pass points in the same area for a given model are rejected the solution is terminated.

The vector sum of all *x* and *y* parallaxes is printed out for each pass point which appears on three photographs. Only the *y* parallaxes are output for all other image points. A single root-mean-square value of all the residual parallaxes for the 18 pass points in a triplet is also output. This serves as a reliability number for the triplet.

The NRC strip computation has provision for correcting the measured photographic coordinates for the effects of differential film shrinkage, radial lens distortion, earth curvature, and atmospheric refraction.\* Two correction factors for differential film shrinkage are applied in the *x* and *y* directions. This single set of values is applied to all the photographs in the flight strip. Any number of image points may be used for relative orientation and an experimental weighing equation may be applied if the photographs have been obtained with a Wild 6-inch Aviogon lens. With this equation, image points near the principal point are given more weight in the relative orientation solution than those located near the corners of the photograph. Up to 10 image points may be used for scaling by use of an appropriate signal on each scaling point card. Equal weight is given to each scaling point. There are also four standard patterns for the scaling points that can be used depending upon a number punched in the first data card of the strip. This same pattern of image points is used throughout the triangulated strip, but a maximum of four scaling points is permitted with the standard patterns. The program has provision for discarding anomalous scale transfer points.

The measured photographic coordinates for input to the NRC program are arranged in groups according to models. The first card of each model contains the coordinates of the principal points of the two photographs. On this card is a number which determines the number of points to be used in relative orientation. The coordinates of corresponding image points appear on each of the subsequent cards, The cards for the relative orientation points follow immediately after the first card. All other object point cards come last. Residual *y* parallaxes are printed out for each image point. Providing the value of the base component *bx* in the first model has been set equal to the actual distance on the photo-

\* The coordinate refinement portion of the NRC program was not utilized in this investigation.

### 700 PHOTOGRAMMETRIC ENGINEERING



**• VERTICAL - PICTURE POINT**

FIG. 2. Ground control distribution for the 1:4,800-scale photography.

graphs, the residual parallaxes will be at photograph scale.

The input data for the CGS and NRC strip adjustment programs are similar. This includes: (a) the strip coordinates and surveyed ground control data; (b) strip coordinates of all the points whose ground coordinates are needed; (c) the *x* and *y* strip coordinates of two points near each end of the flight strip for defining the axis-of-flight; and (d) a card containing the degrees of polynomials to be used. The CGS and NRC methods provide for first, second and third degree polynomials for correcting the horizontal and vertical coordinates. The NRC program also allows for higher degree polynomials and the use of a separate degree polynomial for correcting (a) scale and azimuth, (b) longitudinal tilt, and (c) transversal tilt. The NRC program can also be used for a block adjustment of parallel overlapping flight strips.

#### EVALUATION SCHEME

In testing the accuracy of the analytic computations by the two methods the following procedure was employed:

- (a) The measured *x* and *y* plate coordinates for the 1:4,800- and 1:9,600-scale photographic strips were corrected for film and radial lens distortion.<sup>†</sup>
- (b) The strip coordinates for the two flight strips were computed by the NRC and CGS methods using the same set of refined coordinates.<br>Twelve image points in each model were used to compute the relative orientation. Two of these image points were located in each of the six conventional pass point locations.

t The mathematical formulation is described in Reference 9. This method of coordinate refinement is included as an integral part of the Three-Photo Aerotriangulation program.'

- (c) The computed strip coordinates for each scale of photographs were adjusted and transformed to ground coordinates by the NRC and CGS strip adjustment methods. Second and third degree polynomial adjustments were employed for each of four combinations. These four combinations of strip coordinates and strip adjustments were:
	- (1) NRC strip coordinates and NRC strip adjustment (NRC-NRC)
	- $(2)$  CGs strip coordinates and CGs strip adjustment (cGS-CGS)
	- (3) NRC strip coordinates and CGS strip adjustment (NRC-CGS)
	- $(4)$  CGs strip coordinates and NRC strip adjustment (CGS-NRC).

The foregoing procedure was designed to permit a comparison to be made between two independent methods of analytic triangulation and to enable a determination to be made of the differences due either to the strip adjustments or possibly due to the strip coordinate computation.

(d) The standard errors for the X, Y and Z coordinates were computed for all ground surveyed test points which were withheld from the strip adjustment solution.

Figure 2 shows the ground control distribution used for controlling and testing the computations for the  $1:4,800$ -scale photographs. Four horizontal and seven vertical control points were used for strip adjustment. The horizontal ground control points used are identified in the figure as  $T-26$ ,  $AT-9$ ,  $T-29$ and T-4. The vertical control is designated as AT-5, T-26, T-11, T-13, T-28, T-2 and T-3. The ground control distribution for the 1: 9,600-scale photographs is shown in Figure 3. Four horizontal control points (AT-3, AT-9, AT-7, T-4) and eight vertical control points (AT-4, T-17, T-9, T-11, T-28, T-1, 1'-3) were used to adjust this strip.

The  $X$  and  $Y$  ground coordinates of 14 points and the elevations of 13 points were



**• VERTICAL - PICTURE POINT**

FIG. 3. Ground control distribution for the 1:9,600-scale photography.

available for testing the accuracy of the analytically computed coordinates for the 1:4,800-scale flight strip. The horizontal position of 25 ground points and the elevations of 23 points were available for testing the computed coordinates for the 1:9,600-scale strip.

#### DISCUSSION OF RESULTS

PHOTO-TRIANGULATION-1:4,800-SCALE GRAPHS

Tables 1 and 2 show the standard errors obtained strip coor



TABLE 1 GROUND COORDINATES FROM SECOND DEGREE STRIP ADJUSTMENTS

putations for the 1 :4,800-scale photographs. Comparison No. 1 in Table 1 shows no significant differences in the computed *X* and *Z* coordinates by the two independent methods using second degree polynomials. The standard error of the *Y* coordinates for the CGS method is slightly less than that for the NRC method. Similarly, the second comparison shows only a slight reduction in the standard error of *Y* due to the CGS adjustment. Comparison No.3 shows no significant differences due to the strip adjustments.

urth and fifth comparisons do not





show any differences that can be attributed solely to the strip computations. Any minor differences that may have existed due to the strip coordinates alone are likely to have been compensated by the adjustment procedure. In all trial combinations the standard errors for *Y* were less than those for *X.*

**In** the first comparison shown in Table 2, the CGS computation resulted in standard errors that were greater in X and lesser in Y and *Z* than for the NRC method. The standard errors for Z in comparisons No.2, 3, 4 and 5 suggest that the slight improvement in the standard error of the Z-coordinates in comparison No.1 may have been due to the CGS strip computation rather than to the strip adjustment. A similar analysis of comparisons Nos. 2, 3, 4 and 5 shows that the more accurate *Y* coordinates of the CGS method were due largely to the CGS strip adjustment. The lower standard error for X in comparison No. 1 was largely due to the NRC strip adjustment.

For all the trials which were adjusted by the NRC method, the standard errors for *X* were about the same as those for *Y.* On the other hand, the *Y* coordinates from the CGS strip adjustments were about twice as accurate as the *X* coordinates.

Table 3 shows the differences in standard errors resulting from the quadratic and cubic polynomials used in the two strip adjust-Inents.

For both the NRC and CGS methods the second degree strip adjustments gave the better overall results. This would normally be expected for a relatively short flight strip and relatively dense ground control.

#### TRIANGULATION-1:9,600-SCALE PHOTO-GRAPHS

The standard errors for computed ground coordinates using the  $1:9,600$ -scale flight strip appear in Tables 4 and 5. Strip adjustments were performed using second degree polynomials for the comparisons shown in Table 4.

TABLE 3. ERROR DIFFERENCES RESULTING FROM QUADRATIC AND CUBIC POLYNOMIALS AT J :4,800 SCALE

		X (ft) Y (ft) Z (ft)	
NRC-NRC (quadratic) NRC-NRC (cubic)		$0.49$ $0.38$ $0.34$ $0.46$ $0.52$ $0.44$	
CGS-CGS (quadratic) CGS-CGS (cubic)	0.51	0.27 $0.61$ $0.31$ $0.37$	0.32



TABLE 4. STANDARD ERRORS FOR COMPUTED



The CGS method gave slightly lower standard errors for  $Y$  and  $Z$  in comparison No. 1. The second comparison reveals no significant differences between the computed coordinates resulting from the two strip adjustments. The results shown for comparison No.3 do not corroborate those of the previous comparison since the CGS strip adjustment gave slightly lower standard errors for X, Y and Z. The fourth comparison shows no significant differences due to the method of strip computation. Comparison No. 5 does suggest, however,

#### TABLE 5. STANDARD ERRORS FOR COMPUTED GROUND COORDINATES FROM THIRD DEGREE STRIP ADJUSTMENTS



		X (ft) Y (ft) Z (ft)	
NRC-NRC (quadratic)		$0.58$ $0.61$ $0.62$	
NRC-NRC (cubic)		$0.89$ $0.92$ $0.66$	
CGS-CGS (quadratic)		$0.54 \quad 0.51$	0.48
CGS-CGS (cubic)		$0.55$ $0.45$	0.56

TABLE 6. ERROR DIFFERENCES RESULTING FROM QUADRATIC AND CUBIC POLYNOMIALS AT 1:9,600 SCALE

that the CGS strip computation gave slightly more accurate elevations.

Standard errors for X and Y for both the NRC and CGS strip adjustments were about equal. This is unlike the standard errors from second degree CGS adjustments for the 1:4,800scale photographs (Table 1) in which the *Y* coordinates were more accurately computed than the *X.*

Table 5 shows the standard errors for the ground coordinates using third degree polynomials. In comparison No.1, the CGS method, gave markedly improved X and Y coordinates and only slightly improved *Z* coordinates. The second and third comparisons show the improvements in  $X$  and  $Y$  are due mainly to the CGS strip adjustment. The data for the Z coordinates in these comparisons also suggests the improvement was a result of the CGS strip adjustment. In comparison No.4 there appears only a slight improvement in X from the NRC strip computation. Comparison No. 5 gives a similar indication for  $X$ , but also shows improvements for  $Y$  and Z due to the NRC strip coordinates. Comparisons No. 4 and No. 5 in Tables 4 and 5 suggest to the writer that the third degree NRC strip adjustment has simply *adjusted* out any differences which may have existed between the strip coordinates. The CGS adjustment does this to a lesser extent. It is also possible that the NRC strip computations were slightly more accurate in the case of the 1 :9,600-scale strip, but this improvement was insignificant in terms of the improvements in coordinates resulting from the CGS strip adjustment (comparison No. 1). Comparisons for the 1:4,800scale strip (Tables 1 and 2) do not, however, substantiate this conclusion. Evidence from Table 1 indicates no significant differences between the two methods of strip computations while data from Table 2 suggests that the slight improvement in Z coordinates was due to the CGS strip computation.

Both the NRC and CGS strip adjustments re-

sulted in standard errors for  $X$  which were about equal to those for  $Y$ . This is the same relationship obtained with the quadratic adjustments.

Table 6 shows the differences obtained from quadratic and cubic polynomial adjustments for the two independent methods of triangulation.

A significant difference occurred in the standard errors of the X and Y coordinates due to the degree of polynomial for the XRC adjustment. There is little or no difference for the horizontal coordinates whether a second or third degree CGS adjustment was employed. For both the NRC and CGS methods, slightly lower standard errors for Z were obtained with second degree adjustments.

#### PHOTOGRAPHIC IMAGES

The discussion in this section largely reflects the observations made by the stereocomparator operator during the measurement of the two photographic flight strips used in this investigation.

Figure 1 illustrates the target designs used in the investigation. Based on the use of a 40 micron diameter measuring mark (black dot) and  $11 \times$  magnification, the center of target type C was found to be both too large and too dark for precise measurement. This was true for both the 1 :4,800- and 1 :9,600-scale photographs, There was a tendency for the black measuring mark to disappear from view within the target center. Under these circumstances target type C does not appear suitable for analytic triangulation.

The colored wedges forming the center of target type B lacked tonal contrast in the aerial photograph. The center of this type of target had a rather uniform gray photographic tone and appeared more like target type C except for the lighter tone. Target type  $B$  is preferred to type  $C$  because the black floating mark can be seen within the target center.

The center of target type *A* also appeared too large on both scales of photographs for optimum measuring accuracy. [t has been suggested that reduction of the leg widths would provide a more suitable target for the two scales of photographs tested.

In general the picture points that were selected bv the field crew were found acceptable for coordinate measuring. It was impossible to establish any definite correlation between the type of images (picture points or target types) and the errors in position and elevation at these points.

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#### SUMMARY AND CONCLUSIONS

Differences in computed ground coordinates from the two analytic systems are due largely to the strip adjustments. The degree of polynomial used produces significantly different results with the NRC strip adjustment and to a lesser extent with the CGS adjustment. Second degree polynomials in both methods gave the better overall results. When using the NRC strip adjustment, polynomials higher than second degree appear unwarranted. For the CGS strip adjustment, the degree polynomial used should depend on the scale of photographs, length of flight strip and density and distribution of ground control.

Alt hough the error propagation within each strip computation is undoubtedly different, the resulting computed ground coordinates are not significantly affected.

The magnitude of the standard errors obtained for the computed ground coordinates in this investigation do not necessarily represent the utmost in accuracy that can be expected for the scales of photographs and the distributions and densities of ground control used. No conclusions are possible regarding optimum densities and distributions of ground control of the effect of using different photographic target designs.

In the writer's opinion the CGS method for film distortion compensation is superior to that used in the NRC program. In the latter program, the same average linear scale factors are applied to the entire flight strip. The CGS method, however, is applicable only to cameras with four corner fiducials, or eight fiducials if additional fiducials are present along the mid-points of the sides. A separate in-house program was written to provide input to the NRC strip computation program whenever photographs with side fiducials are used. This program transforms the origin of the plate coordinates to the principal point and applies linear film distortion compensation to each photograph of the flight strip.

The choice of a particular method of analytic aerial triangulation will depend on: (a) the personal preferences of the user for specific program features; (b) the design of aerial  $camera(s)$ ; and  $(c)$  the size and speed of the available computer. The NRC strip triangulation program requires less computer storage and runs more efficiently than the CGS strip coordinate computation.

For all practical purposes either the CGS or RC method of analytic aerial triangulation will provide acceptable results for highway location and design purposes.

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