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Selenodetic Control from Ranger Photos

This computational system incorporates data from earth-based telescopes and also Ranger spacecraft photos of lunar features.

Introduction

The Department of Geodesy, Army Map Service (AMS), is developing an extraterrestrial geodetic control model which will be used to establish a fundamental lunar control using orbital as well as earth-based photography. This model is an extension of the earth-based photography model previously used to establish the AMS-64,[1] Group NASA,[2] and DOD-66[3] fundamental control systems. The new, more general model permits up to nine parameters to vary for each plate whereas the previous, more specialized systems treated as unknown only four parameters for each plate and three parameters for each feature. The first attempt to determine control using the new model employs data from six Ranger VIII and four Lick Observatory plates. The goal of this effort is to establish a system of coordinates of selected features with *rms* uncertainties smaller than those obtained from previous controls using earth-based photography alone.

Data

No attempt was made to measure all 200 to 300 craters visible on each plate. Fifty craters were found to be the maximum that could reasonably be measured by a operator in a single measurement session. Of the total 50 craters measured, an average of 40 craters were measured on each of six Ranger plates. Ranger and Lick plates were measured with a Mann Comparator. The 35-mm Ranger negatives were contact printed to 9-by-9-inch glass plates. A grid system was superimposed on the outer perimeter of each plate during the reproduction stage. All measurements were referred to this grid system whose origin was selected in the upper left-hand corner. The coordinates of the geometric center of each feature were obtained by measuring and averaging the maximum and minimum ordinates and abscissas of the feature according to its orientation on the plate. For zero-degree orientation the horizontal grid lines on the plate were oriented parallel to the

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X-axis of the engine. The physical corrections to parallel were made by a rotation adjustment on the plate holder. After all craters were measured in the zero degree orientation the plate was rotated approximately 180 degrees and the measurements repeated. The zero-degree and 180-degree measurements were combined to remove systematic error using the method described in *AMS Technical Report No. 29 (Part One: Methods)*.[4] A spread of 25 μ among any set of rim measurements was cause for rejection of the measurements of the crater for that plate.

ABSTRACT: *Reduction procedures are derived for use on extraterrestrial photography to obtain a precise horizontal and vertical control. A least-squares method is used to adjust observations on terrestrial and extraterrestrial photography simultaneously. The model treats nine parameters for each plate and three parameters for each feature as unknown in the final adjustment. However, it allows for any of the parameters to be treated as known if they are sufficiently established by other means.*

All Ranger VIII crater images identifiable on the Lick Observatory phase-photography (an average of nine unknown craters per plate) were measured in the same manner on the earth-based plates. Of these common craters three had coordinates established by the DOD-66[3] selenodetic control.

The first approximations of the spacecraft data required, such as the longitude, latitude, altitude, range, and the longitude and latitude of the principal point, were obtained from Jet Propulsion Laboratory (*JPL*) listings.[5] A preliminary plate adjustment was performed on a Bendix G-15 computer to give approximate values to those plate parameters not available or less well known. The measurements in demi microns (one μ equalling two demi microns), the spacecraft data, and the plate data were transposed from cards to tape to await reduction on the AMS Honeywell-800 computer with 16-K core. The prototype program was written in Fortran through open-shop arrangements.

As the problem of analyzing the mathematical model is a complex one, it was decided that the use of real Ranger data for this purpose would only complicate the analysis. For this reason the real data have been stored and experiments are in progress using idealized Ranger-type data which were generated for the purpose of analyzing the model. The idealized data were determined by selecting Ranger-type plate parameters and crater coordinates. The plate measurements were then generated using the geometry of the mathematical model. These ideal data were used to determine whether the linearized observation equations do, in fact, represent the non-linear observation equations, and to establish the best method of writing the linear equations to obtain the fastest rate of convergence. By randomly incrementing the various parameters, the model's ability to recover the correct parameters is tested and evaluated. The results of these experiments and the techniques learned will be employed in reducing the present Ranger data and other orbital photographic data when available.

Mathematical Model

Internal Translations. The image coordinates of a lunar feature are determined with respect to the origin of the measuring engine system. The measurement coordinates are translated to a coordinate system centered at the point of intersection of the optical axes on the plate (Figure 1) by

$$\begin{pmatrix} X_{kpi} \\ Y_{kpi} \end{pmatrix} = \begin{pmatrix} x_{kp} \\ y_{kp} \end{pmatrix} + \begin{pmatrix} \Delta x_{pi}^{vii} \\ \Delta y_{pi}^{vii} \end{pmatrix} \quad (1)$$

where

X_{kpi}, Y_{kpi} = i-th approximation of measurement coordinates of image k on plate p referred to the optical center of plate c

Δx_{pi}^{vii}, Δy_{pi}^{vii} = i-th approximation of the origin A, on plate p with respect to the optical center of plate c.

Scaling. The images on the plate are projected by scaling onto a plane which bisects the moon and is perpendicular to the optical axis of the camera (the plane of the scaled plate). The plate measurements are referred to the coordinates of the intersection of the principal ray passing through the feature and the plane of the scaled plate (Figure 2) by

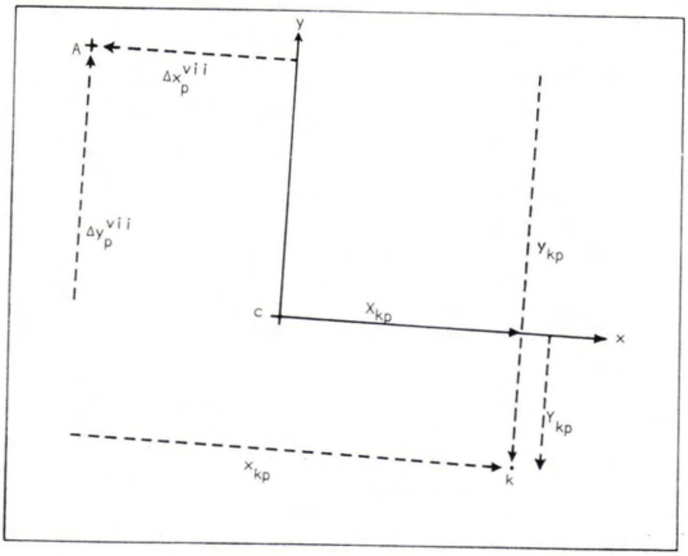

FIG. 1. Coordinate system of the measuring engine. Legend: c, intersection on plate p of the optical axes of the camera; k, crater or feature image; A, origin of measuring engine system; x, y, axes parallel to measuring engine system and whose origin is at A; Δx_p^{vii}, Δy_p^{vii}, coordinates of A in the coordinate system centered at c parallel to the measuring engine system; x_{kp}, y_{kp}, measured coordinates of image k on plate p with respect to origin of measuring system A with systematic error removed; X_{kp}, Y_{kp}, measurement coordinates of k referred to c.

$$\begin{pmatrix} x_{kpi}^{vii} \\ y_{kpi}^{vii} \end{pmatrix} = \frac{1}{r_{pi}} \cdot \begin{pmatrix} X_{kpi} \\ Y_{kpi} \end{pmatrix} \quad (2)$$

where

x_{kpi}^{vii}, y_{kpi}^{vii} = i-th approximation of X_{kp}, Y_{kp} projected onto the plane of the scaled plate

r_{pi} = i-th approximation of semi-diameter of moon on plate p.

Projection Correction. To obtain the orthogonal coordinates of the feature k re-

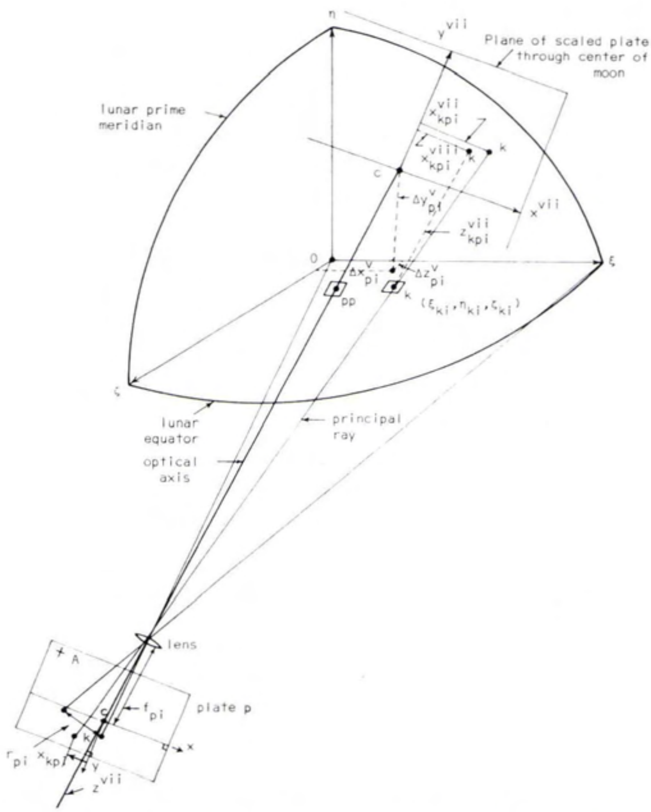

FIG. 2. Projection correction geometry. Legend: ξ, η, ζ; selenodetic coordinate axes system described in AMS TR 29, Part One[4]; f_{pi}, i-th approximation of camera's effective focal length for plate p; ξ_{ki}, η_{ki}, ζ_{ki}, i-th approximation of the coordinates of feature k in the ξ, η, ζ-coordinate system; pp, principal point on lunar mean sphere; x^{vii}, y^{vii}, projected coordinate system on plane of scaled plate parallel to measuring engine system; z_{kpi}^{vii}, i-th approximation of perpendicular distance from feature k to plane of scaled plate; Δx_{pi}^{v}, Δy_{pi}^{v}, Δz_{pi}^{v}, i-th approximation of the intersection of the optical axis with the plane of the scaled plate in the ξ, η, ζ-system; r_{pi}, i-th approximation of the semi-diameter of moon on plate p.

ferred to the x^{vii}-, y^{vii}-system (Figure 2), the following projection correction must be applied:

$$\begin{pmatrix} \overset{viii}{x_{kpi}} \\ \overset{viii}{y_{kpi}} \\ \overset{viii}{z_{kpi}} \end{pmatrix} = \begin{pmatrix} \overset{vii}{x_{kpi}} \\ \overset{vii}{y_{kpi}} \\ \overset{vii}{z_{kpi}} \end{pmatrix} - \begin{pmatrix} \overset{vii}{x_{kpi}} \cdot \overset{vii}{z_{kpi}} \cdot r_{pi}/f_{pi} \\ \overset{vii}{y_{kpi}} \cdot \overset{vii}{z_{kpi}} \cdot r_{pi}/f_{pi} \\ 0 \end{pmatrix} \tag{3}$$

where

x_{kpi}^{viii}, y_{kpi}^{viii}, $z_{kpi}^{viii} = i$-th approximation of x_{kp}^{vii}, y_{kp}^{vii}, z_{kp}^{vii} corrected for projection.

Translation to Center of Moon. Three translations are applied to x_{kpi}^{viii}, y_{kpi}^{viii}, z_{kpi}^{viii} to transform them to a coordinate system whose center coincides with the moon's center and whose axes are parallel to the measuring engine system, as in Figures 3*a*, . . . , 3*c* which illustrate the transformations from orthogonal coordinates to coordinates in the final comparison system.

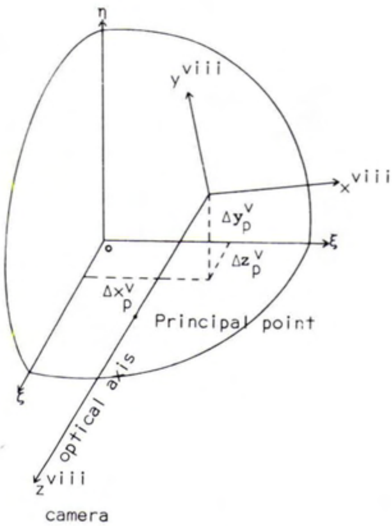

FIG. 3a. Orthogonalized coordinates in x^{viii}, y^{viii}-system.

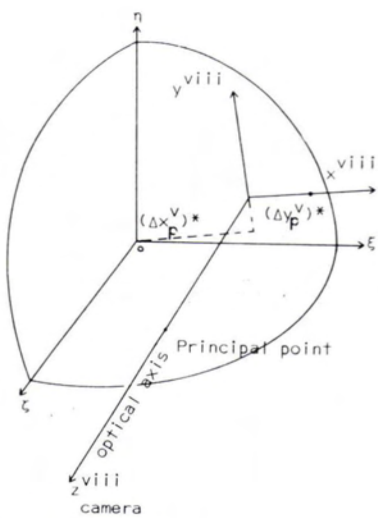

FIG. 3b. Δx_p^v, Δy_p^v, Δz_p^v rotated into the plane of the scaled plate.

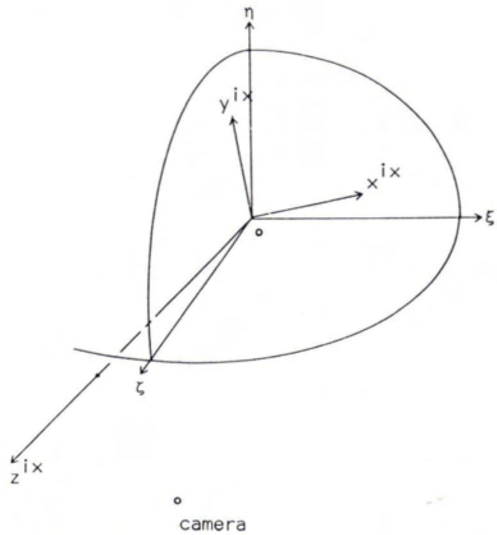

FIG. 3c. Translation of the x^{viii}, y^{viii}, z^{viii}-coordinate system through $(\Delta x_p^v)^*$, $(\Delta y_p^v)^*$ to the center of the moon.

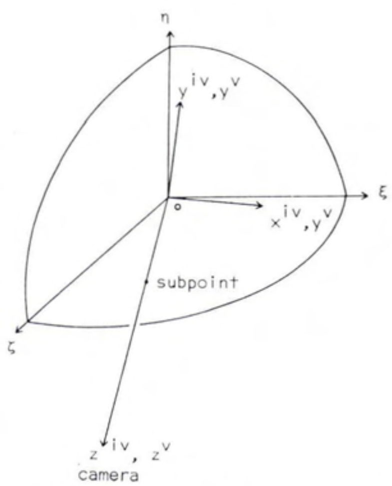

FIG. 3d. Rotations into the comparison system.

$$\begin{pmatrix} \overset{ix}{x}_{kpi} \\ \overset{ix}{y}_{kpi} \\ \overset{ix}{z}_{kpi} \end{pmatrix} = \begin{pmatrix} \overset{viii}{x}_{kpi} \\ \overset{viii}{y}_{kpi} \\ \overset{viii}{z}_{kpi} \end{pmatrix} + \begin{pmatrix} (\Delta \overset{v}{x}_{pi})^* \\ (\Delta \overset{v}{y}_{pi})^* \\ (\Delta \overset{v}{z}_{pi})^* \end{pmatrix} \tag{4}$$

where

x_{kpi}^{ix}, y_{kpi}^{ix}, z_{kpi}^{ix} = i-th approximation of coordinates of feature k as determined from measurements on plate p referred to a coordinate system whose center coincides with center of moon and whose axes are parallel to measuring engine system

$(\Delta x_{pi}^{v})^*$, $(\Delta y_{pi}^{v})^*$, $(\Delta z_{pi}^{v})^* = i$-th approximation of coordinates of optical center of scaled plate in x_p^{ix}, y_p^{ix}, z_p^{ix}, coordinate system

or

$$\begin{pmatrix} (\Delta \overset{v}{x}_{pi})^* \\ (\Delta \overset{v}{y}_{pi})^* \\ (\Delta \overset{v}{z}_{pi})^* \end{pmatrix} = \overset{ix}{R}_{L_{pi}} \cdot \cdot \overset{ix}{R}_{B_{pi}} \cdot \overset{T}{R}_{P_{pi}} \begin{pmatrix} \Delta \overset{v}{x}_{pi} \\ \Delta \overset{v}{y}_{pi} \\ \Delta \overset{v}{z}_{pi} \end{pmatrix} \tag{5}$$

where

Δx_{pi}^{v}, Δy_{pi}^{v}, $\Delta z_{pi}^{v} = i$-th approximation of coordinates of center of scaled plate in ξ, η, ζ, system.

$$\overset{T}{R}_{P_{pi}} = \begin{pmatrix} \cos P_{pi} & \sin P_{pi} & 0 \\ -\sin P_{pi} & \cos P_{pi} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{6}$$

where

$P_{pi} = i$-th approximation of the counterclockwise angle measured from the projection of η axis on the x_p^{ix}-, y_p^{ix}-plane to the y_p^{ix} axis.

$$\overset{ix}{R}_{B_{pi}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \overset{ix}{B}_{pi} & -\sin \overset{ix}{B}_{pi} \\ 0 & \sin \overset{ix}{B}_{pi} & \cos \overset{ix}{B}_{pi} \end{pmatrix} \tag{7}$$

$$\overset{ix}{R}_{L_{pi}} = \begin{pmatrix} \cos \overset{ix}{L}_{pi} & 0 & -\sin \overset{ix}{L}_{pi} \\ 0 & 1 & 0 \\ \sin \overset{ix}{L}_{pi} & 0 & \cos \overset{ix}{L}_{pi} \end{pmatrix} \tag{8}$$

where

$L_{pi}^{ix} = i$-th approximation of the counterclockwise angle measured from ζ axis to the projection on the lunar equator of the vector passing through the moon's center parallel to the camera's optical axis

$B_{pi}^{ix} = i$-th approximation of the counterclockwise angle measured in the plane of the L^{ix}-th meridian from the lunar equator to the vector passing through the moon's center parallel to optical axis of the camera.

Final Rotation. The axes to which the coordinates are referred are rotated around the z_p^{ix}-axis, the ξ-axis, and the η-axis to obtain the coordinates of the feature (Figure 4). These coordinates are determined from the measurement of the plate referred to the coordinate system in which comparison with the assumed coordinates of the feature is made by

$$\begin{pmatrix} \overset{iv}{x}_{kpi} \\ \overset{iv}{y}_{kpi} \\ \overset{iv}{z}_{kpi} \end{pmatrix} = R_{L_{cpi}} \cdot R_{B_{cpi}} \cdot R_{P_{pi}} \cdot \begin{pmatrix} \overset{ix}{x}_{kpi} \\ \overset{ix}{y}_{kpi} \\ \overset{ix}{z}_{kpi} \end{pmatrix} \tag{9}$$

where

x_{kpi}^{iv}, y_{kpi}^{ix}, $z_{kpi}^{ix} = i$-th approximation of coordinates of feature k as determined

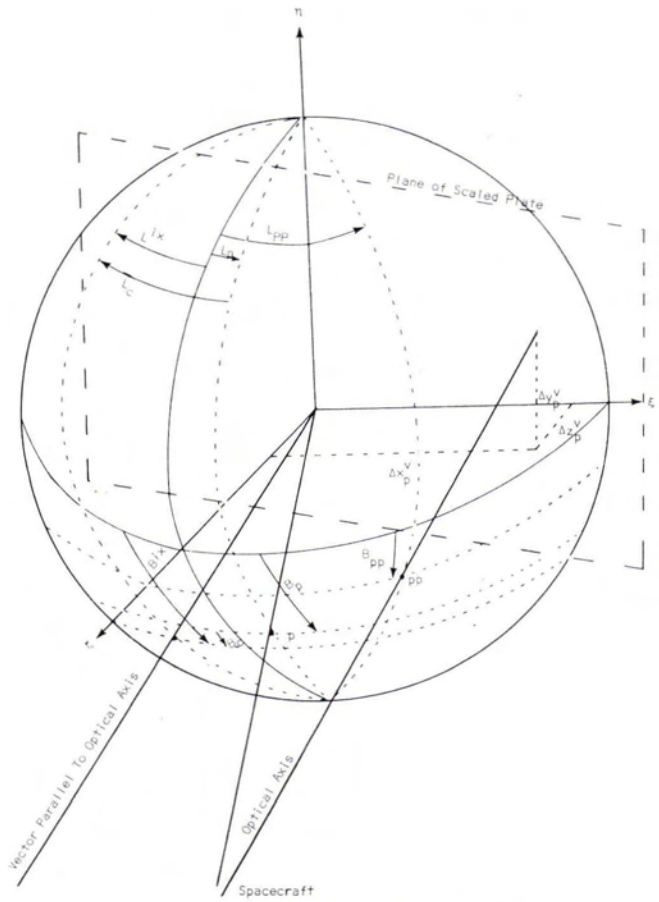

FIG. 4. Coordinate transformations.

from measurements of plate p referred to the coordinate system of final comparison

and

$$R_{L_{c_{pi}}} = \begin{Bmatrix} \cos L_{c_{pi}} & -\sin B_{p_{pi}} \cdot \sin L_{c_{pi}} & \sin L_{c_{pi}} \cdot \cos B_{p_{pi}} \\ \sin B_{p_{pi}} \cdot \sin L_{c_{pi}} & \cos^2 B_{p_{pi}} \cdot (1 - \cos L_{c_{pi}}) + \cos L_{c_{pi}} & \cos B_{p_{pi}} \cdot \sin B_{p_{pi}} \cdot (1 - \cos L_{c_{pi}}) \\ -\cos B_{p_{pi}} \cdot \sin L_{c_{pi}} & \cos B_{p_{pi}} \cdot \sin B_{p_{pi}} \cdot (1 - \cos L_{c_{pi}}) & \cos L_{c_{pi}} + \sin^2 B_{p_{pi}} \cdot (1 - \cos L_{c_{pi}}) \end{Bmatrix}, \quad (10)$$

$$R_{B_{c_{pi}}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos B_{c_{pi}} & \sin B_{c_{pi}} \\ 0 & -\sin B_{c_{pi}} & \cos B_{c_{pi}} \end{pmatrix} \quad (11)$$

and

$$R_{P_{pi}} = \begin{pmatrix} \cos P_{pi} & -\sin P_{pi} & 0 \\ \sin P_{pi} & \cos P_{pi} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (12)$$

where

$B_{p_{pi}}$ = approximation of latitude of center of plate p in ξ, η, ζ-system
$L_{p_{pi}}$ = i-th approximation of longitude of center of plate p in ξ, η, ζ-system
$L_{c_{pi}}$ = i-th approximation of $\overset{ix}{L}_{pi} - L_{p_{pi}}$
$B_{c_{pi}}$ = i-th approximation of $\overset{ix}{B}_{pi} - B_{p_{pi}}$.

Combining Equations 1, 2, 3, 4, and 9:

$$\begin{pmatrix} \overset{iv}{x}_{kpi} \\ \overset{iv}{y}_{kpi} \\ \overset{iv}{z}_{kpi} \end{pmatrix} = R_{L_{c_{pi}}} \cdot R_{B_{c_{pi}}} \cdot R_{P_{pi}} \cdot \left[\begin{pmatrix} (x_{kp} + \Delta \overset{vii}{x}_{pi}) \cdot \left[\dfrac{1}{r_{pi}} - \dfrac{\overset{vii}{z}_{kpi}}{f_{pi}} \right] \\ (y_{kp} + \Delta \overset{vii}{y}_{pi}) \cdot \left[\dfrac{1}{r_{pi}} - \dfrac{\overset{vii}{z}_{kpi}}{f_{pi}} \right] \\ \overset{vii}{z}_{kpi} \cdot (1) \end{pmatrix} \right.$$

$$\left. + \overset{ix}{R}_{L_{pi}} \cdot \overset{ix}{R}_{B_{pi}} \cdot R^T_{P_{pi}} \cdot \begin{pmatrix} \Delta \overset{v}{x}_{pi} \\ \Delta \overset{v}{y}_{pi} \\ \Delta \overset{v}{z}_{pi} \end{pmatrix} \right]. \tag{13}$$

Rotations of Assumed Coordinates of Features. The assumed coordinates of the features are rotated into the comparison coordinate system by

$$\begin{pmatrix} \overset{v}{x}_{kpi} \\ \overset{v}{y}_{kpi} \\ \overset{v}{z}_{kpi} \end{pmatrix} = R_{B_{p_{pi}}} \cdot R_{L_{p_{pi}}} \cdot \begin{pmatrix} \xi_{ki} \\ \eta_{ki} \\ \zeta_{ki} \end{pmatrix} \tag{14}$$

where

$\overset{v}{x}_{kpi}, \overset{v}{y}_{kpi}, \overset{v}{z}_{kpi}$ = i-th approximation of coordinates of feature k in the comparison system as determined from assumed coordinates of the feature
$\xi_{ki}, \eta_{ki}, \zeta_{ki}$ = i-th approximation of the coordinates of feature k in the ξ, η, ζ-coordinate system.

$$R_{B_{p_{pi}}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos B_{p_{pi}} & -\sin B_{p_{pi}} \\ 0 & \sin B_{p_{pi}} & \cos B_{p_{pi}} \end{pmatrix} \tag{15}$$

$$R_{L_{p_{pi}}} = \begin{pmatrix} \cos L_{p_{pi}} & 0 & -\sin L_{p_{pi}} \\ 0 & 1 & 0 \\ \sin L_{p_{pi}} & 0 & \cos L_{p_{pi}} \end{pmatrix}. \tag{16}$$

The Observation Equations. Let the plate parameters be

$$\begin{array}{lll} \phi_1 = L_{p_p} & \phi_4 = \Delta \overset{v}{x}_p & \phi_7 = \Delta \overset{vii}{x}_p \\ \phi_2 = B_{p_p} & \phi_5 = \Delta \overset{v}{y}_p & \phi_8 = \Delta \overset{vii}{y}_p \\ \phi_3 = P_p & \phi_6 = \Delta \overset{v}{z}_p & \phi_9 = r_p \end{array}$$

and the crater parameters

$$\phi_{10} = \xi_k$$
$$\phi_{11} = \eta_k$$
$$\phi_{12} = \zeta_k.$$

The observation equations are then

$$\left\{\begin{matrix} \sum_{j=1}^{12} \left(\frac{\partial x^v}{\partial \phi_j} - \frac{\partial x^{iv}}{\partial \phi_j}\right) \cdot \Delta\phi_j \cdot S_j \\ \sum_{j=1}^{12} \left(\frac{\partial y^v}{\partial \phi_j} - \frac{\partial y^{iv}}{\partial \phi_j}\right) \cdot \Delta\phi_j \cdot S_j \end{matrix}\right\} = \begin{bmatrix} x^{iv} - x^v \\ y^{iv} - y^v \end{bmatrix} \tag{17}$$

where

$S_j = 0$ if parameter is known; 1 if parameter is to be determined
$\Delta\phi_j$ = Correction to be added to j-th parameter.

A least-squares method is used to determine the corrections to the unknown parameters. The parameters are updated and the process is repeated until the corrections are less than 10^{-6} in the unit of their measurement.

Conclusions

With the model described above it will be possible to obtain a precise horizontal and vertical control from a combination of extraterrestrial and terrestrial photography or from extraterrestrial photography alone. The model permits the treatment of any feature or plate as completely or partially known and determines the longitude and latitude of the camera at the time the plate was exposed as free parameters in the adjustment. The prototype program being used for experimentation determines the altitude of the camera above the mean sphere for each plate. The spacecraft position in addition to the feature position is, therefore, completely determined at the time of exposure for each plate.

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