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Principles and Applications of Analytical Mirror Photogrammetry

Useful for dentistry and lunar Surveyor I mission.

(Abstract on next page)

INTRODUCTION

IN CONVENTIONAL PHOTOGRAMMETRY the basic condition for the spatial determination of an object is that it must be recorded in two nonidentical photographs. This is a fundamental concept of stereophotogrammetry for which either the camera is moved between exposures, or the camera is held stationary and the object itself is displaced. The net outcome is that two photographs are obtained which represent two different perspective views of the object.

There is another way of obtaining two different perspective views without having to move either the camera or the object. This is achieved by placing a plane mirror close to the object in such a way that the camera would register not only the object but also its mirror reflection. Hence, on the one plate two different perspective views are obtained with an amount of separation depending on the relative positions of the object, mirror, and camera. The only point to note is that the mirror-reflection of the object is left-right reversed.

In the early application of this concept of mirror photogrammetry (see references 2 & 3), the image of the mirror-reflection, heretofore referred to as the satellite image, was *rectified*† by analog means. This entailed the use of certain photographic processes to obtain two plates that were compatible. Furthermore, these plates were then utilized in a photogrammetric instrument for optical-mechanical restitution. The end product was normally a topographic map.

* Prepared for presentation in the program of Commission III at the Eleventh Congress on Photogrammetry, Lausanne, Switzerland, July 1968.

† The word *rectified* is used here in a broad sense to mean reversing the left-right image to a correct positioning.

In this paper we will attempt to show that, although the basic concept was last mentioned over a decade ago, it is still of considerable interest and usefulness. Unlike the early applications, however, the present treatment has as its basis the use of analytical, rather than analog, techniques.

GEOMETRY OF MIRROR PHOTOGRAMMETRY

The basic idea of mirror photogrammetry is fairly simple, and its uses are also simple if the mirror plane and the camera axis are parallel. If, however, the position of the mirror changes or is not accurately known beforehand, the geometry gets progressively complex. For this reason, in the subsequent paragraphs four different cases will be treated depending on the relative position of the mirror with respect to the camera axis. In all these cases, the camera axis is assumed to be horizontal and the object space coordinate system has its origin at the camera center, its X -axis toward the mirror, and its Y -axis along the optical axis.

CASE A: MIRROR VERTICAL AND PARALLEL TO CAMERA AXIS

As mentioned above, this is the simplest case which is shown in Figure 1. A triangular pyramid 1, 2, 3, 4 is photographed by a camera C such that the base 1, 2, 3 lies in the same horizontal plane as the camera axis (i.e., the XY -plane). The plane mirror is placed at a distance d from the camera and is both vertical and parallel to the optical axis. The mirror reflection of the pyramid is shown as $\bar{1}$, $\bar{2}$, $\bar{3}$, $\bar{4}$ and is photographed as the satellite image $1'$, $2'$, $3'$, $4'$, while the pyramid itself is photographed as the main image 1, 2, 3, 4. These two images are then used as conjugate images for spatial restitution of the object.

To illustrate the applicability of stereo-

photogrammetry, we seek to show that these conjugate images are equivalent to photographing the object from two camera stations. In Figure 1, \bar{C} represents the mirror-reflection of the camera C , which in turn would photograph the object at $\bar{1}, \bar{2}, \bar{3}, \bar{4}$. These image points would then be referred to the *reflected* coordinate axes, \bar{x}, \bar{z} , which constitute a left-handed system. Since such a

system is not compatible with the coordinate system of the original camera (i.e., left camera in the stereopair), a reflection matrix \mathbf{R} need be applied to yield the correct system, x_r, z_r , for the right camera. Hence, the coordinates of images on the right plate (shown in negative position in Figure 1) are obtained from the satellite images by the reflection transformation

ABSTRACT: *The basic principle underlying stereophotogrammetry is the fact that the object space is recorded on two photographic plates representing two different perspective views. Although the majority of conventional applications of this principle have depended on different photographs of the object itself, recent applications entail photographs of mirror reflections of the object. These may be used by themselves or in conjunction with photographs of the object as well. This paper presents the geometric principles of "mirror photogrammetry." First, different cases of photo-pairs each composed of one photograph of the object and one of its mirror-reflection, are studied. Next, the case of stereo-pairs each formed from two photographs of two different mirror-reflections of the object is discussed. Throughout the development, analytical methods of reduction are emphasized. "Reflection" matrices necessary for operating on image-coordinates on photographs of mirror-reflections are derived. One of the possible applications discussed is the use of such photographic systems to close-range photogrammetry particularly as applied to dentistry. This is followed by a more important application, namely that involved with analytical reduction of lunar photographs acquired by soflanded vehicles. These latter photographs are all taken of mirror-reflections of lunar surface and not the surface itself.*

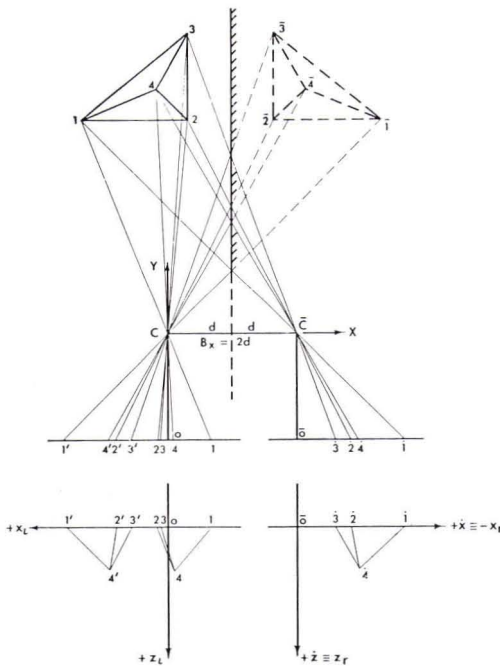


FIG. 1. Case A—mirror vertical and parallel to axis of the camera.

$$\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ z' \end{bmatrix}, \quad (1)$$

where (x', z') are the plate coordinates of the satellite images. With the two sets of coordinates $(x, z)_l$ and $(x, z)_r$, the spatial object coordinates can be easily determined realizing that Figure 1 represents the conventional *normal case* of terrestrial photogrammetry. The coordinates of the right camera \bar{C} , which are also the base components, are given by:

$$\begin{aligned} X_{\bar{c}} &= B_x = 2d \\ Y_{\bar{c}} &= B_y = 0 \\ Z_{\bar{c}} &= B_z = 0. \end{aligned} \quad (2)$$

Also, the orientation angles of the right camera into the object system are all zero with a corresponding unit orientation matrix.

CASE B: MIRROR VERTICAL AND CONVERGENT TO CAMERA AXIS

This case is illustrated in Figure 2. The plane of the mirror is vertical and makes an angle θ with the camera axis. The *virtual* right camera is shown as \bar{C} and its axis makes an angle 2θ with the actual, or left, camera axis. It can be easily ascertained that the reflection

transformation of Equation 1 holds true for this case. Furthermore, the coordinates of the right camera (base components) can be obtained, by inspection, from Figure 2 as

$$\begin{aligned} X_{\bar{c}} &= B_x = 2d \cos \theta \\ Y_{\bar{c}} &= B_y = 2d \sin \theta \\ Z_{\bar{c}} &= B_z = 0 \end{aligned} \quad (3)$$

where d is the distance between the camera C and the mirror. Knowing the left plate coordinates, the reflected satellite coordinates, the base components, and the axes convergence angle 2θ , the computation of object spatial coordinates becomes a straightforward matter.

CASE C: MIRROR INCLINED TO THE HORIZONTAL PLANE ($90^\circ - \gamma$) BUT PARALLEL TO CAMERA AXIS

Figure 3, in plan view and a number of auxiliary projections, shows the geometry of this case. Here, the plane of the mirror deviates an angle γ from the vertical plane but still remains parallel to the optical axis. Because the x -axis x_l of the left negative makes an angle ($90^\circ - \gamma$) with the plane of the mirror, its reflection, or x -axis (also the negative x_r -axis), would make the same angle with the mirror. Hence, the angle between x_r and x_l axes (and correspondingly z_r and z_l axes) is 2γ . This is equivalent to having a relative swing angle of 2γ (in the vertical plane of the two negatives) between the two camera systems.

If the distance between the left camera C and the mirror is d , the coordinates of the

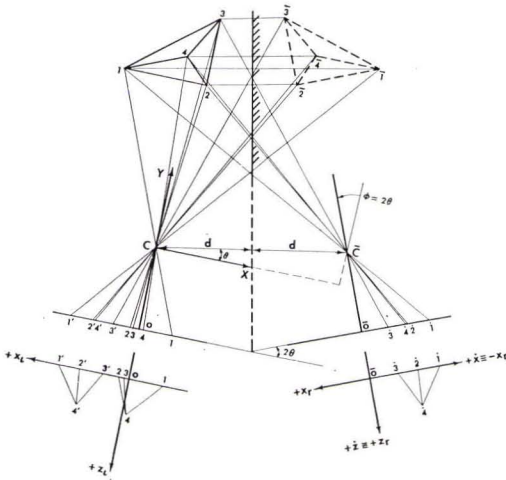


FIG. 2. Case B—mirror vertical and convergent toward the axis of the camera.

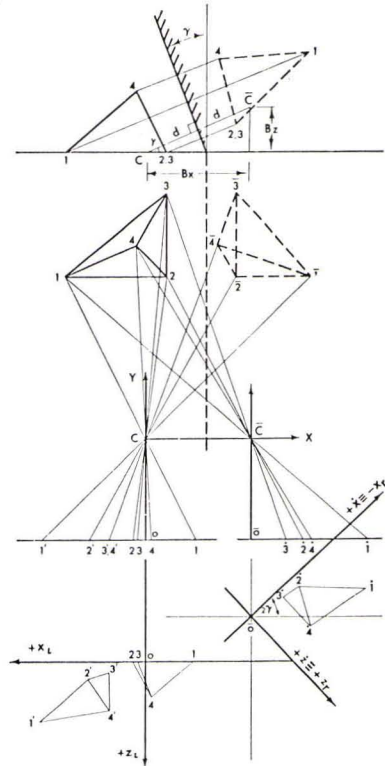


FIG. 3. Case C—mirror inclined to the horizontal plane ($90^\circ - \gamma$) but parallel to the axis of the camera.

right camera (i.e., the base components) would be:

$$\begin{aligned} X_{\bar{c}} &= B_x = 2d \cos \gamma \\ Y_{\bar{c}} &= B_y = 0 \\ Z_{\bar{c}} &= B_z = 2d \sin \gamma. \end{aligned} \quad (4)$$

Furthermore, it is easily seen from Figure 3 that the right plate coordinates are obtained from the satellite image coordinates using the same reflection transformation given by Equation 1.

Thus, if d and γ are known, all the elements of exterior orientation can be computed. With the proper plate coordinates, one can then compute the coordinates of all object points.

CASE D: MIRROR PLANE NEITHER VERTICAL NOR PARALLEL TO CAMERA AXIS—GENERAL CASE

In the three cases previously discussed, it was possible to illustrate the geometry through projection views. This case, however, is considerably more complex so that it would require too many auxiliary projections which

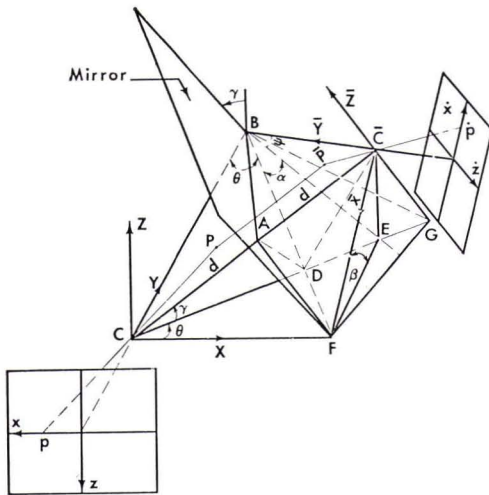


FIG. 4. Case D—mirror plane neither vertical nor parallel to the camera axis: the general case.

might be somewhat confusing. Consequently, Figure 4 represents a three-dimensional view of the case where the mirror plane makes an angle $(90^\circ - \gamma)$ with the horizontal plane $(X - Y)$, and its line of intersection with it makes an angle θ with the camera axis.

If d denotes the length of the perpendicular from the camera station C to the plane of the mirror, the coordinates of the right station \bar{C} are obtained as follows:

$$\bar{C}A = \bar{C}C = d$$

and

$$\bar{C}E = 2d \cos \gamma.$$

Hence,

$$\begin{aligned} X_{\bar{C}} &= B_x = \bar{C}E \cos \theta = 2d \cos \gamma \cos \theta \\ Y_{\bar{C}} &= B_y = \bar{C}E \sin \theta = 2d \cos \gamma \sin \theta \\ Z_{\bar{C}} &= B_z = \bar{C}C \sin \gamma = 2d \sin \gamma. \end{aligned} \quad (5)$$

As a check on these equations, if $\gamma = \theta = 0$, they reduce the Equation 2; if $\gamma = 0$, they reduce to Equation 3; and if $\theta = 0$, they reduce to Equation 4.

To obtain the orientation of the right camera system with respect to the given left system, we first note that the former $\bar{X}\bar{Y}\bar{Z}$ is a left-handed system, whereas the latter XYZ is right-handed system. Therefore, to arrive at $\bar{X}\bar{Y}\bar{Z}$ from XYZ we perform a sequence of rotations, then a reflection of one axis, which we will choose to be the X -axis. In order to obtain the values of the sequential rotations (in this case there are three of them) we start by deriving the following auxiliaries: Referring to Figure 4,

$$\begin{aligned} \bar{C}\bar{D} &= \bar{D}\bar{C} = d/\cos \gamma \\ \bar{C}\bar{B} &= \bar{B}\bar{C} = \bar{C}\bar{D}/\sin \theta = d/\cos \gamma \sin \theta. \end{aligned}$$

Thus

$$\sin \psi = \frac{\bar{C}\bar{E}}{\bar{B}\bar{C}} = \frac{Z_{\bar{C}}}{\bar{B}\bar{C}} = \frac{2d \sin \gamma}{d/\cos \gamma \sin \theta}$$

or

$$\sin \psi = \sin 2 \gamma \sin \theta \quad (6)$$

$$\begin{aligned} \bar{D}\bar{E} &= \bar{C}\bar{E} - \bar{C}\bar{D} = 2d \cos \gamma - d/\cos \theta \\ &= d (2 \cos^2 \gamma - 1)/\cos \theta \end{aligned}$$

$$\bar{B}\bar{D} = \bar{C}\bar{D} \cot \theta = d \cot \theta / \cos \gamma.$$

Thus,

$$\tan \alpha = \bar{D}\bar{E}/\bar{B}\bar{D} = d (2 \cos^2 \gamma - 1)/d \cot \theta$$

or

$$\tan \alpha = \tan \theta (2 \cos^2 \gamma - 1) \quad (7)$$

$$\bar{C}\bar{F} = \bar{C}\bar{F} = \bar{C}\bar{D}/\cos \theta = d/\cos \theta \cos \gamma,$$

then

$$\sin \beta = \frac{\bar{C}\bar{E}}{\bar{C}\bar{F}} = \frac{Z_{\bar{C}}}{\bar{C}\bar{F}} = \frac{2d \sin \gamma}{d/\cos \theta \cos \gamma}$$

or

$$\sin \beta = \sin 2 \gamma \cos \theta. \quad (8)$$

With the three angles ψ , α and β evaluated by Equations 6, 7, and 8, the following are the three sequential rotations:

- First rotation, $(\theta + \alpha)$ about Z $X \rightarrow Y$
- Second rotation, ψ about X_1 (once rotated) $Z_1 \rightarrow Y_1$
- Third rotation, β about Y_2 (twice rotated) $X_2 \rightarrow Y_2$

After these rotations are performed, the X_3 -axis is reflected thus leading to the $\bar{X}\bar{Y}\bar{Z}$ -system shown.

As this is the general case, we will show that all the other three cases can be deduced from this by enforcing the proper conditions.

(i) If $\theta = \gamma = 0$, then $\psi = \alpha = \beta = 0$ and the three rotation angles are zero. That is, the right camera system is parallel to that of the left which can be ascertained from Figure 1 representing Case A.

(ii) If $\gamma = 0$, but $\theta \neq 0$, then from Equation 6 $\psi = 0$, from Equation 7 $\alpha = \theta$, from Equation 8 $\beta = 0$.

Thus, the first rotation is $(\alpha + \theta = 2\theta)$, the second rotation is 0 as is the third rotation which is Case B as shown in Figure 2.

(iii) If $\gamma \neq 0$, but $\theta = 0$, then from Equation 6 $\psi = 0$, $\alpha = 0$, $\beta = 2\gamma$.

Hence, the first rotation is $(\alpha + \theta = 0)$, the second rotation is $\psi = 0$, and the third rota-

tion is $\beta = 2\gamma$. This is the same as Case C as demonstrated by Figure 3.

Through all the four cases discussed so far, the six elements of exterior orientation of the right photograph were shown to be determined from at most three parameters, γ , θ , and d . If these parameters are known, then the photogrammetric problem reduces to a simple determination of the intersections of conjugate rays. If, however, these parameters are not sufficiently known, the mathematical model that is used would only involve three unknowns which would be determined from control points in the object space. It is interesting to note, therefore, that the use of a mirror reduces the number of unknowns to three in place of six where the camera is moved to take the second photograph. Another remark worth pointing out is that complete determination of the object space is possible without need for control if the mirror position relative to the camera axis is known.

Up until now, the discussion has been limited to cases of pairs of photographs, one of the object itself and the other of its mirror reflection. There is nothing to limit us, however, to this particular configuration. It is quite possible to consider pairs composed of photographs of two different mirror-reflections of the same object. One such configuration is shown in Figure 5.

The pyramid $defg$ is placed in front of a mirror M_1 which makes an angle θ_1 with the axis of the camera C . Its mirror-reflection $d_1e_1f_1g_1$ is then photographed by C . Next, the mirror is rotated to position M_2 , changing θ_1 to θ_2 , and the mirror-reflection $d_2e_2f_2g_2$ is photographed on a different plate. The two plates thus obtained contain sufficient information for spatial determination of the object.

The first operation necessary is to apply the reflection transformation of Equation 1 to the coordinates measured on the two negative plates. The new coordinates would be equivalent to those obtained if we were able to place the camera at C_1 and C_2 and photograph the pyramid itself. Hence, the positions of C_1 and C_2 as well as the orientation of the coordinate axes systems associated with them are required.

From Figure 5, it is easily seen that the coordinates of C_1 are

$$\begin{aligned} X_{c_1} &= 2S_1 \cos \theta_1 \\ Y_{c_1} &= 2S_1 \sin \theta_1 \\ Z_{c_1} &= 0 \end{aligned} \tag{9}$$

where S_1 is the distance between C and M_1 . The coordinates for C_2 are obtained in the

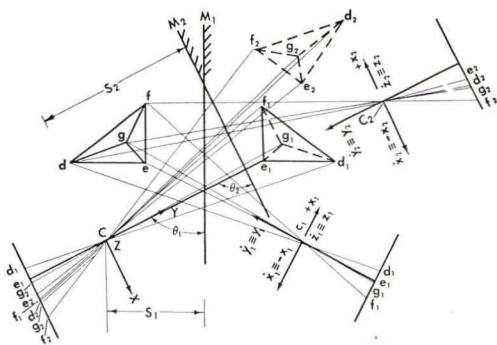


FIG. 5. Images of two mirror reflections forming a stereopair.

same manner replacing S_1 and θ_1 in Equation 9 by S_2 and θ_2 , respectively, where S_2 is the distance between C and M_2 . The orientation of the $X_1Y_1Z_1$ - (or $X_2Y_2Z_2$ -) system with respect to the XYZ -system is obtained by rotating the latter about Z by an angle $2\theta_1$ (or $2\theta_2$) taking X toward Y .

Figure 5 represented the case when the mirror was rotated about an axis perpendicular to the horizontal plane containing the camera axis. The more general case would arise when the axis of rotation of the mirror is inclined to the said horizontal plane. In this situation, the total rotation angle would have two components thus simulating the general case discussed in Case D above but with two mirror reflections.

A progressively more complex situation would be encountered if more than one plane mirror, say two, are used. Here, one perspective view is obtained from a mirror-reflection from the first mirror. Then, the second perspective view is obtained when the first mirror is rotated such that the reflection from the second mirror (which may be fixed in the object space) is reflected again by the first mirror and photographed. This situation, though quite complex, may become necessary when an increase in effective base separation is required as will be discussed later.

APPLICATIONS

We have, thus far, discussed the basic geometric concepts of mirror photogrammetry and corresponding procedures for analytical reduction. To illustrate the usefulness of these concepts we will attempt in the following paragraphs to discuss some applications.

CLOSE-RANGE PHOTOGRAMMETRY

The author's first encounter with the sub-

ject was during a research project concerned with the application of photogrammetry to dentistry (Reference 4). The use of plane mirrors in that project suggested itself, not so much for the purpose of obtaining stereo-coverage, but to overcome the serious problem of *blind spots* when photographing dental casts. By *blind spots* is meant hidden areas which cannot adequately be registered in both photographs. In these circumstances the dental cast to be photographed was surrounded by as many as three high quality front-surfaced mirrors. One photograph was taken on which appeared one parent and three satellite images of the dental cast. Then the camera was displaced by an amount representing the base (which must be carefully computed taking account of the position of the mirrors) and another photograph was exposed. Consequently, from one pair of exposed plates it was possible to obtain as many as four stereomodels. The only factor to be carefully observed is that the satellite images are photographic records of mirror reflections of the cast and hence are left-right reversed.

It is obvious that the mirror technique discussed above need not be restricted to applications in dentistry. As a matter of fact, any application in the field of close-range photogrammetry could easily make use of this technique. Such use may prove advantageous when problems of *blind spots* arise or when it is more economical to register more than one image on the same negative.

EXTRATERRESTRIAL MAPPING FROM SOFT-LANDED VEHICLES

Another, and most interesting, field of application for the principles of mirror photogrammetry is large scale mapping from photographs acquired by soft-landed vehicles on moons and planets. The accent in our discussion will be placed on mapping the lunar surface, although the techniques can easily be applied to other planetary surfaces.

By virtue of the fact that the landed vehicle is normally stationary, the photographic system aboard the vehicle utilizes mirrors which, while rotating, makes possible the scanning of the surface surrounding the vehicle. In the following sections we will discuss the elements of the photographic system and the means for reducing the resulting photographic records. This discussion will center on the Surveyor I mission because of the writer's experience with it while acting as a consultant to the U. S. Army Map Service (see Reference 5).

The Photographic System. The camera sys-

tem is composed of three components: a vidicon tube representing the negative plane, a lens, and a plane mirror. The distance between the lens and the vidicon tube is equal to the principal distance of 25 millimeters. There are 25 reseau points on the surface of the vidicon used to relate the photographs obtained to the camera coordinate system. The optical axis of the camera is defined as the line connecting the center reseau point to the center of the plane mirror and passing through the optical center of the mirror. This line is assumed to be fixed in space and makes a constant angle γ with the local zenith vertical.

The plane of the mirror is inclined to the surface of the vidicon tube by a variable angle m (Figure 6). For any given value of m the reflection of the optical axis makes a constant angle m' with the vidicon tube for all azimuthal positions of the mirror. The value of m' is obtained from m by the simple relation

$$m' = 90^\circ - 2m. \quad (10)$$

This angle m' is referred to in the mission data as the elevation angle. It has a value of zero when the reflected optical axis is parallel to the vidicon plane. The elevation angle is taken as positive if it is upward from the zero position, and negative when downward.

The azimuth angle of the mirror β is measured about the fixed optical axis. The plane of zero azimuth contains the optical axis and intersects the lunar horizontal at 89.6 ± 0.4 degrees east of lunar north (Figure 7). Positive azimuths are taken counterclockwise, as viewed from above, from the zero to a stop located at $+132$ degrees. Negative azimuths increase clockwise from zero to a stop at -222 degrees (total of 354 degrees).

Stereo Coverage. Although there is only one camera whose lens is fixed with respect to the object space, the rotation of the mirror in azimuth and elevation enables the attaining of varying perspective views. The existence of the stereo-effect, at least theoretically, between two exposures is illustrated in Figure 6. The front view shows the projection on the vertical plane through the inclined optical axis (i.e., the plane containing γ). The mirror is inclined at an angle m to the vidicon, and the corresponding angle m' , which the reflected optical axis makes with the vidicon, is also shown. The angle θ is that angle which the reflected axis makes with the lunar horizontal plane. For a given value of m , it is important to note that while m' remains constant, θ varies according to the mirror azimuth. It lies between the limits

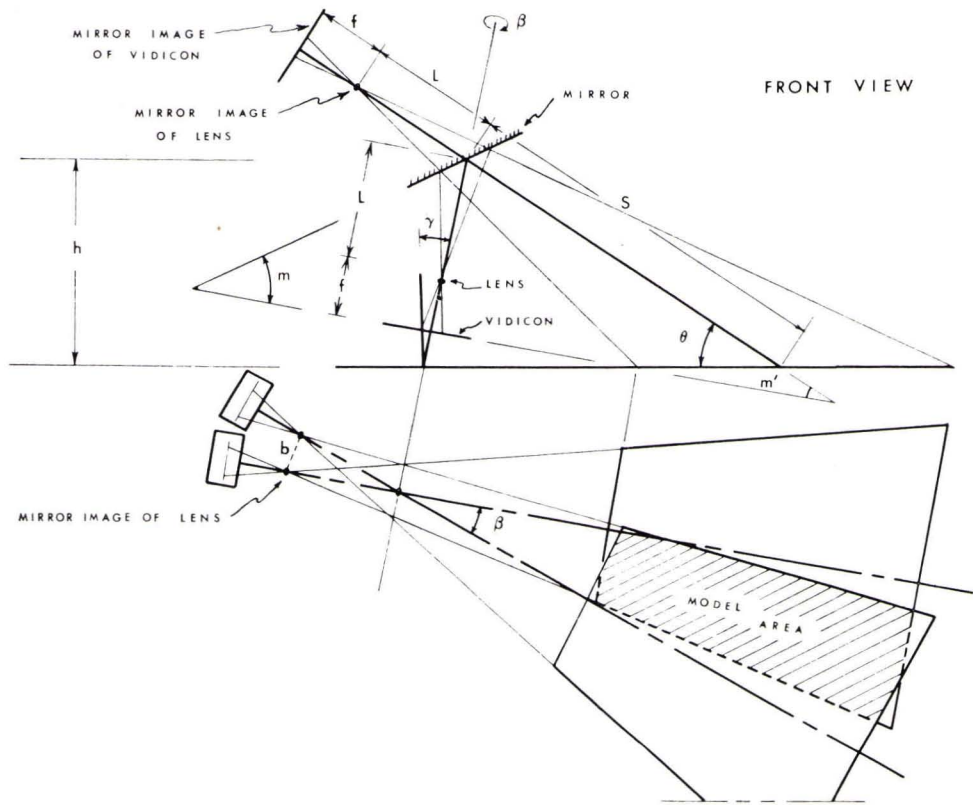


FIG. 6. Demonstration of stereo coverage in the lunar Surveyor I mission. The lower portion of the drawing represents the auxiliary projection on the plane of the vidicon.

$$(m' - \gamma) \leq \theta \leq (m' + \gamma) \quad (11)$$

Also shown in the front view are the following distances:

- h is the height of the mirror center above the lunar horizontal plane.
- f is the principal distance, or the distance between the lens and the vidicon.
- L is the distance between the lens and mirror center measured along optical axis.

The mirror is so positioned (both in elevation and azimuth) that an area of detail on the lunar surface is reflected by the mirror, the reflected rays pass through the lens, and the image formed on vidicon is scanned from beneath and transmitted to the earth. It is evident from the front view that the picture received is equivalent to a photograph taken from a position in space coincident with the mirror reflection of the lens (Figure 6).

To demonstrate the existence of stereo-coverage, an auxiliary projection on the plane of the vidicon is also shown in Figure 6. There, a trapezoidal area of coverage is shown for the first position of the mirror. Next, the

mirror is rotated in azimuth (at the same elevation angle) about the optical axis by an angle β . The new trapezoid of coverage is shown to overlap the first by a shaded area designated the model area. Also shown in the auxiliary projection is the effective base b representing the distance between the two positions of the mirror-reflections of the lens. Thus, at least theoretically, the use of the mirror affords two different perspective views, hence stereo-coverage.

Analytical Reduction. So far we have established the existence of stereoscopic coverage between pairs of photographs. Each pair will yield a stereo-model of the lunar terrain when both interior and exterior orientation of each camera have been regained. In conventional applications of photogrammetry (whether analog or analytical) the correct positioning and orientation of a stereo model with respect to the object space inevitably requires the use of control points. It must be obvious that in the present situation there is no known control on the lunar surface for absolute orientation purposes. Consequently, in order to obtain information

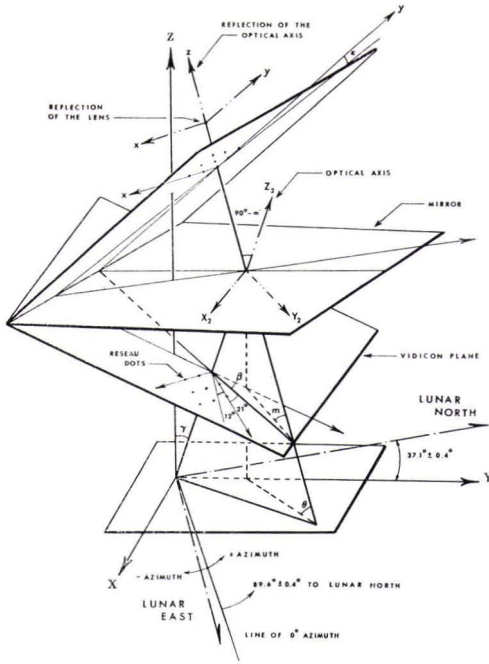


FIG. 7. Geometric elements of mirror photographs obtained during the lunar Surveyor I mission.

that has true terrain significance, other means of deriving the elements of absolute orientation must be found. In the following sections it will be shown that this is possible.

Coordinate Axes. Figure 7 shows the complete geometry of one photograph, including the relationship between the lunar and photographic systems. The vertical plane containing the optical axis intersects the lunar horizontal 37.1 ± 0.4 degrees east of lunar north. The control Y -axis is chosen to coincide with the trace of that vertical plane on the lunar horizontal. The origin of the control system is the point of intersection of the extended optical axis and the lunar horizontal. The Z -axis is taken as positive upward and the X -axis is fixed to form a right-handed coordinate system.

The photo-coordinate system associated with a photograph in the positive position is also shown in Figure 7. The square pattern of the reseau dots on the vidicon is oriented such that one direction makes an angle of $+12$ degrees from the zero azimuth direction. This direction is taken to represent the positive y -direction. The photo z -axis is taken as positive along the reflected optical axis upward, and the x -axis is fixed to form a right-handed system at the virtual position of the photograph, as shown in Figure 7.

Orientation Matrix of a Given Photograph. For every photograph, the elevation angle m'

and the azimuth of the mirror are given. Knowing also the angle γ , the orientation matrix relating the two coordinate systems XYZ and xyz , as defined above, can be readily derived by a set of four sequential rotations. Next, as the relationship between the arbitrary system XYZ and the lunar East-North system is known (see Figure 7) the absolute orientation matrix of each photograph can be easily computed.

Coordinates of Photographic Exposure Station. Given the distance L and h (see Figure 6), the coordinates of the exposure station (i.e., the mirror image of the lens) with respect to the XYZ -system may be computed for every given pair of elevation and azimuth angles. These coordinates can in turn be transformed to corresponding absolute values with respect to the lunar East-North system. Hence, the absolute position of each virtual exposure station can be computed from known parameters of the photographic system.

It has been shown in the preceding sections that for the photographic system utilizing mirrors, it is possible to derive both the position and orientation of each photograph without the need for any control in the object space. This unique possibility is rather fortunate in lunar and planetary applications because of the absence of otherwise necessary control points.

In essence, we have demonstrated that all twelve elements of exterior orientation of a stereo-pair can be computed beforehand from the parameters of the photographic system. With these known, the question of determining spatial positions of object points is rather simple. For a detailed discussion of the various analytical schemes used, the results obtained, and error propagation consideration, the reader is referred to Reference 5.

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