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Tree Heights and Upper Stem Diameters

A single 35-mm. terrestrial photographic determination offers a practical field method.

Abstract: *Foresters need to know the upper stem diameters and heights of standing trees to study tree form, taper, growth, and volume. These dimensions are usually measured at regular intervals up to the merchantable limit of the main stem. Currently used methods of measurement are discussed to provide a comparison for a proposed photogrammetric technique. A single-photo terrestrial photogrammetric method makes use of a 35 mm. camera and a special frame which attaches to the tree, and results indicate that it is practical, economical, and accurate. Additional advantages of the photogrammetric method are also discussed.*

Introduction

Foresters need to know the upper stem diameters and heights of standing trees to study tree form, taper, growth, and volume. Upper stem diameters and heights are at positions on the main trunk which cannot be directly reached and measured from the ground. These measurements are commonly made at four-foot intervals up to the merchantable limit of the main stem. Furthermore, accurate upper stem diameter measurements are necessary for volume table construction, log grading and quality appraisal, sampling schemes such as 3-p sampling where only a relatively few trees are measured, and stem taper and form analysis after prescribed silvicultural practices.[6]

Today's increasing costs and timber values make it desirable to obtain inventory information on timber value and condition through the most efficient and accurate methods possible, within cost limitations. Individual trees may be valued at more than $300 and upper stem measurements are necessary to evaluate their volume, grade and quality.

Industry and research both use repeated measurements over time to assess changes in tree characteristics, such as volume or form. Industry alone spends many thousands of dollars each year on periodic remeasurement of trees on permanent sample plots located throughout their timber holdings. Each tree on these sample plots is uniquely identified and several measurements, including upper stem diameters and heights, are made on it. The period of remeasurement varies but is usually 5 years.

Currently Used Methods

Upper stem diameter and height measurements can be made directly or indirectly. The indirect methods use more complex and diverse instrumentation and are the most widely applied.

Direct Measurement

For direct measurement, the person taking the measurements actually climbs the tree to obtain the desired information. Diameters are commonly measured with either a diam-

* Presented at the Spring Technical Meeting, Great Lakes Region, American Society of Photogrammetry, South Bend, Ind., May 1968 under the title "Photogrammetric Determination of Upper Stem Diameters and Heights."

eter tape or calipers, and heights are read from a tape lowered to the ground. Climbing irons, ladders or Swiss Tree Grippers are a few of the devices used to climb the trees. There are severe drawbacks in obtaining direct measurements. Climbing irons can injure the tree's cambium which is undesirable where repeated measurements are to be made as taper and growth may be influenced by such injury. Safety is another critical factor. If several trees are measured in a day, the climber will become fatigued and very accident prone. Also, a tree may be too tall, have too many limbs, or have too great a diameter to be measured directly.

INDIRECT MEASUREMENT

Indirect measurements of the upper stem are obtained by some instrumentation system remote to the tree. The systems currently used are all terrestrial and can be categorized as hypsometers, instruments used only to measure heights; dendrometers, instruments used only to obtain diameter; and instruments designed to measure both heights and diameters. Only the latter group will be discussed here.

Dendrometer-Hypsometer Combinations. Instruments with which both upper stem diameters and heights can be measured are the transit, Liljenstrom dendrometer, optical calipers (vertical arc, optical wedge), Abney level, Spiegel-Relascope, Barr and Stroud dendrometer, and the terrestrial camera. This list, although not complete, represents the principal types of instruments in use.

The Spiegal-Relascope is used to measure diameters and heights by reading the instruments slope-actuated scales. It has no magnification and must be positioned some fixed horizontal distance from the tree. Field experience has shown that accuracy better than 1 inch for diameters cannot be expected. Tree lean will in some way affect both diamter and height measurements (Figure 1). Bouchon[1] using an instrument of similar design, found that without calibration of the instrument, heights could be measured within 5 or 10 percent, but with calibration the error was only about 1 percent.

FIG. 1. Error in height measurement introduced by tree lean or sway.

The Liljenstrom dendrometer is a telescopic instrument with a scale etched in the eyepiece. It is equipped with a level and vertical arc. The etchings in the eyepiece function similarly to stadia hairs in a transit. In a field test, Winters[16] concluded: (1) errors in height and diameter measurement increase with height; (2) the instrument's design makes it difficult to obtain accurate and repeatable readings on the crosshair scale; (3) tree sway from the wind makes upper stem diameter measurements very inaccurate; and (4) diameters were generally measured with an accuracy of ± 10 percent whereas heights were measured to ± 3 percent. An additional contribution to the errors could have resulted from the dendrometer not being perfectly leveled and the trees measured not being vertical.

The transit is capable of accurate vertical and horizontal angular measurements. Leary[9] however, found that it had several field limitations in determining upper stem diameters. The standard deviation of individual diameter measurements taken at four heights was ± 0.177 inches, but large irregularities were noted when the measurements were taken on swaying trees, on trees with large bark plates, and where the stem was partially obscured by branches or foilage. Leary also noted that tree lean introduced some inaccuracy into the results.

The optical wedge-Abney level, or other clinometer combination, provides relatively good results for the initial cost of the instrument. Miller[12] has shown how a circular optical wedge in combination with an Abney level can be used to obtain diameters. The horizontal distance to the tree must be known for height determinations. Miller[13] states that under ideal conditions, diameters can be estimated within ± 0.2 inches. Tree lean will introduce error in height measurement, but diameter measurements will not be influenced if, when the prism is being moved from the sighting position for the diameter to the target position, the prism is moved parallel to the axis of the tree.

The optical calipers-inclinometer is an instrument which used optical calipers to measure diameters and the inclinometer to obtain heights. This device has not in actuality been constructed and is only a proposal. The calipers have an advantage over those dendrometers depending upon isosceles triangles to

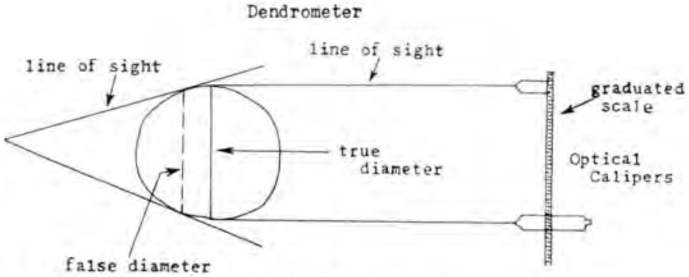

FIG. 2a. Lines of sight of optical calipers and of a dendrometer using an isosceles triangle to solve for diameters.

derive diameters as the lines of sight are parallel and tangent to the true diameter of the tree (Figure 2a). The other dendrometers measure a false diameter which is something slightly less than the true. Diameter measurement is not biased by tree lean but the range of measureable diameters is limited by the distance which the penta-prisms can be separated. Wheeler[15] has a 95 percent confidence interval of ± 0.5 inches in a field test on ten trees. Heights are calculated from angles generated by the inclinometer with the instrument set up at some given distance from the tree. As with the Abney level, sway and accuracy of the distance measurement from the tree will affect the accuracy of the determined height.

The Barr and Stroud dendrometer is a modified rangefinder with magnification and has an inclinometer from which heights can be calculated. Jeffers[8] has an excellent basic discussion of the instrument. He also reports the results of a field test involving 408 sets of readings of diameters and heights. He points out that the manufacturers have shown that under laboratory conditions diameters can be measured to within ± 0.1 inches for diameters between 1.5 and 10.0 inches and within ± 1 percent for diameters from 10.0 to 30.0 inches. The field test data indicated that approximately 95 percent of all individual measurements would be within ± 0.6 inches for diameter, ± 1.0 feet for height, and ± 2 feet for range. Jeffers noted that any obscuration of the point being measured resulted in readings giving standard deviations three times those listed above. Bruse[2] and Grosenbaugh[5] also cite field tests which indicate results comparable to those of Jeffers. However, the cost of the Barr and Stroud is about $2,900. The other instruments with the exception of the transit cost less than $150.

The terrestrial camera should provide a very reliable system of obtaining upper stem measurements. However, in a study conducted by Marsh[10], diameter accuracy of only ± 2.5 inches for tilted oblique and ± 0.8 inches for horizontal photography was obtained. He also states that 17 percent of the measurements on horizontal photos were obscured. Apparently he had set up the camera some distance from the tree to take the photographs. Grosenbaugh[5] dismisses the camera as slow, expensive and inaccurate. The latter part of this paper will present a different approach and refined technique by which inexpensive accurate measurements can be obtained.

The object of this background has been to familiarize the reader at least partially with those instruments that have been or are in use, and to provide a guide for comparison with the results of a study on a relatively untried instrument, the camera.

Photogrammetric Application

Forestry literature covering tree height and diameter measurements, as mentioned earlier, often does not consider photogrammetry as a possible method because it is expensive, inaccurate, or too complicated, although little discussion appears to substanti-

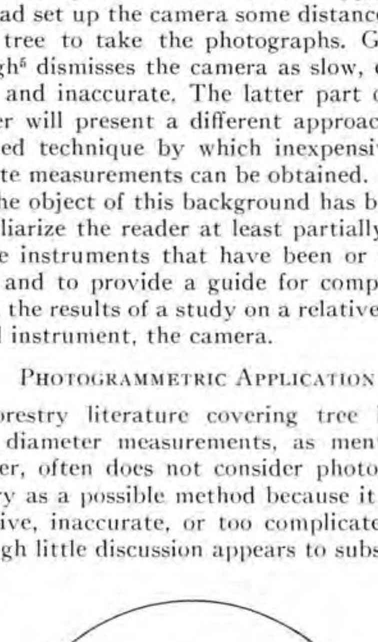

FIG. 2b. Sighting picture for the optical calipers.

ate these claims. The purpose of this paper is to evaluate photogrammetric methods which are not only accurate enough, but which are also practical and economical. There is no question that elaborate photogrammetric equipment and methods can produce the required results. However, unless the equipment is already on hand, this would not be an economical possibility. The interest, then, is in determining whether or not a simple photogrammetric solution using a minimum of equipment is possible.

Aerial methods are eliminated from consideration because of expense and inability to distinguish the tree stem through the canopy. Possible methods to be discussed are all ground methods.

STEREOSCOPIC METHODS

Perhaps the first method which comes to mind is to use a fixed-base stereometric camera. However, the camera is expensive and usually quite heavy, and two photos are required for each tree which will increase operational costs over single-photo methods. Tree lean or sway, though, will not be a problem, and no external measurements are required for a solution. An error propagation of the formulas suggests that the solution might be at least as accurate as presently used methods. However, a fairly expensive coordinate measuring instrument is required if an analytical solution is desired, or an expensive restitution instrument if the solution is to be analogue. The analytical solution is very simple and can be done on a desk calculator if a computer is not available.

In order to reduce original expense, a single camera can be mounted on a bar and moved between exposures to obtain the required base. It can be designed so that the camera axes are convergent, thus giving more coverage. However, accuracy might be reduced because of the loss of the stability of the fixed characteristics, and the analytical solution, if used, will be more complicated. Tree sway between photos will also become a factor.

If the camera is placed closer to the tree the angle of inclination of the camera axis must be increased. As a result the angular coverage and depth of field of the lens become critical.

SINGLE-PHOTO METHODS

The major disadvantage of a stereoscopic method is that the equipment is expensive and the reduction of the data can be somewhat complicated. Because of these factors, it was considered that a single-photo technique would be inexpensive and uncomplicated by comparison. The major problem with this method is whether or not enough accuracy can be obtained.

If the camera is placed at a distance with its axis horizontal, a simple scale relationship can be used for the computations. If a rod of known length is fastened to the tree, and appears in the photo, the camera focal length need not be calibrated and the distance to the tree does not have to be measured. The basic assumption for this case is that the camera axis is perpendicular to the stem of the tree. Tree lean will affect the accuracy. Also, with the camera axis horizontal, the camera will have to be a great distance from the tree in order to obtain coverage of the whole stem unless the lens has a very wide angle of coverage. This large distance will increase the error and also make indentification of select points more difficult.

If the camera axis is inclined to increase the coverage and decrease the distance from the tree, the simple scale formula cannot be used because the photo will now be oblique with respect to the axis of the tree. There are several methods of solving the oblique geometry. It can be done analytically, graphically, or with instruments, such as an oblique sketchmaster or other simple rectifier. If the solution is to be by instrument there are several types of simple rectifiers which could be purchased, or built from surplus material if necessary. An oblique sketchmaster would be simple and economical, the points of interest on the tree being marked on a grid sheet and the desired distances scaled from it.

If a graphical solution is used, a perspective grid overlay can be constructed and placed over the photograph. The planimetric distances (diameters and heights) at desired points can then be transferred to a scale line on the grid and the actual distances scaled from it. This is an extremely economical solution as no special instrumentation is required, and it is quite fast. This is one of the methods chosen for further testing, and it will be discussed more thoroughly presently.

The analytical solution uses photo distances between points as input data. The formulas, although not overly complicated, are probably better suited for use with computers if a large amount of data is to be processed. The formulas do, however, break down quite nicely for use with a desk calculator. The photo distances must be measured to be relatively high precision. Either the negative or a print can be used. There are several methods of obtaining these photo distances. A comparator can be used to provide the highest precision, but the instrument is expensive. A good glass scale can also be used, or the negative can be projected onto a grid sheet and the distances scaled from that. The analytical solution will also be discussed further.

METHOD OF PHOTOGRAPHY

It is proposed that a 35 mm. camera be used because the operational expense is low and the camera itself is small, rugged, and light. Up to 36 trees can be covered on one roll of film and the film is easily stored, which is important because the studies involved are usually time studies and the measurements are repeated every few years, making it desirable to store the film so that future comparisons can be made. As only one photograph is taken per tree, the measurements will be determined from photo distances and scales. Because the photo will be oblique to the tree axis, the geometry will be greatly simplified if the center-line of the tree is in the principal plane of the photograph and if the depression angle is defined as being measured from a plane which is parallel to the longitudinal axis of the stem. This means that the diameters will lie along lines of constant scale and the heights will be measured along the principal line.

Perhaps the simplest way to ensure the above conditions is to mount the camera on a frame which will attach to the tree (Figure 3). The frame can be in the shape of a right angle, one leg fastened to the tree parallel with the tree axis (lying in the principal plane) and the other at right angles to the tree axis. The camera is mounted on a tilted platform at the end of the latter leg. Targets can be attached to the base leg (the one parallel to the tree axis) in such a way that they define two known distances which are perpendicular to the principal plane, thus having a constant scale along them. These distances also lie in a plane containing the tree axis, which is the plane in which the measurements are to be determined. The frame can be built from aluminum, and the sections hinged so that they fold up, making it easily portable.

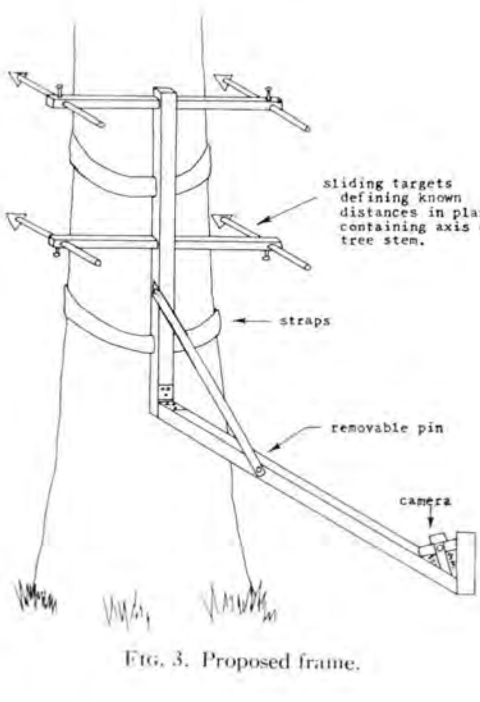

FIG. 3. Proposed frame.

The depth of field of the camera lens becomes important if the camera is close to the tree, as it is when a frame is used as above. The limits of the depth of field can be calculated from the following formulas[4]:

$$D' = D/(1 + \mathrm{Dcf}/F^2), \qquad D'' = D/(1 - \mathrm{Dcf}/F^2)$$

where

- D' = near limit, feet
- D'' = far limit, feet
- D = focused distance, feet
- c = limiting circle of confusion, normally $F/1{,}000$
- f = f-number of lens aperture.
- F = focal length of lens, feet

For example, if $F=50$ mm, $c=F/1{,}000$, $f=16$, and $D=12$ feet, then the depth of field would extend from 5.5 feet to infinity.

A very fine-grain film should be used. It is proposed that enlargements of the negatives be used for obtaining the measurements, although the negatives themselves can be used. However, we have found that enlargements give better results.

MATHEMATICAL FORMULATION—Diameters D_i

Refer to Figures 4 and 5. Let i be any line perpendicular to the principal line and

- S_i = photo scale for x-direction, line i
- d_i = photo distance for line i
- D_i = actual distance for line i
- H = perpendicular distance of camera from tree axis, $H=L+r$ where L = frame constant and r = radius of tree at the base of the frame
- θ = depression angle
- Y_i = photo y-coordinate of line i, measured from true horizon.

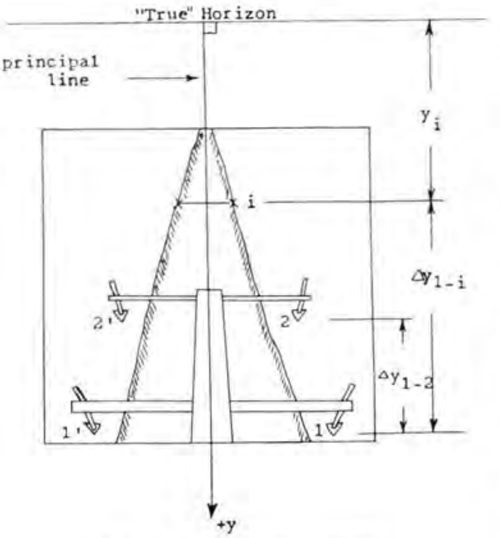

FIG. 4. Perspective geometry.

It can be shown that $S_i = Y_i \cos \theta/H$ (See appendix for proof). Since θ and H are constant for the photograph, let $A = \cos \theta/H$. Then $S_i = Y_i A$, and for lines 1 and 2, $S_1 = d_1/D_1$ and $S_2 = d_2/D_2$ where d_1, d_2 are photo distances between the targets on the frame and D_1, D_2 are known. Thus

$$S_1 = Y_1 A$$
$$S_2 = Y_2 A,$$

and subtracting,

$$S_1 - S_2 = A(Y_1 - Y_2).$$

Let $Y_1 - Y_2 = \Delta Y_{1-2}$ which can be measured on the photo. Thus

$$A = (S_1 - S_2)/\Delta Y_{1-2} \quad \text{and} \quad Y_1 = S_1/A,$$

and for any point i

$$Y_i = Y_1 - \Delta Y_{1-i}$$

where as before ΔY_{1-i} can be measured on the photograph. Also

$$S_i = Y_i \cdot A = d_i/D_i \qquad D_i = d_i/(Y_i \cdot A)$$

which is the desired formula for the diameter at point i. It can be rearranged into many other forms, but perhaps the easiest to use is

$$D_i = d_i/(S_1 - A\Delta Y_{1-i})$$

or

$$D_i = d_i/S_i.$$

MATHEMATICAL FORMULATION—Heights M_i.

Let M_i be the height from a reference point on the frame to a point i on the tree (Figure 5). Because the derivation is rather lengthy it will not be presented here, but is given in the appendix. However, it can be shown that

$$M_i = (M_1 S_1 + \Delta Y_{1-i} \sin\theta)/S_i.$$

This formula can also be changed to the form

$$M_i = -M_1 + S_2 \Delta M_{1-2}(1 + S_1/S_i)/(S_1 - S_2)$$
$$M_i = ((1/A^2) - H^2)^{1/2} + S_1 S_2 \Delta M_{1-2}/(S_1 - S_2)S_i.$$

Because these formulas involve different variables with different accuracies, their error propagations are slightly different. In the first form ΔY_{1-i} and S_i are the only factors which change for the entire photograph, whereas in the second and third forms only S_i changes. The factor A can be computed from several different formulas, either $A = \cos \theta/H$, where

$$\theta = \arccos (H(S_1 - S_2)/\Delta Y_{1-2})$$

or can be determined from calibration of the frame, or

$$A = (S_1 - S_2)/\Delta Y_{1-2}, \quad \text{or}$$
$$A^2 = 1/(H^2 + (M_1 - S_2 \cdot \Delta M_{1-2}/(S_1 - S_2))^2).$$

(See appendix for derivation.) Again, the last two formulas have different error propagations, the latter one being the more favorable.

It is interesting to note from the formulas for A, M_i and D_i that the diameter and height calculations are not affected by a shrinkage of the enlargement or negative as long as the shrinkage is linear and equal in both the x and y directions. If it is not equal in both

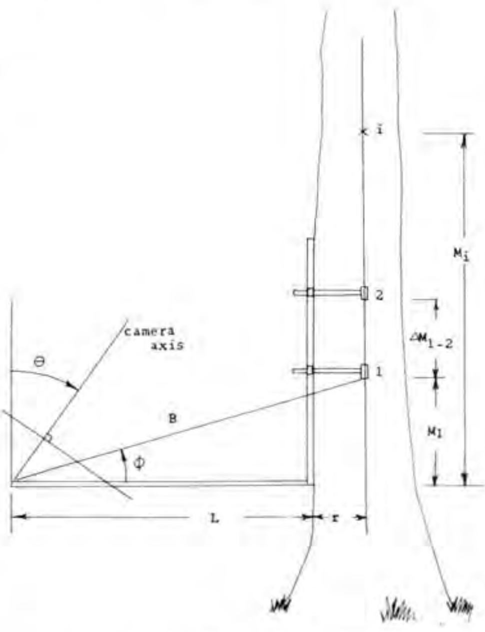

FIG. 5. Side view of frame mounted on a tree.

directions the appropriate formula for A can be used which will still have no effect on the diameters, and only a slight effect on the heights.

A suggested mathematical procedure for each photograph is as follows: Compute

$$S_1 = d_1/D_1 \quad \text{and} \quad S_2 = d_2/D_2$$
$$H = L + r$$
$$N = S_2 \Delta M_{1-2}/(S_1 - S_2)$$
$$A = (1/(H^2 + (M_1 - N)^2))^{1/2}$$

and check by computing

$$A = (S_1 - S_2)/\Delta Y_{1-2}$$
$$P = -((1/A^2) - H^2)^{1/2}$$
$$Q = S_1 N$$

where

d_1, d_2 are photo measurements, D_1, D_2, L, M_1, ΔM_{1-2} are frame constants, and r is the radius of the tree at the base of the frame, measured manually at the time of photography.

Then, for each point i where the diameter and corresponding height is desired, compute

$$S_i = S_1 - A \Delta Y_{1-i}$$
$$D_i = d_i/S_i$$
$$M_i = P + Q/S_i$$

where ΔY_{1-i} is a photo measurement. This mathematical solution would be strengthened considerably by the addition of a third known distance on the frame which would provide a redundancy of the data.

Two important features of this method are:

(1) The only measurements involved other than r are photo distances which can be made in a laboratory with a scale, and which can be repeated as often as necessary. If, in the future, a different spacing of measurements is desired, or additional information needed, they can be made as the tree shape is preserved on a photographic record.

(2) Because the scales S_1 and S_2 are determined from photo measurements, and similarly ΔY_{1-i}, any linear film shrinkage or other similar systematic errors do not affect the computed diameters. In fact, in one of the test cases the enlargement had considerably more y-shrinkage than x-shrinkage, but this did not affect the diameters.

In addition, a small degree of tree bend will not affect the method as long as the bend is in one plane, and the frame is strapped to the tree in such a position that the camera leg is perpendicular to that plane. Most trees of interest that do bend usually do so in one plane only. Even though the tree is bent so that it is no longer parallel to the principal line, the angle that it makes with the principal line can be easily computed and a correction can be made to the computed diameters in that region of the stem.

GRAPHICAL REDUCTION

The theory of perspective grids can also be utilized for a graphical reduction of the diameters and heights. One of the known scale-lines (usually line 1) is used as a base. The principal line is constructed along the base leg of the frame and up the middle of the tree, and the *true horizon* or vanishing line is constructed perpendicular to it at a distance Y_1 from the scale-line, Y_1 being computed as in the mathematical method. This point of intersection of the vanishing line and principal line is also the intersection of the

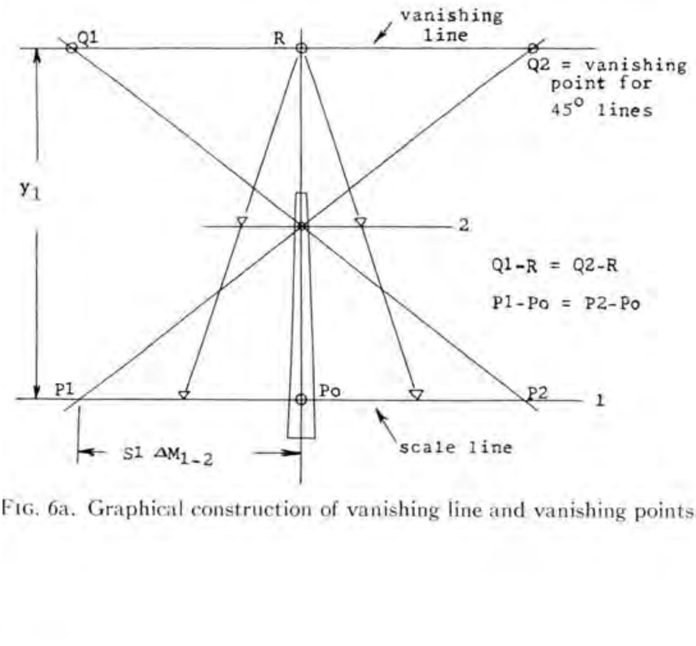

FIG. 6a. Graphical construction of vanishing line and vanishing points.

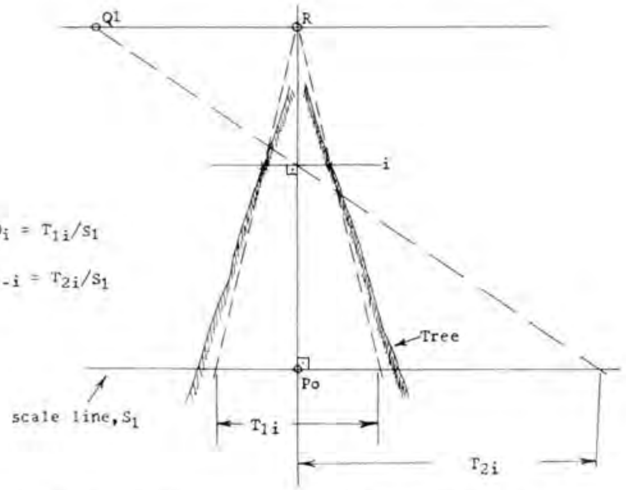

FIG. 6b. Graphical determination of diameters and heights.

lines connecting the two targets on each side of the tree (See Figure 6a) if the two known distances are equal. The vanishing points can be located using the known scale of the scale-line and the known distance ΔM_{1-2} on the frame, as illustrated in Figure 6a. Vanishing points for 45°-lines were used, although points for smaller angles may be more convenient. All diameters are transferred to the scale-line for measuring. Any line of known scale may be used. The heights can similarly be transferred to the scale-line using the vanishing points, and their values measured there (See Figure 6b). Tests have shown good accuracy for both diameters and heights. There are several advantages which are characteristic of the graphical method:

- The perspective base for the reduction (principal line, scale-line, vanishing line) can be pre-drawn on a plastic overlay for particular diameter trees and enlargements, since the only variable involved in the grid is Y_1, which depends on the radius of the tree at the point where the camera leg of the frame touches the tree and on the enlargement distance d_1. This shifts the position of the vanishing line only, and an *all-diameter-scale* grid can be constructed as shown in Figure 7. The lines D_1, D_2, etc. represent different combinations of diameters and enlargement distances d_1.
- The only extra equipment or material needed other than camera and frame is an enlargement of each photo and a scale. It seems from the tests that a 4×5-inch size is sufficient. The cost of an enlargement of this size is negligible if a sufficient number are done.
- Other than the darkroom technician, no special skills are required, as the reduction is very simple and quick.
- The camera does not have to have fiducial marks, and the principal distance does not have to be known.
- Tree sway and/or lean do not affect the measurements, and irregularities in stem form can be smoothed very easily.

The latter four advantages pertain to the mathematical reduction also.

EXPERIMENTATION AND RESULTS

In order to confirm the theoretical investigation, some laboratory tests were performed using targets at known distances away and known distances apart. Pairs of targets spaced 1 and 3 feet apart were placed on a

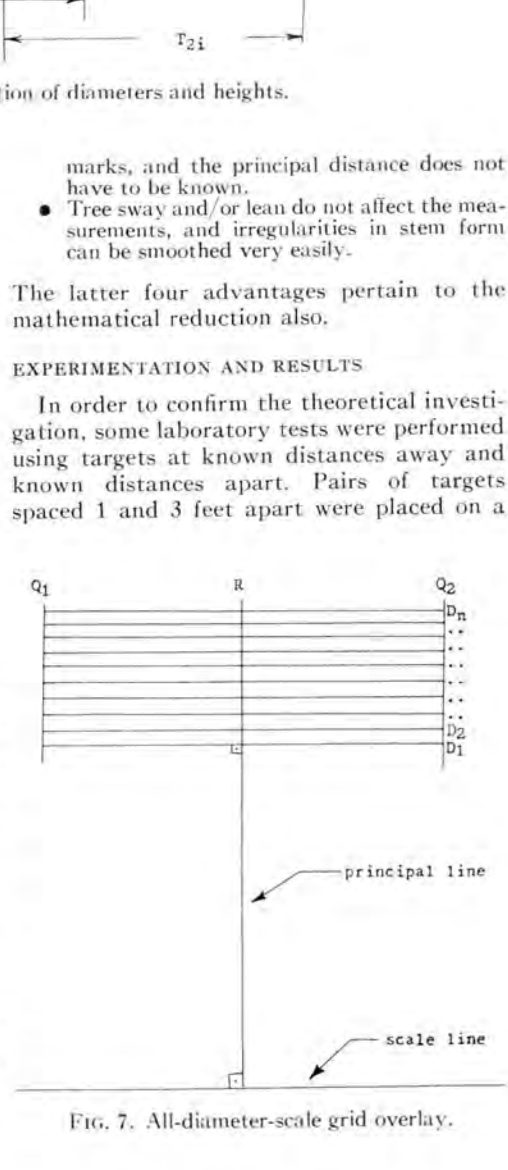

FIG. 7. All-diameter-scale grid overlay.

flat surface at 5-foot intervals up to 100 feet from the camera position, even though the merchantable limit of the majority of trees on which upper stem measurements are made is only about 40 feet. The results were very good and have led to the conclusion that this type of photogrammetric solution can indeed be competitive with currently used methods. The camera had a focal length of about 50 mm. and an angular field of about 39°. It cost less than $50 new. The negatives were enlarged about 4 times, giving a 4×5 print to work from. This was a convenient size to use as the detail was large enough to see easily, yet the cost of a 4×5 print is very small. An interesting aspect of this method is that the exact size of the enlargement is not needed, thus giving the darkroom technician freedom from matching sizes or making measurements.

The photo measurements were made with a Gurley comparator (glass scale) with a least reading of 0.005 inch. The reduction was done both mathematically and graphically, both methods giving similar results. The diameters were determined with a standard error of about 0.3 inch and the heights with a standard error of about ½ percent of the height. However, if the test had been conducted to only 40 feet, these errors would have been much smaller. Because of some unfavorable test conditions it was believed that even better determinations could have been made. In other words, these figures do not represent the limits of the system, but merely indicate that sufficient accuracy is inherent in the method. Even though these results are from laboratory tests, it is expected that results in the field should not be too much different because of the fixed characteristics of the system. Field tests are being scheduled in order to confirm this.

As mentioned previously, linear photo shrinkage has no effect on diameter measurements and very little effect on heights because the photo images of the known control also have the same systematic error. In fact, the true simplicity of this photogrammetric method does not become apparent until the method is actually used. Only then does the speed, accuracy, economy, and simplicity of the method completely reveal itself.

The main purpose of this paper was to demonstrate the feasibility of applying photogrammetry to the problem of determining upper stem diameters and heights. The results of the investigation show that a photogrammetric method can be practical, economical, and accurate, and that it should be regarded as an approach worthy of consideration. In addition, to a photogrammetric solution's advantages, which have either been mentioned or are otherwise readily apparent, one in particular seems quite important: the fact that one obtains for each tree a permanent photographic record, which could be invaluable when the time-growth characteristics of the tree are evaluated at the conclusion of the study period. No other method has the capability of obtaining and storing so much information with so little effort.

The actual methods and equipment used in the investigation should be considered as examples only. Certain modifications and refinements may be necessary in order to improve the technique and results. For example, a correction table or formula can be calculated to correct the diameter measurements so that true diameters are obtained, because the diameters computed are not actual diameters but are distances between two tangent planes. This also happens with many of the currently used methods and was discussed in the first section of this paper. The assembled apparatus should be completely tested and calibrated. This would undoubtedly improve results. In addition, actual use should expose shortcuts and modifications which will improve the execution of the method.

Appendix

A. Derivation of $S_i = Y_i \cos\theta / H$

Refer to Figure 8. Let $\overline{LP} = c$, $\overline{Li} = C$, $\overline{KP} = Y_i$, and $S_i = c/C$ by definition. Construct PP' parallel to Ni and LK, and $L'P$ parallel to LP'. Then $c/C = LP'/H$ by similar triangles. As $LP' = L'P$, $\cos\theta = L'P/KP = LP'/Y_i$ or $LP' = Y_i \cos\theta$. Thus, $S_i = c/C = LP'/H = Y_i \cos\theta / H$.

B. Derivation of Formulas for M_i and A

Refer to Figure 8. By the laws of sines,

$$\frac{Z + \Delta y_{1-i}}{\sin\phi_c} = \frac{c}{\sin\theta} \tag{1}$$

$$\frac{Z}{\sin\phi} = \frac{b}{\sin\theta}$$

$$Z = \frac{b \sin\phi}{\sin\theta}$$

By definition of scale, $S_1 = b/B$ and $S_i = c/C$ or $b = BS_1$ and $c = CS_i$. Therefore $Z = BS_1 \sin\phi / \sin\theta$. From triangle $LN1$, $M_1 = B \sin\phi$, thus $Z = M_1 S_1 / \sin\theta$ (2). Substituting Equation 2 into 1 and multiplying by $\sin\theta$ gives

$$(M_1 S_1 + \Delta y_{1-i} \sin\theta)/\sin\phi_c = c. \tag{3}$$

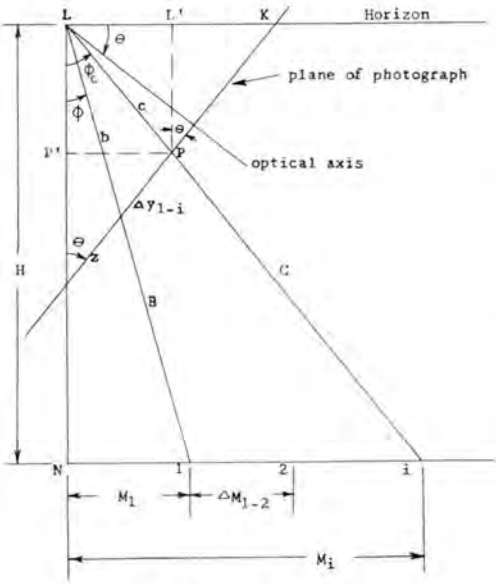

FIG. 8. Oblique photo geometry.

But $c = CS_i$, and from triangle LNi, $\sin \phi_c = M_i/C$ or $\sin \phi_c = M_iS_i/c$ which, when substituted into Equation 3, gives

$$(M_1S_1 + \Delta y_{1-i} \sin \theta)c/M_iS_i = c.$$

Rearranging and dividing both sides by c gives

$$M_i = (M_1S_1 + \Delta y_{1-i} \sin \theta)/S_i. \qquad (4)$$

Writing Equation 4 for point 2 gives

$$M_2 = (M_1S_1 + \Delta y_{1-2} \sin \theta)/S_2.$$

But

$$M_2 = M_1 + \Delta M_{1-2}$$

so that

$$(M_1 + \Delta M_{1-2})S_2 = M_1S_1 + \Delta y_{1-2} \sin \theta$$

or

$$M_1(S_2 - S_1) = \Delta y_{1-2} \sin \theta - S_2 \Delta M_{1-2}$$

$$M_1 = (\Delta y_{1-2} \sin \theta - S_2 \Delta M_{1-2})/(S_2 - S_1). \qquad (5)$$

From the original definition of A in the text, $A = (\cos \theta)/H$, we get $\cos \theta = AH$ and $\sin \theta = (1 - \cos^2 \theta)^{\frac{1}{2}}$. Thus $\sin \theta = (1 - A^2H^2)^{\frac{1}{2}} = A(1/A^2 - H^2)^{\frac{1}{2}}$ (6). Substituting Equation 6 into 5 gives

$$M_1 = (A \Delta y_{1-2}(1/A^2 - H^2)^{1/2} - S_2 \Delta M_{1-2})/(S_2 - S_1). \qquad (7)$$

But we also had in the paper that

$$A = \frac{S_1 - S_2}{\Delta y_{1-2}} \quad \frac{S_1 - S_i}{\Delta y_{1-i}}$$

or

$$A \Delta y_{1-2} = S_1 - S_2 = -(S_2 - S_1) \qquad (8)$$

which, when substituted into Equation 7 gives

$$M_1 = -(1/A^2 - H^2)^{1/2} + \frac{S_2 \Delta M_{1-2}}{(S_1 - S_2)}. \qquad (9)$$

Substituting 9, 8, and 6 into Equation 4 gives

$$S_iM_i = -(1/A^2 - H^2)^{1/2}S_1 + \frac{S_1S_2\Delta M_{1-2}}{S_1 - S_2} + (1/A^2 - H^2)^{1/2}(S_1 - S_i)$$

which, when rearranged, becomes

$$M_i = -(1/A^2 - H^2)^{1/2} + \frac{S_1S_2\Delta M_{1-2}}{(S_1 - S_2)S_i}. \qquad (10)$$

Equation 9 can be arranged to solve for A as follows:

$$M_1 - \frac{S_2\Delta M_{1-2}}{S_1 - S_2} = -(1/A^2 - H^2)^{1/2}.$$

Square both sides,

$$\left(M_1 - \frac{S_2\Delta M_{1-2}}{S_1 - S_2}\right)^2 = 1/A^2 - H^2$$

or

$$A^2 = 1/\left(H^2 + \left(M_1 - \frac{S_2\Delta M_{1-2}}{S_1 - S_2}\right)^2\right). \qquad (11)$$

The term $(1/A^2 - H^2)$ from Equation 10 can be changed using Equation 11 to give

$$1/A^2 - H^2 = \left(M_1 - \frac{S_2\Delta M_{1-2}}{S_1 - S_2}\right)^2$$

and

$$(1/A^2 - H^2)^{1/2} = M_1 - \frac{S_2\Delta M_{1-2}}{S_1 - S_2}. \qquad (12)$$

Substituting Equation 12 into 10 gives

$$M_i = -M_1 + \frac{S_2\Delta M_{1-2}}{S_1 - S_2} + \frac{S_1S_2\Delta M_{1-2}}{(S_1 - S_2)S_i}$$

which, when rearranged, gives

$$M_i = -M_1 + \frac{S_2\Delta M_{1-2}}{S_1 - S_2}\left(1 + \frac{S_1}{S_i}\right). \qquad (13)$$

Equations 4, 10, 11 and 13 are those given in the text of the paper.

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The Adoption of The Metric System by The Ordnance Survey

The essence of the Ordnance Survey's proposals for the adoption of the metric system was that the metric system should be adopted for all future productions of maps at the scale of 1:25 000 and larger, and that lists of bench marks should give heights in metres instead of in feet.

It had originally been intended to defer for the time being any change from the present scale (1:10 560) to the decimal scale of 1:10 000, but preference has been widely expressed for a map on which linear measurements may be simply made with a metric rule, and which stands in a simple relationship as regards scale to the larger scale maps (1:1 250 and 1:2 500). Those who have expressed this preference accept that the change would take many years to become fully effective, and that in the meanwhile, maps at both 1:10 560 and 1:10 000 scales would be current in different areas.

Investigation has shown that a change in scale to 1:10 000 for future production can be carried out without extensive disruption to the Ordnance Survey's mapping programme, and it is now proposed to put this change in hand concurrently with the introduction of contours at a metric interval. The advantage of doing this now, when only some 1 500 of the 10 000 maps in the regular series have been published, is obvious.

The effect of this proposal would be that until conversion to the 1:10 000 scale is completed, six inch scale maps with contours at a vertical interval of twenty-five feet would co-exist with 1:10 000 scale maps with metric contours. The division between the two will be coincident with sheet lines of 1:25 000 mapping, and uniformity of contours will be maintained within the area covered by any single 1:25 000 map. The Ordnance Survey will publish maps in the one style or the other, taking into account the need to avoid, as far as possible, a change of scale in administratively or geographically awkward places. In order to minimise inconvenience the Ordnance Survey is also looking into the possibility of providing monochrome enlargements or reductions to enable composite maps to be made up where a change in scale occurs.

July, 1968.

(*Reprinted from* The Photogrammetric Record, *Vol. VI. No. 32, October 1968. Also included in that issue was* "Metrification in The Photogrammetric Record," *pages 230–234.*)