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Graphic Determination of Slope and of Dip and Strike

This stereographic method is applicable not only to vertical aerial photographs but also to tilted and oblique views.

ABSTRACT: *This stereographic method for determining strike and dip requires only that the two geologic outcrop lines of the bedding be observed. They need not be the same level nor from the same bedding plane. The method can also be applied to the determination of slope. The attitude of only one line is required and considerably less work is involved than for the strike and dip problem. No corrections for relief displacement or vertical exaggeration are required. The method can also be applied to oblique photographs.*

INTRODUCTION

THE METHOD OF DETERMINING dip and strike from aerial photographs described by Wallace (1950) is widely quoted in text-books of photogeology and structural geology (e.g., Ray 1960, p. 64, Badgley 1959, p. 202–206). The method is probably not widely used, and this may account for its having escaped criticism for so long.

However, it has advantages over other methods of determining dip and strike from aerial photographs. No specialised equipment is required other than a Wulff net. The method can be used when stereometer and mirror stereoscope are not available and it can be applied to more limited data than required by other methods.

The stereometer method requires that the strike and direction of true dip be estimated in stereoscopic viewing before the parallax measurements can be utilised. (e.g., Hemphill, 1958). In many terrains the evidence is generally inadequate for this purpose. The stereographic method described here requires only that two outcrop lines of the bedding be observed. They need not be at the same level nor from the same bedding plane, although they must be close enough so that it can be assumed that they belong to the same bedding attitude. Both dip and strike are derived from the determination.

The stereographic method can also be applied to *slope* determination. The attitude of only one line is then required and considerably less work is involved than for strike and dip determination. No corrections for relief displacement or vertical exaggeration are necessary. This method of determining slopes may be valuable to geomorphologists, geologists and others as an alternative to methods using specialized equipment, such as the stereo-slope comparator (Heckman 1956). It may also be used for determining true slopes as a basis for determining the vertical exaggeration factor in stereoscopic viewing.

Finally, unlike the other methods, the stereographic method can be applied to oblique photographs.

Familiarity with stereographic projection methods (e.g., Phillips 1954) is required, but the principles are simple and most geology students receive training in these methods. This paper therefore attempts to eliminate the errors in the method as described by Wallace, and to present a procedure which is applicable to both vertical and oblique photographs.

DISCUSSION OF THE METHOD

The principle of the method is simple (cf. Wallace, op. cit. p. 270). Figure 1 illustrates the fact that for any short feature BC on the ground surface (which may be an outcrop line or a line between two points at top and

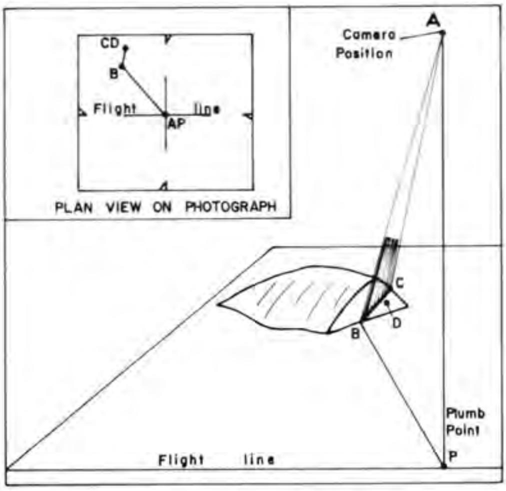

FIG. 1. Relationship between an outcrop line on the ground and its image on the aerial photograph. *BC*, the outcrop line on the ground surface, will be seen as the rake line *BD* on the photograph owing to relief displacement. For further explanation see text.

bottom of a slope), a plane *ABD* in which the feature is observed can be defined by the sight line *AB* and the rake line *BD* (which represents the projection along the sight line of the feature *BC* on the photograph). This plane must contain the true position of the required linear feature. Thus, if the same feature is observed along the different sight lines of two photographs of a stereographic pair, two planes can be obtained, each of which contains the true position of the required feature. If these two planes are plotted in stereographic projection, then their intersection provides the true orientation of the required outcrop line or slope line. This is all that is necessary for determination of slopes.

In the case of dip and strike determinations, the process must be carried out for two outcrop lines in a locality where the bedding plane orientation appears to be constant, e.g., outcrops on either side of a valley. Thus two apparent dips can be obtained. The plane containing these two apparent dips can be obtained. The plane containing these two apparent dips in the stereographic projection is the plane of bte bedding and gives the true dip and strike.

Wallace (1950) used this principle for the determination of dip and strike both from indirect observations in the field and from aerial photographs. In all cases he plots the measured angles to the rake lines in the plane normal to the sight line on the stereographic projection. This is correct for direct observations made in the field because then the angles to the rake lines can be measured in the plane normal to the line of sight whatever the orientation of the line of sight (Wallace op. cit. problem 1, pp. 270–272). In aerial photographs, however, the plane in which angles are measured is the plane normal to the optical axis of the camera system, and not the plane at right angles to the line of sight. For example, in vertical aerial photographs the sight line is normal to this plane only at the principal point of the photograph. Elsewhere on the photographs the sight line will usually have a plunge between 90° and 60°. The measured angles to the rake lines should therefore be plotted in the horizontal plane in stereographic projection and not in the plane normal to the sight line. This is the procedure adopted in the revised method given below.

In the method given here the fundamental plane of the stereographic projection is not necessarily horizontal. It is taken to be the plane of the photograph, which is normal to the optical axis. The optical axis may suffer slight tilt from the vertical in *vertical* air photographs, or it may be strongly tilted in oblique air photographs. In either case, correction for the tilt can be made at the end of the procedure, which is therefore applicable to both vertical and oblique photographs.

The geographical orientation of the projection is found in different ways for vertical and oblique photographs. In the case of vertical photographs, the north line through the principal point can be plotted in the fundamental plane as the zero meridian. For oblique photographs the line normal to the axis of tilt in the plane of the photograph is chosen as zero meridian. Its azimuth, the direction of tilt, must be known.

In order to plot the attitudes of any rake line and its sight line in the stereographic projection, it is necessary to choose a suitable reference line from which the angles to the required sight and rake lines can be easily measured in the plane of the photograph. This may be the zero meridian or the projection of the sight line itself on the photograph (*BP* in Figure 1). The latter is the line joining the principal point to the observed feature and is here preferred.

For vertical photographs, the projection of the sight line can now be plotted on the primitive circle of the stereographic projection at the appropriate horizontal angle, measured on the aerial photograph, from the north line (the zero meridian); and the trends of the rake lines can also be plotted on the

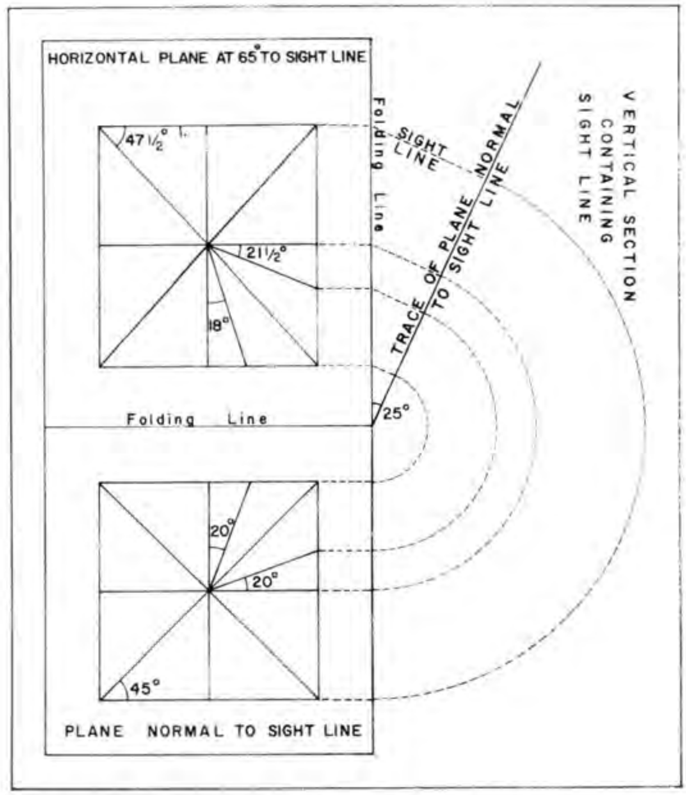

FIG. 2. Diagram showing the angular relations between lines in the plane normal to the sight line and between the same lines projected along the sight line onto a horizontal plane.

primitive circle at the measured angles from the sight line trend. For oblique photographs, the projection of the sight line is plotted, with reference to the zero meridian (whose azimuth is the direction of tilt) in just the same way as for vertical photographs. The angle between the projection of the sight line and the zero meridian is measured in the plane of the photograph and this angle is plotted on the primitive circle of the projection. Again, the projections of the rake lines in the plane of the photograph can be plotted on the primitive circle of the stereographic projection at the measured angles from the projection of the sight line. The true geographic orientation of the latter is not required and can be ignored.

The angles measured from the projection of the sight line (or from any other reference line in the plane of the photograph) to the selected rake lines in the plane of the photograph will not be the same as the corresponding angles in the plane normal to the sight line. This is illustrated graphically in Figure 2, which shows the relationship of angles between lines seen in the plane normal to the sight line and angles between the same lines projected by orthographic projection along the sight line onto a horizontal plane. The construction is made by the standard method of preparing *right sections* in structural geology (see Badgley 1959, pp. 51–55) and is self-explanatory. It is easily seen that horizontal lines and their normals in the plane normal to the sight line retain their orthogonal relationship when projected onto a horizontal plane. All other lines make a larger angle to the horizontal when projected onto the horizontal plane.

It is clear, therefore, that angles measured in the plane of the aerial photograph cannot be plotted in stereographic projection in the plane normal to the sight line as Wallace (op. cit.) recommends. However, *if the angles are correctly plotted in the fundamental plane of the projection as recommended above, the procedure is actually simplified.*

The plunge of the sight line can be determined for both vertical and oblique photographs with reference to the optical axis (which is normal to the fundamental plane of the stereographic projection) if the lens focal length is known. The construction of Figure 3 illustrates the principle for constructing a scale for the direct measurement of angles from the principal point on the photograph. Thus the trend and plunge of each sight line can now be plotted on the stereographic net for either vertical or oblique photographs with reference to their respective fundamental planes and zero meridians.

The method assumes that the plunge of the sight line can be regarded as constant over the area of observation of any one rake or slope line. If the observed rake lines are only a few millimeters long on photographs of about 1:25,000 scale, then the true plunge of the sight line over the measured area will not vary by more than about 1°. Such variation will not produce significant errors in the measured angles.

Upper hemisphere projections have been used.

Procedure for Vertical Photographs

The north line is taken as the zero meridian. One end of the observed feature is taken as reference point, and the sight line to this point is plotted on the stereographic projection, the ternd and plunge being obtained as explained above (e.g., Figures 4c and 4d). The position of the rake line is plotted on the primitive circle with reference to the trend of the sight line. Assuming the plunge of the sight line to be constant over the area of observation, a plane is constructed containing the sight line and rake line. (The rake line may be regarded as the strike of a plane which contains the plunge line as a pitching linear

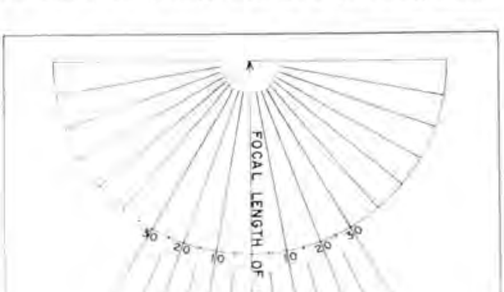

FIG. 3. Diagram showing the relationship of the angle of plunge of the sight line to the distance from the principal point on the photograph and to the focal length of the lens.

feature). The rake line is therefore placed on the *N-S* axis of the net and the great circle is completed which passes through the point marking the plunge line. This plane must contain the true position of the observed linear feature. The procedure is then repeated for the same feature on the second photograph of the stereographic pair, and a second plane containing the true position of the observed feature is found. Clearly, the intersection of the two planes gives the true position of the observed linear feature with reference to the coordinates of the stereographic projection (Figure 4e). If a slope is being determined one may now proceed immediately to any correction for tilt. In the determination of dip and strike, the procedure must be repeated for a second rake line on each photograph of a stereographic pair so that a second apparent dip is determined. The two apparent dips then define the required bedding plane (Figure 4f). This procedure is somewhat simplified if the two rake lines on the photographs have a common point which can be used as the sight line for both, as in Figure 4. This is not essential, however, provided it can be assumed that the orientation of the bedding does not change between the two rake lines.

For truly vertical photographs, the true slope or dip and strike can now be read directly from the stereographic projection. It is necessary, however, to correct for the tilt of oblique photographs and it may be necessary to correct for a small tilt on vertical photographs. This is simply done by rotating the required planes or lines on the stereographic net, using the axis of tilt as the axis of rotation. The rotation is made in the same direction and by the same amount as the tilt of the photograph. This brings the fundamental plane of the original projection into its true geographic position so that lines and planes plotted with reference to it also assume their true orientation.

Application to Oblique Photographs

As already noted, the method described above is also applicable to oblique photographs, since the projection is made with the plane of the photograph as the fundamental plane, and correction for tilt is made subsequently. This procedure differs greatly from the method described by Wallace (*op. cit.*, pp. 272–274) for oblique photographs.

The geometrical relations are indicated in Figure 5a. It should be noted that the line of sight is a pitching line in an inclined plane

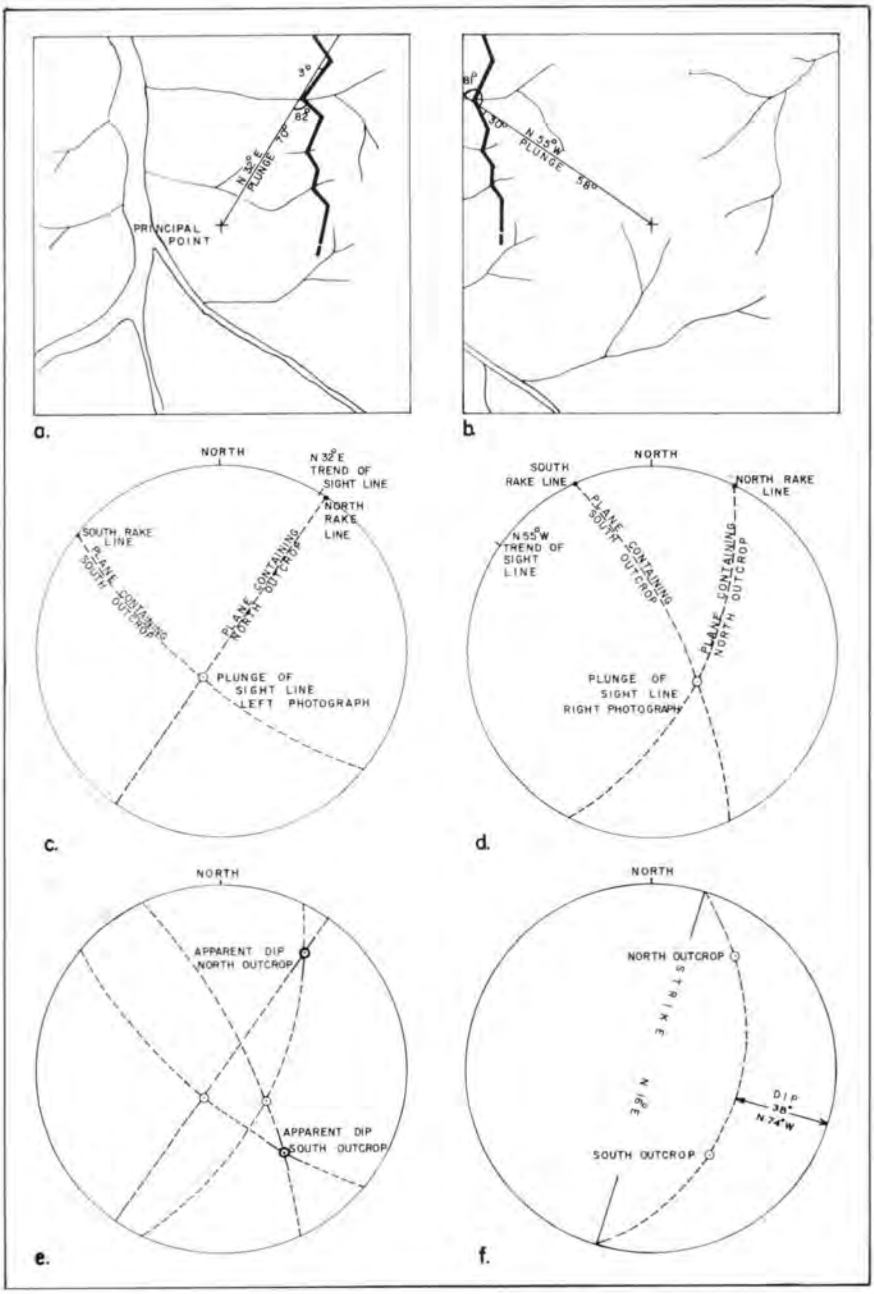

FIG. 4. Illustrations of the revised method of determining dip and strike for vertical photographs. Upper hemisphere projections have been used. Data after Wallace (1950), Fig. 4, p. 275.

striking N 40° E. The latter is therefore not the bearing of the line of sight, nor is 25° the true plunge, but an angle measured in the inclined plane (cf. Wallace *op. cit.*, pp. 272–277).

The plunge of the sight line is determined with reference to the optical axis, and the rake angles are measured with reference to the projections of the sight lines on the photographs. The zero meridian is taken as the normal to the axis of tilt in the plane of the photograph and in the example, has a bearing of N 20° E. Figure 5 illustrates the procedure, which follows that for vertical photographs. The attitude of the required bedding plane

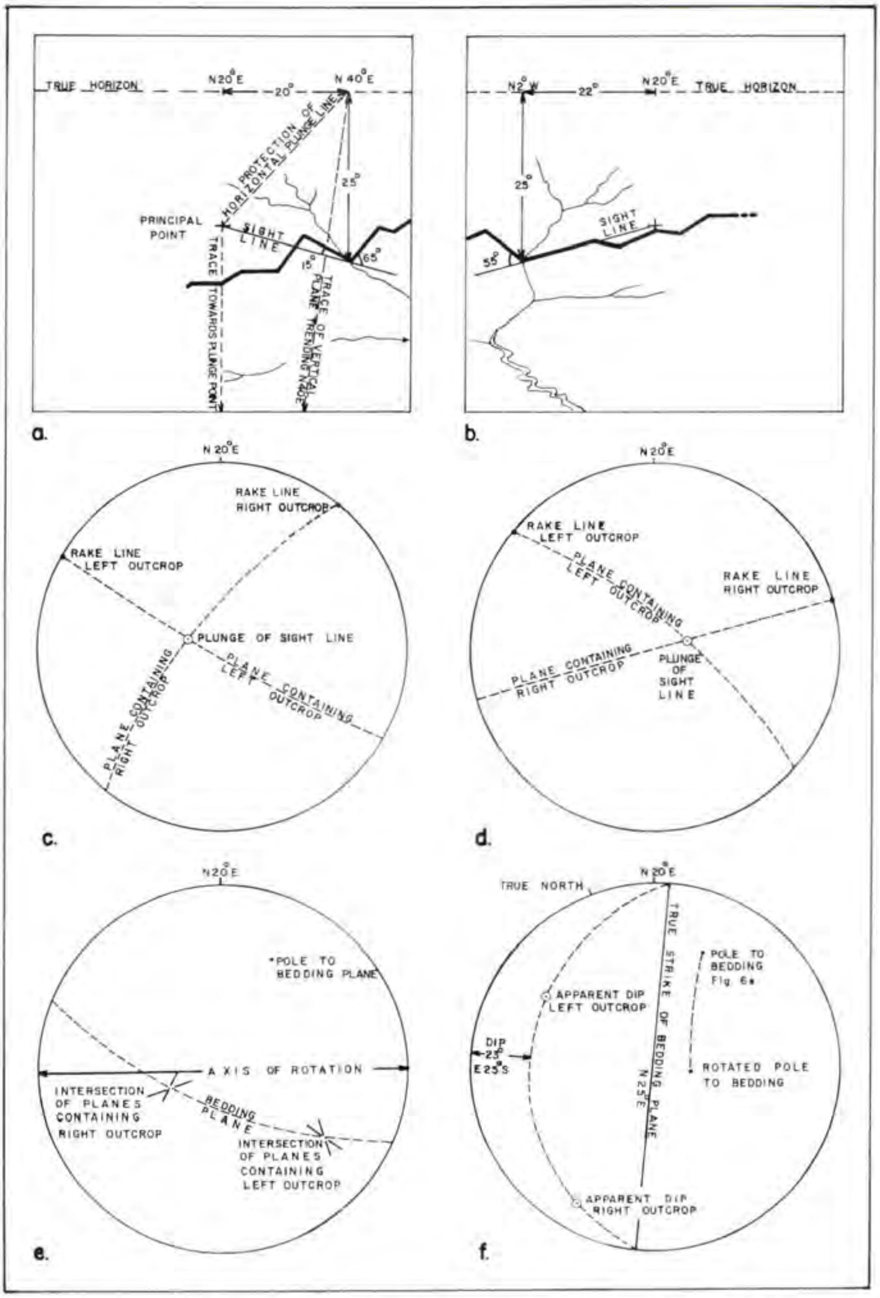

FIG. 5. Illustrations of the revised method of determining dip and strike from oblique photographs. Upper hemisphere projections have been used. The photograph has been assumed to cover an angle of 57°. Some reference lines have been added to Fig. 5a to assist in visualising the geometry (cf. Wallace (1950), Fig. 3, p. 273).

with reference to the coordinates of the projection is obtained in Figure 5e. This is rotated 69° in Figure 5f in the direction of tilt to give the true geographic orientation. The resulting strike of N 28° E and dip of 22° compare with a strike of N 64° E and dip of 52° found by Wallace. It seems unlikely that this is a real example because the apparent dip of the right outcrop proves to be away from the river. However, it serves to show that large errors are involved in the method described by Wallace for oblique photographs.

Concluding Discussion

The method described in this paper eliminates the errors of the method previously described, but uses the same principle. It is applicable to both vertical and oblique photographs because it uses the plane of the photograph as the fundamental plane of the projection.

However, the method is probably too complicated and of too limited applicability to be commonly used in the determination of dip and strike on aerial photographs. It will be most valuable for steep dips where the vertical exaggeration and distortion in stereoscopic viewing make estimated dips less reliable. Further possible uses, however, lie in the determination of slopes on aerial photographs so that vertical exaggeration can be estimated, or for geomorphological studies.

Slopes can be determined in exactly the same way as dip and strike by determining the apparent dip of any two divergent lines on a uniform slope. The maximum slope is the dip of the plane containing the two apparent dips. If the line of maximum slope can be recognized on the photograph, however, the amount of work is much reduced as the apparent dip of only the one line has to be determined. It is necessary to be able to recognize definite linear features on uniform slopes or definite points at the tops and bottoms of slopes, so that the rake lines measured on each photograph of a stereo pair correspond to the same ground line. Narrow gullies, for example, would provide suitable features, because they are evidently lines of maximum slope. If the line of maximum slope has to be estimated, cognizance should be taken of the fact that distortions are radial from the perspective center of a stereo pair (see for example Miller, 1961, pp. 32–50). The estimate of maximum slope should therefore be made on slopes close to the perspective center, with strikes normal to the radial lines from the perspective center.

When the correct angles for a number of slopes have been determined they may be compared with the angles estimated in stereoscopic viewing. The ratio of the tangent of actual angle to the tangent of the estimated angle gives the vertical exaggeration factor.

List of References

Badgley, P. C., 1959. *Structural Methods for the Exploration Geologist*, Harper and Brothers, New York.

Hackman, R. J., 1956. The stereo-slope comparator—an instrument for measuring angles of slope in stereoscopic models. Photogrammetric Engineering 22, 387–391.

Hemphill, W. R., 1958. Determination of quantitative geologic data with stereometer-type instruments. *U. S. Geol. Survey Bull.* 1043-C, 35–56.

Miller, V. C., 1961. *Photogeology*, McGraw Hill.

Phillips, F. C., 1954. *Stereographic Projection in Structural Geology*. E. Arnold.

Ray, R. G., 1960. Aerial Photographs in Geologic Interpretation and Mapping. *U. S. Geol. Survey Prof. Paper* 373.

Wallace, R. E., 1950. Determination of Dip and Strike by Indirect Observations in the Field and from Aerial Photographs: A Solution by Stereographic Projection. *Journal of Geology* 58, 269–280.

Meetings Calendar

February 2–7. 1969 Alaska Surveying & Mapping Convention, Anchorage, Alaska.

March 9–14. 1969 ASP/ACSM Annual Convention, Washington-Hilton Hotel, Washington, D. C., Mr. George L. Loelkes, Jr., Director. 8608 Cherry Valley Lane, Alexandria, Virginia 22309.

April 28–29. Seminar in Depth: Photo-optical Techniques in Simulators, South Fallsbury, N. Y. C/o SPIE, P.O. Box 288, Redondo, Calif. 90277.

June 2–13. Ninth Annual Photogrammetric Short Course. Harold W. Wecke, 120 Illini Hall, University of Illinois, Urbana, Ill. 61801.

June 2–20. Fundamentals of Infrared Technology, Advanced Infrared Technology, Infrared Radiometry, Fundamentals of Remote Sensing. Engineering Summer Conferences, Chrysler Center, Dept. 181, University of Michigan, Ann Arbor, Mich. 48105.

June 7–15. Photo Expo 69: The Universe of Photography. ASP Color Committee; SPIE and PRS Pattern Recognition Studies. New York City. Write ASP, 105 N. Virginia Ave., Falls Church, Va. 22046.

July 21–25. ASCE Transportation Engineering Meeting. Washington, D. C.

Sept. 23–27. ASP/ACSM Semi Annual Convention, Portland, Oregon, Write ASP Headquarters.