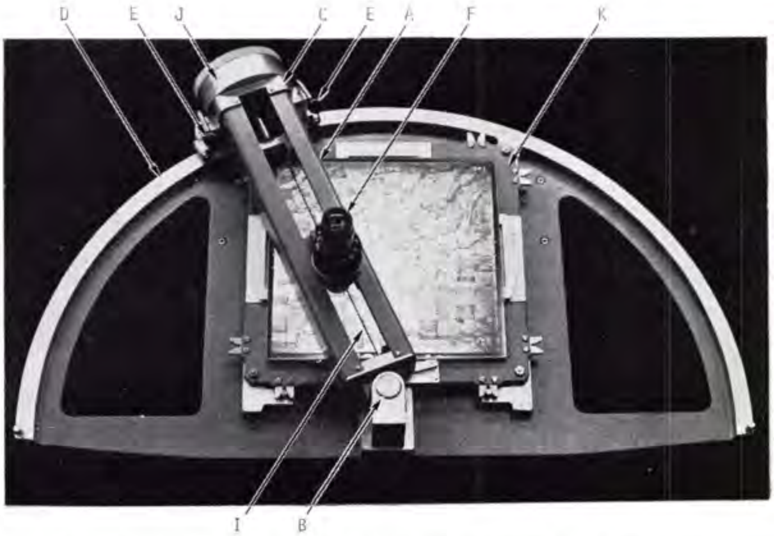

FRONTISPIECE. Full front view of the Multilaterative Comparator.

DUANE C. BROWN*
D. Brown Associates, Inc.
Melbourne, Fla. 32901

Computational Tradeoffs in the Design of a Comparator

A simple and economical one-micron plate comparator results from the exploitation of the computer as a primary principle of the systems design.

(*Abstract on next page*)

INTRODUCTION

THE SELF-CALIBRATING Multilaterative Comparator began about four years ago as a thought experiment intended to prove a point. Previous to this, in our work in the data reduction of missile and satellite tracking systems, we had developed a reduction called EMBET (Error Model Best Estimate of Trajectory). EMBET is designed to reconcile the conflicting trajectories produced by different tracking systems or combinations of tracking systems. Such conflicts are the result of unknown systematic errors in the various observational channels. Previous reductions had ignored the existence of unknown systematic errors and had determined the trajectory by means of independent point-by-point least-squares adjustments of the tracking observations. Thus, if the coordinates X, Y, Z of n trajectory points were to be determined from $m \geqq 3$ observational channels per point, such reductions would entail simply the formation and solution of n independent sets of normal equations of order 3×3. Accordingly, the solution for any given point would be un-

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affected by, and likewise would have no effect on, the solution for any other point.

The EMBET reduction, on the other hand, recognizes that systematic errors do exist in the observations and that, though unknown, they can (for the most part) be mathematically modelled. It then attempts to recover in a single, massive least-squares adjustment the coordinates of all trajectory points and the unknown parameters of the error models of the various observational channels. Inasmuch as each trajectory point introduces three unknowns, the normal equations arising from the simultaneous recovery of n trajectory points and p error coefficients are of order $(p+3n)\times(p+3n)$. Hence if several hundred trajectory points are to be determined, the normal equations can become very large indeed. However, the normal equations have a highly patterned structure that is exploited in the EMBET reduction to develop a special algorithm that renders their solution practical no matter how many points are to be carried. By virtue of this algorithm, the computational effort for the formation and solution of the normal equations increases only linearly with the number of points being carried. Moreover, the largest matrix operation to be performed is the inversion of a matrix of order equal to the number of error parameters being recovered (i.e., a $p\times p$ matrix).

Inasmuch as the error coefficients are recovered without recourse to external standards of calibration, the EMBET reduction can be said to consitute a process of self-calibration. The practical effectiveness of EMBET depends mainly on three factors: (a) the adequacy of the error models; (b) the degree of observational redundancy; (c) the strength of the geometry. Where these can be adequately controlled, self-calibration can successfully suppress the ultimate effect of systematic errors to insignificance.

As practical experience with EMBET was gained, it became increasingly apparent that many tracking systems were unduly complicated and that many functions performed by hardware could be performed equally well, if not better, by software soundly based on principles of self-calibration. In due course, we began preaching the gospel of self-calibration through designed redundancy—that simpler, more effective and less expensive systems could be developed if the power of self-calibration by means of software were duly exploited as a primary principle of systems design. To our way of thinking, advanced concepts of data reduction should direct development rather than emerge as afterthoughts.

It was for the purpose of providing a concrete demonstration of these principles, then, that we originally undertook the conceptual development of the Multilaterative Comparator. As conceived, it was to be a radical departure from conventional comparators. It was to be capable of determining coordinates within a large format (245×245 mm.) to accuracies on the order of one micron. Yet instead of being massive (200 to 300 kg., typical of conventional comparators), it was to be lightweight (about 10 kg.). Instead of being bulky, yet delicate, it was to be compact, rugged and readily transportable in a hand-held carrying case. Instead of requiring precisely linear and precisely orthogonal ways it was to do away with these troublesome elements altogether. Instead of requiring a controlled environment, it was to be relatively insensitive to environment and hence well-suited to field measurements of ballistic camera plates. In addition, it was to be fully self-checking and self-calibrating, simple in construction, simple to operate, capable of being digitized, and immune to effects of constant personal biases or even to systematically changing personal biases. Moreover, it was to provide sound estimates of the accuracies of final plate coordinates as well as other measures suitable as indices for quality control. Finally, it was to be comparable in

ABSTRACT: *Principles of self-calibration through designed redundancy have been successfully applied to the design of a unique, large format (245×245 mm.), lightweight (10 kg.), self-calibrating and self-checking comparator of one-micron accuracy. The comparator achieves extreme instrumental simplicity and enhanced accuracy through judicious tradeoffs of mechanical and computational factors, the general availability of modern digital computers being a decisive element of the design.*

over-all measuring speed with conventional large-format comparators and was to cost substantially less.

About two years ago, it became apparent that an actual need and market existed for such a device. Accordingly, we undertook an in-house project to turn the concept (shortly to be described) into reality. Before developing an engineering test model, we performed a series of computer simulations to ascertain theoretical feasibility of the conceptual approach. The simulations demonstrated that if the approach could be successfully mechanized, the recovery of plate coordinates and other parameters could indeed be performed to the required accuracies. Guided by the results of the computer simulations, we then designed an engineering test model. The engineering test model was completed a year ago. Insamuch as its performance matched our fondest expectations, we are now manufacturing the comparator in quantity.

In this paper we shall trace the development of the comparator, placing particular emphasis on the dominant role played by data reduction.

The Concept

Our point of departure was to consider the problem of measuring the precise coordinates of a set of points on a plate as being equivalent to a two-dimensional tracking problem. At its most primitive and most abstract level, the problem was formulated in terms of a four station ranging system as in Figure 1. Here we imagine that the coordinates x_j, y_j of an arbitrary point on a plate are to be determined from measurements of distance (or range) r_{ij} from four fixed external points ($i = 1,2,3,4$) whose coordinates x_i^c, y_i^c we assume temporarily to be known. This gives rise to the following set of four observational equations:

$$\begin{aligned} r_{1j}^2 &= (x_j - x_1^c)^2 + (y_j - y_1^c)^2 \\ r_{2j}^2 &= (x_j - x_2^c)^2 + (y_j - y_2^c)^2 \\ r_{3j}^2 &= (x_j - x_3^c)^2 + (y_j - y_3^c)^2 \\ r_{4j}^2 &= (x_j - x_4^c)^2 + (y_j - y_4^c)^2. \end{aligned} \tag{1}$$

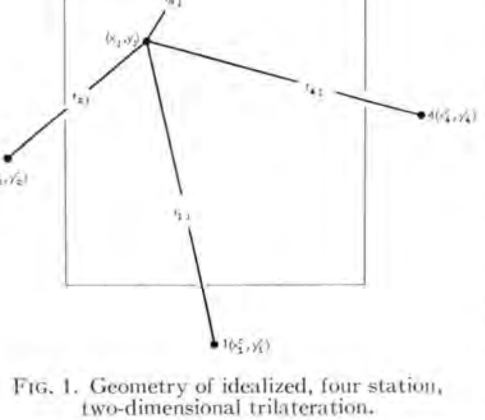

FIG. 1. Geometry of idealized, four station, two-dimensional trilateration.

Because the external points, or *tracking stations* as we shall call them, are considered to be of known location, and the ranges r_{ij} are considered to be known by virtue of measurement, the above constitutes a system of four equations in but two unknowns, the desired coordinates x_j, y_j. We may thus perform a simple least-squares adjustment to minimize the effects of random measuring errors. A by-product of such an adjustment is the error propagation associated with the adjusted coordinates. If the plate format were square and if the tracking stations were located a short distance from the midsides of the format, we would find from numerical simulation that, for a specified standard deviation σ of ranging, the standard deviations of x_j, y_j obtained from a least-squares adjustment would range in magnitude from 0.71 σ for centrally located points to 0.85 σ for points near the edges of the format. This means that, throughout the format, basic measuring accuracies are geometrically enhanced, rather than diluted, and that geometrical variation of accuracies of plate coordinates is acceptably small.

The foregoing is, of course, an abstraction and it does represent a severe limiting case in that the coordinates of all tracking stations are assumed to be perfectly known. Yet had we been unable to obtain satisfactory results from this primitive limiting case, there would have been no point to proceeding further along the above lines.

The next logical matter to be considered is the effect of abandoning the assumption that the coordinates of the tracking stations are perfectly known. Now if these coordinates were considered to be known, the coordinate system is thereby implicitly defined. In abandoning this knowledge, we gain the prerogative of defining the coordinate system to be employed. Several logical choices present themselves. We could, for example, choose the origin to coincide with some particular image on the plate and define the positive y-axis to pass through some other image. The system then becomes uniquely defined once we decide whether it is to be right or left handed. Alternatively, we could choose the origin to be at, say, Station 1 thereby rendering its coordinates x_1^c, y_1^c equal to 0, 0. In addition, we could define the system to be right handed with the positive y-axis passing through Station 3 thereby rendering its x-coordinate x_3^c equal to zero. This, too, would uniquely define the coordinate system. In still another choice, the system may be defined as right handed with the positive y-axis passing through points 1 and 3, thereby rendering $x_1^c = x_3^c = 0$, and with the origin chosen so that the y-coordinates of Stations 2 and 4 are equal in magnitude but of opposite sign (thus $y_2^c = -y_4^c$). This has the advantage over the previous choice of placing the origin near the center of the plate and is the particular choice we decided to adopt.

Returning to the set of four Equations 1 arising from the measurements of the j-th point, we now regard not only x_j, y_j as unknown but also the following five coordinates of the tracking stations: y_1^c, x_2^c, y_2^c, y_3^c, x_4^c ($x_1^c = x_3^c = 0$ and $y_4^c = -y_2^c$ by definition of the coordinate system). Accordingly, we now have a system of four equations in seven unknowns, and a unique solution does not exist. On the other hand, if we direct our attention to the entire system of equations generated by the measurements of n distinct points, we see that it involves $4n$ equations in a total $5+2n$ unknowns (the five station coordinates and the x_j, y_j for each of the n points). Hence if $n \geqq 3$ there will be more equations than unknowns, and a solution can be attempted. The system of normal equations generated by the simultaneous adjustment of the observations from all n points is of order $2n+5$ and hence increases with increasing n.

However, the normal equations are of precisely the same form as those arising in an EMBET reduction; in fact, they may be viewed as a special, four-station, two-dimensional case of a general EMBET reduction for an m-station ranging system. In our problem, the error parameters to be recovered consist of the five unknown coordinates of the tracking stations. Accordingly, by the application of the EMBET algorithm, the general system of normal equations can efficiently be collapsed to a 5×5 system involving only the station coordinates as unknowns. After the station coordinates have been determined, the coordinates of the points x_j, y_j can be established through the solution of n independent 2×2 systems of reduced normal equations. We shall not go into details of the solution at this point. The important matter is that the formation and solution of the normal equations presents no practical difficulties, no matter how many points are to be carried in the reduction. Also, the rigorous error propagation emerges from the reduction.

It remains to be established whether or not the recovery of tracking station coordinates leads to significant dilution of accuracies in the recovery of image coordinates. By numerical simulation, we find that when about 25 well-distributed points are carried in the solution, the expected accuracies of the x_j, y_j are degraded only by from 5 to 15 percent over what they would have had all coordinate of the tracking stations been perfectly known. Moreover, the more points carried in the reduction, the less the dilution of accuracies entailed by the need for recovering station coordinates.

The above result was of signular importance to the development of the comparator, for it meant that, by virtue of a data reduction tradeoff, the design did not need to be concerned with problems relating to precise location of points that would correspond to tracking stations.

The fact that the coordinates of tracking stations need not be known also has a vital consequence relating to the practical mechanization of the measuring process. As a crude first approximation, we could envision the process as one in which a transparent scale is pivoted about its zero point so that it can sweep over the entire plate. With the aid of a device to interpolate between divisions of the scale, one could then determine the distances from the tracking station (i.e., the pivot) to all points of interest on the plate. After all points had thus been measured from one station, the scale could be removed to a second station (pivot), whereupon the process could be repeated. In this manner the required measurements could be obtained from all four stations.

Shifting of the scale from pivot to pivot is obviously a cumbersome procedure and is one that would be awkward to mechanize. Fortunately, because the coordinates of the pivot points need not be known, a geometrically equivalent process becomes feasible. It consists of pivoting the scale about a single point and rotating the plate by nominally 90° between one set of measurements and another. This generates a total set of measurements exactly equivalent to the set that would have been produced from a fixed plate and four separate pivots as discussed above. To see this more clearly, we may imagine a fixed coordinate system $\bar{x}$, $\bar{y}$ attached to the

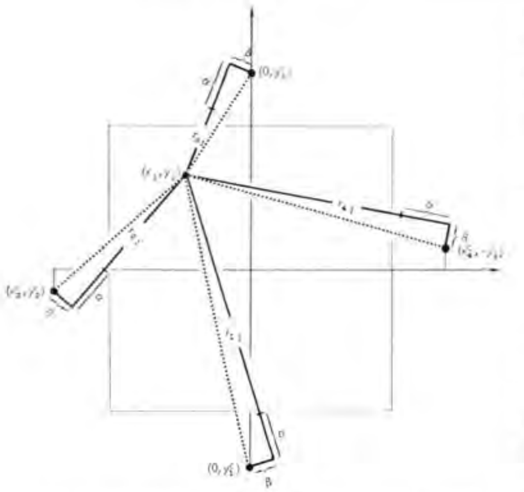

FIG. 2. Illustrating geometrical equivalent of Multilaterative Comparator.

plate. If the plate is in Position 1, the pivot will occupy a position having coordinates $\bar{x}_1^c$, $\bar{y}_1^c$, from which the ranges to the points of interest would be measured. Now we imagine the plate rotated and possibly translated to a new position. The coordinates $\bar{x}_j$, $\bar{y}_j$ of the points on the plate will not be changed by this process, but the pivot will occupy a new position $\bar{x}_2^c$, $\bar{y}_2^c$, from which ranges would again be measured. However, if the plate had not been moved to its new position, precisely the same set of measurements could have been obtained by measuring from a new pivot at $\bar{x}_2^c$, $\bar{y}_2^c$. Accordingly, the two processes are equivalent.

By fixing the pivot and rotating the plate between sets of measurements, we are much closer to a process that is feasible to mechanize. However, as we ultimately aim at micron accuracies, the physical difficulties of pivoting precisely about the zero mark of the scale, or about any precisely known point for that matter, are formidable indeed. We may attempt to get around such difficulties by assuming that the location of the pivot relative to the zero mark is completely unknown and is to be recovered in the reduction. Geometrically, then, our system becomes the equivalent of that pictured in Figure 2. Here we have let α and β denote the radial and tangential components of the offset of the pivot point relative to the zero mark of the scale. We have also recognized that since there is, in reality, only one pivot point and one measuring arm, the same α and β apply to all four measuring stations. The measured range r_{ij} consists now of the distance from the zero mark of the scale to the point x_j, y_j, and the distance from the tracking station to x_j, y_j is given by the hypotenuse of a right triangle having sides of length $r_{ij}+\alpha$ and β, respectively. Considering this and considering our choice of coordinate system, we now obtain the following set of observational equations from the measurements of the j-th point

$$
\begin{aligned}
(r_{1j} + \alpha)^2 + \beta^2 &= x_j^2 + (y_j - y_1^c)^2 \\
(r_{2j} + \alpha)^2 + \beta^2 &= (x_j - x_2^c)^2 + (y_j - y_2^c)^2 \\
(r_{3j} + \alpha)^2 + \beta^2 &= x_j^2 + (y_j - y_3^c)^2 \\
(r_{4j} + \alpha)^2 + \beta^2 &= (x_j - x_4^c)^2 + (y_j + y_2^c)^2.
\end{aligned}
\tag{2}
$$

Except for the additon of the two new parameters α and β, this system is of the same form as the one just considered in which coordinates of the tracking stations were unknown. The measurements from n points again generate $4n$ equations, but now the number of unknowns is increased from $2n+5$ to $2n+7$. The solution has the same properties as before except that the reduced normal equations are now of order 7×7 instead of 5×5. From the numerical simulation of a 25-point pattern we find that the addition of the offset parameters α and β introduce essentially no further dilution of the accuracies of the recovered coordinates x_j, y_j.

In view of the foregoing, we were able to conclude that the design of the comparator need not be concerned with the establishment of the precise offset of the pivot from the zero mark of the scale. Here too, a major simplification was achieved by means of a computational tradeoff. An incidental, but important, benefit gained by the introduction of α as an unknown is that it serves double duty by also compensating automatically for any unknown, constant personal bias. Thus if one were to contaminate a given set of measured ranges by the application of a common additive constant, one would obtain precisely the same coordinates x_j, y_j from independent reductions of both the original and the contaminated sets of measurements; the values obtained for α, however, would differ by an amount equal to the additive constant.

Enough background has been presented at this point to convey a general understanding of the principles underlying the development of the Multilaterative Comparator. Those interested in finer details of the mathematical development are referred to an appendix which can be obtained on request from the author. We shall now direct our attention to consideration of the comparator as it evolved from the above considerations. But before so doing, we would emphasize that the software

FIG. 3. Multilaterative Comparator in normal operating position.

for the comparator had been developed, tested, exercised, refined and retested, well before the go-ahead was given to the design of the hardware.

IMPLEMENTATION OF THE CONCEPT

The comparator is pictured alone in the Frontispiece and in its operational position in Figure 3. The measuring arm A pivots about a point B created by a single captive, top-loaded, self-seating ball bearing having a diameter of 0.75 inches. The micrometer end C of the measuring arm is supported by a pair of rollers that run on the guide rail D. The measuring arm can be locked at any desired point on the guiderail by releasing either of the dual, spring loaded brake levers E.

A closeup of the measuring arm with its side panels removed is shown in Figure 4. The Bausch and Lomb zoom macroscope $(10\times \rightarrow 30\times)$ F is mounted on a carriage G which rolls on sleeve bearings along a pair of stainless steel rods and can be locked at any desired point by releasing the spring loaded brake lever H.

The scale I is 6 mm. thick, 29 mm. wide and 292 mm. long. The bottom side is graduated at one-millimeter intervals over a length of 261 mm. Every even division is numbered. The ends of the scale are attached to supports which ride on sleeve bearings on a pair

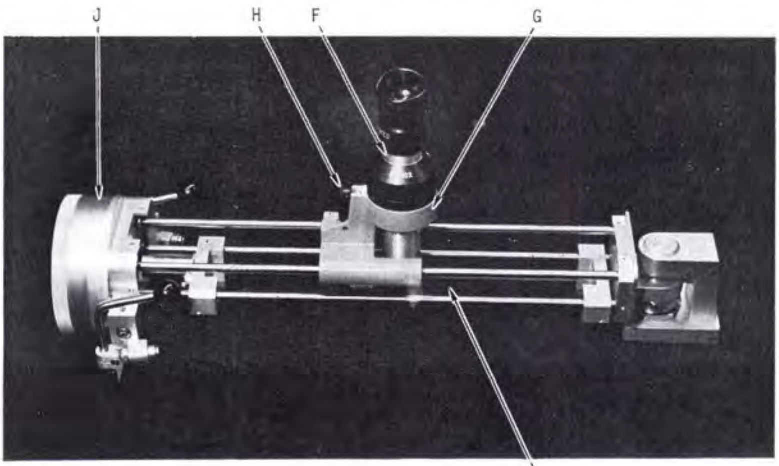

FIG. 4. Measuring arm with side panels removed.

of stainless steel rods. The lower support of the scale is free and the upper support is attached to the nut of the micrometer screw which is 2 centimeters in length. By turning the micrometer drum J, one can translate the scale radially from the pivot by a precisely known amount. Automatic stops in the micrometer head limit this translation to at most one millimeter. The amount of translation can be read by a vernier to the nearest half micron.

The plate to be measured is mounted in the plate holder K (Frontispiece), the four corners of which rest freely on four pads machined to be in the same plane as the upper guiderail. Different plate holders are made to accommodate plates having the following nominal dimensions: 9.5×9.5 inches, 8×10 inches, 290×215 mm. Each plate holder is pre-adjusted to accommodate a standard plate thickness of 0.240 inches. This leaves a gap of 75 to 100 microns between the upper surface of the plate and the bottom surface of the scale. Such a gap is sufficiently small to permit both the plate and the graduations of the scale to be sharply in focus at the upper magnification of $30\times$. Standard plates thinner than 0.240 inches can be accommodated by means of adjustable footscrews fixed to the corners of the plateholder. On each edge of the plateholder a pair of V-blocks, which have been mated to a pair of positioning pins permanently affixed to the mainframe, serve to fix the plateholder in each of the four standard measuring positions.

Illumination of the plate is provided by a built-in light table containing three 8-watt fluorescent lamps. A pair of folding legs built into the back of the mainframe support the comparator in its normal operating position at an angle of 45° from the horizontal.

Because the stainless steel rods of the measuring arm were selected to have very nearly the same coefficient of expansion as glass, the comparator is relatively insensitive to variations in temperature. The glass employed for the scale is of the same type as is used for Kodak plates.

Operation of the Comparator

By rotating the measuring arm and translating the microscope, one can quickly bring any desired point on the plate into the field of the microscope. To measure a point, one first brings it within a circular reticle having a diameter of 400 microns at plate scale. There is no need for precise centering of the image within the reticle, for a lateral offset of as much as 100 microns will cause an error in radial distance of only 0.1 micron in the worst case. The amount of parallactic error caused by the gap between the scale and the plate depends on the effective focal length of the objective; for the B&L Macroscope, a 100-micron radial offset will cause an error of 0.3 micron in radial distance.

The image as seen within the reticle will lie between a pair of millimeter graduations of the scale. The whole millimeter reading is taken directly from the lower graduation. To obtain the fractional reading, one rotates the micrometer drum to translate the lower graduation until it precisely bisects the image. The fractional reading is then made from the micrometer drum to the nearest half micron. The reading is recorded opposite a point number and the process is repeated for all the points to be measured. When all of the measurements have been performed for a given position of the plate, the measuring arm is moved out of the way to a nearly horizontal position and the plate holder is rotated 90° and replaced on the comparator. The points are again measured and their readings are recorded opposite the point number. The operation is completed when the plate has thus been measured in all four positions.

Despite the fact that the plate is measured in four positions, the total measuring time is about the same as that required when double settings are performed on a conventional two-screw comparator. This is attributable mainly to two factors: (a) each setting involves only a single radial bisection, rather than bisection in two directions; (b) slewing is very rapid, for the microscope can be moved into measuring position from any point on the plate to any other point in a matter of about five seconds. An experienced operator can perform the complete measurement of a plate containing 30 images within one hour; if double settings are required (i.e., a total of 8 measurements per point), the time is increased to about 90 minutes.

Calibration

A systems analysis of the comparator shows that aside from stability of the plate during measuring, only three elements of the system are critical to the attainment of the desired measuring accuracy of one micron. They are the *scale*, the *screw* and the *pivot*. All other elements require tolerances from one to two orders of magnitude less demanding. For example, for one micron accuracy the linearity of the translation of the microscope need be good only to about ±300 microns (actually, it is 10 times better than this). Likewise, the axis of the microscope need remain

FIG. 5. Alignment device used in calibration of scales against master scale.

parallel to itself only to within $\pm 0.6°$ (again, the actual tolerance is more than 10 times better than this). Similarly, because the scale is translated by at most one millimeter, the direction of the translation must be parallel to the axis of the scale only to within $\pm 8°$, a tolerance that is easily bettered by a factor of 100.

The scale is indeed a critical item because each measurement will inherit the error of the graduation to which it is referred. This means that, in the absence of other errors, one-micron accuracies demand a scale that is either accurate to one micron or else is calibrated to an accuracy of one micron. The scales employed in the production models of the comparator are manufactured to the following specifications: cumulative non-linearity of spacing is not to exceed one micron over the total length of the scale; error in spacing of successive divisions is not to exceed 0.5 micron. Above and beyond this, we compare each scale with one of the scales selected to serve as a master scale. In making this comparison, the scale to be tested is placed in face-to-face contact with the master, a spacing of nominally 10 microns (or one linewidth) being set between corresponding divisions.

To facilitate the alignment of the two scales, the ends of the master are cemented into position in the device pictured in Figure 5. The scale to be tested is then aligned with the aid of positioning screws, whereupon it is clamped vertically at both ends to prevent further movement relative to the master. The center-to-center spacing between corresponding divisions of the two scales is easily measured to an accuracy of ± 0.2 microns by means of a Watson Image Shearing Micrometer employed at a magnification of $200\times$. If the spacings were perfectly constant for all divisions, the scale to be tested would perfectly match the master. Therefore, the variation of the spacing about the mean is the measure of the discrepancy between the two scales. In this manner all scales are referred to the master scale with an accuracy of ± 0.2 microns. The master scale itself has been calibrated interferometrically to a certified accuracy of ± 0.5 micron by the National Bureau of Standards. By virtue of the comparative process just described, all scales inherit the absolute accuracies of the master with virtually no dilution, for *rms* errors of 0.5 and 0.2 micron combine vectorially to $[(0.5)^2+(0.2)^2]^{\frac{1}{2}}=0.54$ micron. The computer program is designed to apply automatically the calibrated corrections for a given scale.

The second critical element of the comparator, the micrometer screw, is calibrated to an accuracy of ± 0.3 micron over its measuring range of one millimeter. This is accomplished with the aid of a microscope scale calibrated at 50-micron intervals over a length of one millimeter to an accuracy of ± 0.2 micron. Inasmuch as the screw is calibrated when mounted in the comparator, the calibration also accounts for any periodic errors resulting from eccentricity of the measuring drum or from progressive errors in the graduation of the drum (the former may amount to as much as ± 0.5 micron; the latter, by virtue of a dividing accuracy of ± 15 seconds of arc, should not exceed ± 0.12 micron). The calibrated corrections for the screw are automatically applied by the computer program.

The primary requirement of the third critical element of the comparator, the pivot, is that it provide a stationary point of rotation. This point is the center of a 0.75-inch ball bearing having a sphericity of ± 0.25 micron. The center of the ball is located in the plane of the scale thereby making the measurements insensitive to any up-and-down motion of the far end of the scale. Wobble of the bearing is restricted by means of a spring-loaded thrust bearing that provides a controlled loading against the thrust bracket. The thrust is directed precisely along the axis of rotation. With the comparator in its normal operating position and the thrust on the pivot set at 3 kg, a radial *wobble* amounting to as much as 2 microns can be detected over the sweep of the measuring arm; with a thrust of 6 kg, there is no discernible radial wobble (the lateral component of *wobble* has no effect on the measurements). To provide a comfortable margin of safety, the thrust on production models is set at 10 kg.

Results

Various production have models of the comparator undergone extensive testing over the past year. In comparisons employing plates with well defined images measured both on a conventional, two-screw comparator and on the Multilaterative Comparator, an *rms* agreement of final coordinates ranging between ± 2 and ± 3 microns is usually obtained after allowance has been made for a translation and rotation of the one coordinate system to conform to the other. If one were to assume that the discrepancies between the results are equally attributable to both comparators, one would conclude that the *rms* accuracy of the coordinates produced by the Multilaterative Comparator generally range between ± 1.4 and ± 2.1 microns, a range compatible with typical setting accuracies. Thus external evidence indicates that the Multilaterative Comparator is capable of producing accuracies at least comparable with those produced by conventional, one-micron comparators.

A comparison has also been made against a standard consisting of a 23×23 cm. Zeiss grid on which a representative sample of 33 grid intersections had been calibrated by the manufacturer to a stated accuracy of 0.8 micron (*rms*). The discrepancy (after allowance for translation and rotation) between comparator coordinates and grid coordinates of the 33 calibrated points amounted to 1.50 microns (*rms*). Of this, about 1.3 microns (i.e., $[(1.5)^2-(0.8)^2]^{\frac{1}{2}}$ can be attributed to the comparator.

In addition to external evidence concerning accuracies, there is internal evidence arising as a by-product of the reduction of each plate. We refer here to the ranging residuals resulting from the adjustment. By virtue of the redundancy of the measuring process, each point generates four observational residuals. These may be regarded as closures of quadrilateration. As such, they should be consistent with the errors to be expected from combined errors of setting and errors of the comparator. Both the random and the systematic errors of the comparator will be reflected in the residuals. This means that if uncompensated systematic errors are sizeable in comparison with random errors, they will dominate the determination of the residuals and will reveal their presence in a plot of residual vectors. Thus the residuals from the adjustment provide a truly meaningful indication of total measuring accuracy, and the *rms* error of the residuals, representing as it does an *rms* error of closure, provides a particularly suitable criterion for quality control.

The *rms* errors of closure on production models have been found to range typically from 1.5 to 2.0 microns for double settings on points marked by a Wild PUG III. The set of residuals obtained from measurements of a 5×5 array of PUG points evenly spaced at 45 mm. intervals is listed in Table 1. Inasmuch as double settings were made on each point, the *rms* error of setting was also determined. The *rms* error of an individual setting was found to be 1.7 microns which meant that the *rms* error of the mean of each pair of settings amounted to 1.2 microns. Inasmuch as the *rms* error of closure turned out to be 1.6 microns (Table 1), this suggests that the *rms* contribution of the comparator itself is on the order $[(1.6)^2-(1.2)^2]^{\frac{1}{2}} \approx 1.1$ microns.

By-products of the reduction are estimates of the standard deviations of the plate coordinates. In the above example, 80 percent of the standard deviations in x and y ranged between 1.1 and 1.3 microns; 2 points had standard deviations as large as 1.5 microns. These figures consider the total error propagation; that is, the combined effect of the propagation of random measuring errors and the errors remaining in the recovered values of the parameters of the comparator. In general, the standard deviations of the adjusted x, y-coordinates will be somewhat smaller than the *rms* error of closure.

The Computer Program

If developed 15 years ago, the Multilaterative Comparator would have been little more than an interesting academic curiosity, for no one could have tolerated its computational requirements. Accordingly, the digital computer, so commonplace today, is to be regarded as an integral part of the comparator

TABLE 1. RANGING RESIDUALS FROM ADJUSTMENT OF ARRAY OF 25 POINTS MEASURED ON MULTILATERATIVE COMPARATOR

Point	Ranging Residuals			
	Position 1	*Position 2*	*Position 3*	*Position 4*
1	1.1μ	1.5μ	−0.7μ	0.1μ
2	0.3	0.9	−0.6	0.7
3	−0.3	−0.4	−0.2	−0.2
4	0.5	0.0	0.5	−0.5
5	−0.1	0.6	0.5	0.3
6	−0.1	0.6	0.5	0.3
7	1.7	−0.6	1.3	−1.4
8	−0.2	−0.1	−0.1	−0.9
9	−2.0	−2.4	−0.4	−1.3
10	−1.8	1.4	−3.8	3.0
11	0.8	−0.9	1.8	−0.9
12	1.7	0.8	1.2	0.8
13	0.2	0.1	0.2	1.0
14	0.3	0.4	0.5	0.4
15	0.1	0.5	0.6	−0.5
16	−1.0	0.4	−1.5	−0.8
17	−0.8	1.7	0.2	1.6
18	0.9	−0.9	1.0	−0.3
19	−1.8	0.2	−1.3	−0.9
20	1.0	−2.1	2.5	−1.1
21	−0.9	1.0	−0.8	−0.1
22	0.3	0.0	0.0	0.3
23	0.2	−0.5	−0.3	−0.3
24	0.3	−0.6	−0.4	−0.8
25	−0.2	0.2	−0.3	−0.1

Degrees of Freedom $= 2(25) - 7 = 43$
Grand RMS = 1.6 microns

system. For this reason, a fully documented Fortran program is provided with the comparator. The program is designed so that a minimum of modification is required to adapt it to almost any computer. One version of the program is designed specifically to run on a minimal computer configuration having card input and an available core memory equivalent to about 8,000 24-bit words. Another version is designed for medium- to large-scale computers. Both versions feature automatic editing and rigorous error propagation. Typical running time on an IBM 7094 for the reduction of a plate containing 50 measured images is well under 1 minute.

FURTHER DEVELOPMENTS

Optional accessories that have been developed for the comparator include: (*a*) automatic digital readout on punched cards to a least count of 0.5 micron; (*b*) a universal stage designed to accommodate both nonstandard plates and cut film; (*c*) a dual focus microscope designed to focus simultaneously on the scale and on the specimen mounted on the universal stage. Productivity experienced with the digitized version of the comparator has been found to be about double that experienced with the standard model.

We have under development a compact semi-automatic setting device that retrofits to the dual focus microscope. This device is expected to provide an *rms* repeatability of setting to better than 0.5 micron on points of moderately high contrast.

CONCLUSIONS

The successful development of the Multilaterative Comparator provides a striking demonstration of the power and effectiveness of self-calibration through designed redundancy. It clearly shows that through suitable internal contradiction, one can achieve calibration. It also grants a proper role to a new, key factor of systems design—the now commonplace availability of digital computers. By virtue of our success with the Multilaterative Comparator, we now have under development a number of other devices that are specifically designed to exploit principles of self-calibration to achieve improved performance with drastically simplified hardware.

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