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Heights from Simultaneous Radar and Infrared

Images of the type obtainable during both day and night can be used to obtain stereo and to produce elevation contours.

ABSTRACT: *Stereo mapping is possible using two images produced from the same vantage point, provided their parallax displacements are in opposite directions. Side-looking airborne radar (SLAR) and infrared line-scanner make a natural pair for this purpose, because both have the same side-looking geometry and they have similar image recording techniques. A flagpole appears to lean toward the radar, but away from the line scanner; hence, the conditions for single-vantage-point stereo are met. Relations presented show the stereo effect for this sensor pair using both "natural" and "true-ground-range" presentations In neither case is the visual stereo effect perfect except near 45° depression angle, but a distorted visual stereo effect is possible elsewhere. Use of stereometers with proper calibration of stereo displacement vs. look angle should permit quantitative mapping of heights even though the visible effect is somewhat distorted.*

INTRODUCTION

PARALLAX DISPLACES ELEVATED points in different directions on radar and optical systems. Thus, elevated points on simultaneously produced images of the same area are displaced in different directions and, if viewed stereoscopically, show the proper height effects. This paper deals primarily with comparison of line-scan visible and infrared images with side-looking airborne radar images. The geometry is more complex for standard photographs and other kinds of radar images although the same principles could be used there also. As line-scan infrared images and radar images may both be produced at night, this system permits single-pass nighttime contour mapping.

Figure 1 illustrates the fundamental idea of the radar-scanner stereo system. Both radar and optical scanner are assumed to look directly to the side of a flight path; thus, this sketch is a cross-section through a plane perpendicular to the flight path. A plane earth is also assumed for simplicity. The aircraft is located at A and a "flagpole" example is shown with the height of the flagpole, h.

The optical scanner images the bottom of the flagpole correctly if it is set up for the mean ground plane, $OFDC$; that is, the bottom of the flagpole appears properly in the image at point D. The top of the flagpole, however, appears in the image at point C, because the scanner detects ground location by angle, and the angle OAB corresponds to a point on the ground C. Thus, the parallax displacement of the top of the flagpole is the distance DC and is in a direction away from the aircraft.

The radar presents its image in terms of range. Again, point D is imaged properly by measuring the distance AD. The top of the flagpole, however, is at a distance AB; the corresponding slant distance from the aircraft to the ground is AF. Thus, the parallax displacement on the radar image is DF and is in the direction *toward* the aircraft.

Hence, the radar image has a parallax displacement like that for a line scan image produced by an aircraft at point A'.

ANGLE-SCANNER PARALLAX

The angle scanner either may have a linear sweep, in which case displacement on the image is directly proportional to angle with the vertical, or it may have a ground-range-corrected sweep such that the image of a plane surface has the geometry of the surface itself. Figure 2 shows the flagpole example with quantities pertinent to analytic description of the angle scanner.

With a linear sweep the displacement from

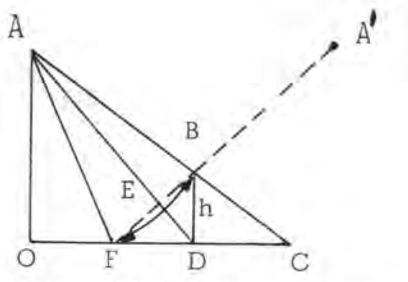

FIG. 1. Principle of radar-scanner stereo.

the nadir point on the image is given by

$$e_{SL} = m\theta = m \tan^{-1}(y/H)$$

$$e_{SL}' = m\theta' = m \tan^{-1}\frac{y}{H-h} = m \tan^{-1}\frac{y+w}{H} \qquad (1)$$

for the bottom and top of a "flagpole", respectively. The coefficient m is a scale factor and the other quantities are identified on the figure. The prime designates quantities associated with the top of the flagpole.

The parallax displacement is given by

$$\Delta e_{SL} = e_{SL}' - e_{SL}$$
$$= m\left(\tan^{-1}\frac{y}{H-h} - \tan^{-1}\frac{y}{H}\right).$$

If we expand $\tan^{-1}[y/(H-h)]$ in a Taylor series we have

$$\tan^{-1}\frac{y}{H-h}$$
$$\approx \tan^{-1}\frac{y}{H} + \frac{h}{2H}\sin 2\theta\left(1 + \frac{h}{H}\cos^2\theta + \cdots\right).$$

Combining these the parallax is seen to be approximately given by

$$\Delta e_{SL} = \frac{mh \sin 2\theta}{2H} \qquad (2)$$

provided h/H is small enough.

With a true ground range sweep the displacement on the image from the nadir point is given by

$$e_{SG} = k \tan\theta$$
$$e_{SG}' = k \tan\theta'$$

where k is another scale factor. The parallax

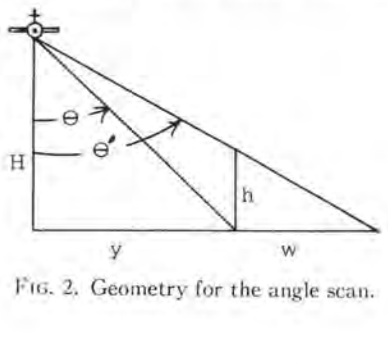

FIG. 2. Geometry for the angle scan.

in this case is given by

$$\Delta e_{SG} = k\left(\frac{y}{H-h} - \frac{y}{H}\right) \approx \frac{kh\tan\theta}{H}\left(1 + \frac{h}{H}\right). \qquad (3)$$

A slit camera produces the same kind of distortion as the true-ground-range sweep, so either may be used along with radar for stereo.

RADAR-SYSTEM PARALLAX

Figure 3 illustrates the geometry for the radar range scan. Again, primes denote the top of the "flagpole". For a linear (slant range) sweep the displacement from the zero slant range point is given by

$$e_{RL} = nR = n\sqrt{(y^2 + H^2)}$$
$$e_{RL}' = nR' = n\sqrt{[y^2 + (H-h)^2]} \qquad (4)$$

where n is a scale factor. Note that the zero-slant-range point will not appear in the image since the minimum slant range that could possibly be of interest in H, and the radar would have poor resolution near the vertical so the minimum actually used will be significantly larger than H. The parallax for the location of the image from the top of the "flagpole" is given by

$$\Delta e_{RL} = e_{RL}' - e_{RL}$$
$$= n\{\sqrt{[y^2 + (H-h)^2]} - \sqrt{[y^2 + H^2]}\}$$
$$\approx -\frac{nHh}{R}\left(1 - \frac{h}{2H}\right) \qquad (5)$$
$$= -nh\cos\theta\left(1 - \frac{h}{2H}\right).$$

For an obstacle with a small enough height, this equation, like its counterpart for the line scanner, Equation 2, shows a linear relation between the parallax displacement and the height of the obstacle.

If the radar circuitry is arranged for a true-ground-range sweep, the image points appear at

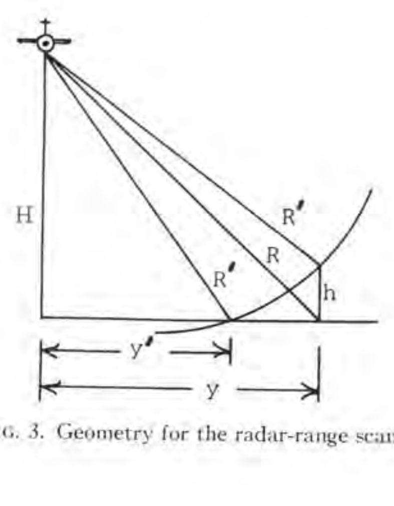

FIG. 3. Geometry for the radar-range scan.

$$\begin{aligned} e_{RG} &= n\sqrt{(R^2 - H^2)} = ny \\ e'_{RG} &= n\sqrt{(R'^2 - H^2)} = n\sqrt{(y^2 - 2hH + h^2)}. \end{aligned} \quad (6)$$

The parallax displacement is therefore given by

$$\Delta e_{RG} = -ny\{1 - \sqrt{[1 - (2h/y)\cot\theta + (h/y)^2]}\}\cdot$$

It is easy to approximate this provided the height of the obstacle is small enough and the distance y' is great enough relative to the height of the aircraft; that is if

$$(h/y) \ll 1, \qquad (2h/y)\cot\theta \ll 1.$$

Making this approximation we find the parallax to be given by

$$\Delta e_{RG} = -nh[\cot\theta + (h/2y)(\cot^2\theta - 1) + \cdots].$$

Again, both this equation and its angle-scan counterpart, Equation 3, show a linear relation between parallax and height, provided $(h/H) \ll \cot\theta$. In both cases, however, this relation is a function of the angle of observation.

Stereo Combination

From Figure 1 it is clear that an object at 45° will experience the same parallax (but in opposite directions) for radar and visible scanner. If the object is closer to the nadir, the radar parallax will be larger and the line-scan parallax smaller and conversely. Thus, to permit viewing the two images with a stereo effect the same as that for two strip camera images taken from opposite sides, the relative locations of the images with respect to the viewing lenses must be different than for the strip photos; and, indeed, the correct location is dependent upon the portion of the image viewed if a large range of angles is covered in a single image.

If the scales are the same, the parallax for each case is proportional to the appropriate ground range. Thus, using the nomenclature of Figure 1,

$$\Delta e_{RG}/FD = \Delta e_{SG}/DC. \quad (8)$$

If $h \ll H$, FB in Figure 1 is approximately normal to ABC, so

$$h \approx FD\tan\theta \approx CD\cot\theta$$

whence

$$\Delta e_{RG} \approx \Delta e_{SG}\cot^2\theta. \quad (9)$$

Figure 4 illustrates positions of a radar and line-scanner at point 1 and the virtual location at point 2 occupied by a line-scanner having displacement equal to that of the radar at point 1. From this

$$\begin{aligned} y_1 &= (H - h)\tan\theta_1, \\ y_2 &= (H - h)\tan\theta_2 = (H - h)\cot\theta_1. \end{aligned}$$

Thus

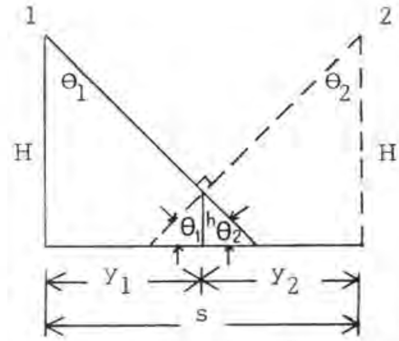

FIG. 4. Equivalent line scanner gives the same displacement as radar.

$$y_2 = (H - h)^2/y_1 \quad (10)$$

so the virtual location of a line-scanner having parallax like that of the radar is a function of the displacement from the nadir, y_1. The total displacement between nadir points is given by

$$s = y_1 + y_2 = y_1/\sin^2\theta. \quad (11)$$

This development shows that no single displacement of images with respect to the viewing point permits normal stereo viewing over the entire image. The proper effect is obtained for any fixed distance, but the scale is different for different parts of the image. For quantitative work, however, the corrections may be easily made and the system should permit adequate scaling of heights.

Since optical/IR images may be black when radar images are white, and vice versa, the similarity required for binocular viewing often may fail to be present. Also, shadows for the radar are always away from the aircraft, but visible shadows depend on location of the illumination; this may cause confusion. Nevertheless, even if binocular viewing fails, quantitative height determination will usually be possible, since major features should be identifiable on both visible/IR and radar images.

Conclusion

Simultaneous images of the type obtainable during both day and night can be used to obtain stereo and consequently to produce elevation contours. For any particular elevation angle it is easy to make the appropriate corrections so that the images may be viewed as if properly taken in *true stereo*, although variations in reflectivity with wavelength and in shadows may make binocular fusion difficult at times. For images having a wide angular spread, the corrections would have to be modified for different angles. No attempt has been made here to obtain exact expressions, or to account for problems of sensor tilt, as this paper merely sets forth the principle.