Discussion Paper

Atmospheric Refraction in Aerial Photogrammetry

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Schut's paper on "Photogrammetric Refraction"[1] pointed out an error in my earlier paper[2] Equation 9. I wish to acknowledge the validity of his observation. Unfortunately, the error is also reflected in Equation 10.62, page 476, in the 1966 edition of the *Manual of Photogrammetry;* the equation represents a good approximation for the refraction problem only for object points at sea level ($h=0$). This note was delayed while I attempted to find a correct replacement for the equation.

My studies revealed a better solution integral—so simple that it makes a *general* equation unnecessary (the equation in the manual was valid only for the ARDC standard atmosphere). To obtain it, my Equation 6 is first generalized to the form

$$\delta\theta = \frac{\tan\theta}{(Z_c - Z_g)c}\int_{Z_g}^{Z_c}(Z - Z_g)dv = K\tan\theta \quad (1)$$

where $\delta\theta$ is the refraction angle; θ is the nominal angle that the ray makes with the vertical; Z_c and Z_g are the camera and ground elevations, respectively; c is the light velocity in free space; and v the velocity as a function of altitude. This is transformed by an integration by parts, followed by the substitution

$$v = c(1 - 0.000226\rho)$$

where ρ is the atmospheric density as a function of altitude. The value of K in Equation 1 can then be written in the form

$$K(Z_c, Z_g) = \frac{226}{Z_c - Z_g}\int_{Z_g}^{Z_c}\rho dZ - 226\rho_c \quad (2)$$

where ρ_c is the density at the camera station and K is given in microradians.

If a table of atmospheric densities is available for a given situation and there are many values of K to be determined, it would be useful to make up two new tables, one listing $R=226\rho$ and the other the integral

$$I(Z) = 226\int_0^Z \rho dZ$$

This permits $K(Z_c, Z_g)$ to be written in the form

$$K = \frac{1}{Z_c - Z_g}[I(Z_c) - I(Z_g)] - R(Z_c). \quad (3)$$

As ρ is a smooth function, the integral is readily evaluated numerically. For example, to calculate K for a camera elevation of 5 kilometers and a ground elevation of 1 kilometer, one has (using an abbreviated table and ρ for the ARDC standard atmosphere[3]):

Z	ρ	R	I
0	1.225	276.8	0
1	1.112	251.3	264.0
2	1.007	227.5	503.4
3	0.909	205.4	719.9
4	0.819	185.1	915.1
5	0.736	166.3	1090.8

(The trapezoidal rule for integration was used to approximate I.)

The desired K is then

$$K(5, 1) = \frac{1090.8 - 264.0}{4} - 166.3 = 40.4.$$

This compares with 40.2 as found in Table 1 of Schut's paper.

The work leading to my paper was prompted by the idea that the refraction problem should be handled by perturbation techniques and that a computer should not be required for the numerical calculation of refraction. Unfortunately, the relatively simple form of the original solution and a bias that suggested that refraction depended upon the rate of change of density caused me to stop short of the solution given in this note.

References

1. G. H. Schut, "Photogrammetric Refraction," Photogrammetric Engineering, Vol. XXXV, No. 1, 1969.
2. S. Bertram, "Atmospheric Refraction," Photogrammetric Engineering, Vol. XXXII, No. 1, 1966.
3. R. A. Minzner, K. S. W. Champion, and H. L. Pond, "The ARDC Model Atmosphere, 1959," *Air Force Surveys in Geophysics No. 115*, Air Force Cambridge Research Center, 1959.