DR. JAMES MCMURRY ANDERSON* *University of California Berkeley, Calif. 94720*

Block Triangulation by ISP Comm. III

Summary of experimental research under the auspices of the International Society of Photogrammetry identifies new principles.

INTRODUCTION

ANALYTICAL AERIAL triangulation and ad- Λ justment of strips and blocks were among the fields of interests specified in a resolution passed by Commission III at the 1964 Congress of the International Society of Photogrammetry. The Commission also called for the formation of a Working Group to undertake experimental research on block triangulation.

In accordance with this resolution Commission President, Mr. G. C. Tewinkel, ap-

adjustments of a specific test block of aerial photography.

A block of simulated aerial photography was prepared and distributed to organizations indicating interest in the investigations. In this way complications which invariably accompany establishing ground control, reproduction and distribution of diapositives, plate coordinate measurement and refinement were avoided.

The Working Group included the following organizations and principal investigators:

ABSTRACT: *The Commission III Working Group consisted of participants* $from$ *U.S., Australia, England, Czechoslovakia, Netherlands and Canada with a total of nine organizations being involved. During the period* 1964-68, *each participant of the Group performed an independent adjustment of all or a portion of a* 9 *by 20-block of simulated near-vertical photography projected from approximately 11,000 meters above terrain, having an approximate scale of 1; 65,000 and 60 percent all-around overlap. Two general groups of methods were used: (a) simultaneous adjustment of photographs,. and (b) sequential adjustment. The first method produced more accurate results at a generally higher cost. However, sequential adjustments yielded results of high enough quality that further refinement in a simultaneous adjustment would not be necessary except for unusually large blocks where extremely high accuracy is required.*

pointed Mr. Charles Theurer as chairman of a Working Group formed to investigate and report on experimental research on block triangulation. The objectives of the Working Group were to be directed toward a practical solution of the problem. In line with these objectives, the Chairman contacted organizations which had operational analytical triangulation procedures and requested that they participate in the experimental studies. The overall plan was to suggest that each participant of the Group perform independent

- 1. Army Map Service, Washington, D. c., U.S.A. Mr. J. R. Barrett and Mr. V. E. Sellers.
- 2. ESSA, Coast & Geodetic Survey, Washing-ton, D. c., U.S.A. Mr. Morton Keller.
- 3. Ohio State niversity, Columbus, Ohio, U.S.A. Dr. S. K. Ghosh.
- 4. International Institute for Aerial Survey and Earth Sciences (lTC), Delft. Dr. K. Kubik.
- 5. University of Illinois, Urbana, Illinois, U.S.A. Dr. H. M. Karara.
- 6. National Research Cou neil of Canada. Mr. G. H. Schut.
- 7. Czechoslovakia, Prague, Czechoslovakia. Dr. V. Kratky.
- 8. Australian Photogrammetric Society. Mr. F. L. Bryant.
- 9. Geodetic Office of Survey Production Centre, R E., Feltham, England. Mr. M. J. Miles and Mr. P. Verral.

^{*} Presented at the Congress of the International Society of Photogrammetry at Lausanne, Switzerland, July 1968.

Reports from these nine participants in the Working Group have been received and analyzed. This paper constitutes a summary and analysis of the significant contributions of the Working Group participant's individual reports.

SIMULATED TEST BLOCK

A 9×20 -block of simulated, near-vertical photography was made available to the Working Group participants.* This simulated block is composed of 180 fictitious photographs taken from approximately 11 000 meters above terrain containing up to 1 000 meters of relief. The camera focal length is 152.00 mm. resulting in an approximate photo scale of 1:65,000. Theoretically perfect (unperturbed) and perturbed plate coordinates are given to microns for an approximately rectangular array of 25 images per photograph. Plate coordinate perturbations consist of random normal deviates having a standard deviation of 6 microns plus a systematic displacement (20 microns maximum) to simulate radial lens distortion, film distortion, and other sources of systematic error. Theoretically perfect and perturbed positions are given for exposure centers and all ground points. Use of the complete set of data results in 9 strips having 60 percent side lap. Selection of odd-numbered or even-numbered flights provides 100 photograph and 80 photograph blocks, respectively, having 20 to 30 percent side lap. All forward overlap is 60 percent.

The guide lines accompanying each data set were minimal. The test block was divided so that nine different sizes of blocks could have been ordered. Participants were requested to report:

- 1. Mathematical formulation
- 2. Details of adjustment procedures
- 3. Results and error analyses
- 4. Time and cost factors
- 5. Conclusions

Participants were free to choose any size block with any combinations of pass points, overlap, and ground control. It was suggested that at least one adjustment be made with minimum control to test and compare procedures.

• The simulated block was generated with a program prepared by the Autometric Operation
of the Raytheon Company for the U.S. Army of the Raytheon Company for the U.S. Army
Engineering Topographic Laboratories (USAETL), and was provided through the courtesy of Autometric-Raytheon Co. (15).

ANALYTICAL TRIANGULATION PROCEDURES TESTED

In general, the procedures tested can be divided into two major groups: (1) simul $taneous adjustment of photographs: and (2)$ sequential adjustment.

GROUP 1. SIMULTANEOUS ADJUSTMENT OF PHOTOGRAPHS

This group includes those triangulation procedures in which the desired parameters are adjusted as a result of one simlutaneous least squares adjustment of strips or blocks by direct or iterative methods (1, 2, 3, 4 in list of participants, above). From the mathematical standpoint, methods of this group are more rigorous and should provide the most accurate results. An outstanding difference between these methods and sequential procedures is the need for estimates of exposures station positions and orientations prior to the simultaneous adjustment of the strip or block. In addition, estimates for coordinates of all pass points may be needed depending on the approach taken. As a consequence, simultaneous adjustment of photographs should include a feasible method for acquiring the required estimated values.

Participants engaging in the simultaneous adjustment are listed in Table 1 where the type of equation and method for the solution of normal equations are also tabulated.

GROUP 2. SEQUENTIAL ADJUSTMENTS

This group includes procedures most easily visualized as analagous to the instrumental process of aerotriangulation (5, 6, 7, 8. 9, 11 in list of participants). Thus, sequential adjustments consist of analytic:

- *a.* Relative orientation and computation of individual basic units† in arbitrary, independent coordinate systems.
- *b.* Assembly of basic units into ^a common coordinate system (this step may be concur- rent with *^a* above).
- *c.* Adjustment of the assembled system to known control points.

Mathematical approaches for sequential adjustments are categorized according to respective relative orientation procedures and the method employed for strip or block adjustment. Further subdivision of the relative orientation procedure is achieved by sub-

t Basic unit, as used in this report, refers in most cases to the traditional stereopair or model. However, it can include 1X3, 3X3 or mXn photographs.

classification according to type of equation, basic unit, method of basic unit assembly, and adjustment procedure. These characteristics are summarized in Tables 2a and 2b for the sequential adjustments.

TEST BLOCK DATA SELECTED AND OBJECTIVES PURSUED

A total of 20 test blocks having variable dimensions and selected from different portions of the simulated block were tested. In addition, 35 different ground control point configurations were used for adjustment of the various test blocks. A total of 72 tests were performed with various combinations of these blocks and control configurations.

Tests performed by the working Group include:

- 1. Studies of the effects of varying the:
	- a. number and distribution of ground control stations;
	- b. ground control position weights;
	-
	- *c.* per-centage of photographic sidelap; d. degree polynomial used for block adjustment.
- 2. Studies of the effectiveness of incorporating horizon data as control to strengthen adjustment of strips and blocks.
- 3. Comparison of:
	- *a.* results from a simultaneous adjustment of photographs with results from a sequential adjustment of a common block and control configuration using the same computer;
	- b. strip extensions performed with a threephoto basic unit with those from a twophoto unit;
	- *c.* two different strip adjustment programs utilizing polynomials. (Schlund and US C&GS).
- 4. Testing and verification of a direct simultaneous adjustment of a strip or block by solving the equation system with function

TABLE 1. SIMULTANEOUS DIRECT ADJUSTMENT SUMMARY MATHEMATICAL PROCEDURES

minimization by conjugate the gradient method.
Block adjustment in three dimensions using a

- linear transformation in a simultaneous least squares adjustment of the entire block.
- 6. Adjustment of heights in a block by the method of linear adjustment of sections using a direct simultaneous least squares adjustment.

TEST RESULTS

Test characteristics and results obtained using the simultaneous and equential adjustments are summarized in Tables 3 and 4, respectively. Extreme caution should be exercised in attempts to compare results obtained by the nine participants. In addition to variable block dimensions and control point configurations, the number of check points utilized to calculate root-mean-square errors differs for practically all tests.

Additional items of interest output from simultaneous adjustments of photographs are: (a) standard deviations of unit weight *So* in plate coordinate residuals for the re-

| Participant | Relative Orientation | | |
|---------------|------------------------------|--------------------------|--------------------------------------|
| | Equation | Basic Unit | Unit Assembly By |
| $\frac{5}{5}$ | Collinearity Collinearity | 2 -photo 3 -photo | Successive Rotation & Scaling |
| 6 | Coplanarity | 2 -photo | Concurrent with Relative Orientation |
| | Not Specified | 2 -photo | Successive Rotation & Scaling |
| 8 | Coplanarity | 2 -photo. | Successive Rotation & Scaling |
| 9 | Coplanarity | 2 -photo | Not Specified |
| | Coplanarity | 2 -photo | Concurrent with Relative Orientation |

TABLE 2a. SEQUENTIAL ADJUSTMENTS: SUMMARY OF MATHEMATICAL PROCEDURES-RELATIVE ORIENTATION

580 PHOTOGRAMMETRIC ENGINEERING

TABLE 2b. SEQUENTIAL ADJUSTMENTS: SUMMARY OF ADJUSTMENT PROCEDURES

spective adjustments (where reported); and (b) iterations required for convergence. Note that values for s_o vary from 1.5 to 4.5 microns and are significantly different from applied perturbations having $\sigma_o = 6$ microns. Consequently, inconsistencies in simulated and/or applied perturbations are indicated. As the tests with unperturbed data yielded reasonable values for s_0 of from 0.3 to 0.8 microns, the most likely cause of inconsistency is in the random normal deviates applied to plate coordinates.

COMPUTER STORAGE AND TIME REQUIREMENTS

Tables Sa and Sb show approximate data for storage and time requirements based on reported figures. The tabulated times *do not* reflect data preparation or manual analyses of results.* As nearly as this reporter can deduce, these times represent only the so called *central processor* time for which the highest rate is usually charged.

• Times/photo as given are averages for all runs where sufficient data were reported.

Maximum available core storage varies from approximately 7,000 words to 131,000 words, where word is a nominal value that can be interpreted differently for various computers. Times vary from of 0.06 to a maximum of 4.0 minutes per photograph. Observe that both of these extremes resulted from simultaneous adjustments of photographs. Also note that the minimum time was from an adjustment on the current generation of an extremely high-speed computer.

Comparisons among procedures are meaningless due to the variable characteristics of the computer systems. However, Participant 1 performed both the sequential and the simultaneous ajdustment of photographs of a common block on the same computing system. This pair of runs (Tests 2 3, and 4, Table 3; Tests 53, 55 and 57, Table 4) provides a valid comparison and reveals that the sequential method requires about one third the computer time needed for the simultaneous adjustment of photographs. This comparison constitutes the only sound, specific conclusion to be reached from data in Tables Sa and Sb.

TABLE 3. ROOT MEAN SQUARE ERRORS IN DISCREPANCIES: SIMULTANEOUS ADJUSTMENTS OF PHOTOGRAPHS

* RMSE (root mean square error) = $(\Sigma v_n^2/n)^{1/2}$ where v_n = true value - adjusted value, and $n =$ number of points checked.

t 60% Side lap.

tt Horizon data used for control.

SUMMARY OF CONCLUSIONS

Major conclusions reached by participants can be summarized as follows:

- 1. Simultaneous adjustment of the one test block on the same computer produced RMS Errors 37 and 64 percent smaller in planimetry and heights than were obtained with a sequential adjustment (Tests 2, 3, 4, Table 3; Tests 53, 55, Table 4). Three times the computer time was required for the simultaneous adjustment.
- 2. Use of 60 percent sidelap improves vertical accuracy (Test 5, 9, Table 3).
- 3. Strong perimeter control is more effective than addition of interior points in a block (Tests 5, 7, Table 3).
- 4. Increasing position weights from 2 to 3 im-

proved fit to control points but did not alter the accuracy of computed additional points.

- 5. The conjugate-gradient method of function minimization requires moderate time on a medium sized computer and resulted in realistic discrepancies for a 3 X20 block (Tests 29, 30,31, Table 3).
- 6. Use of horizon data as control, improves accuracy in both medium sized blocks (15 to 28 photos) and 13-photograph strips. (Tests 26, 28, Table 3).
- 7. The three-photo basic unit yields a signifi-cantly less warped strip than the corresponding strip triangulated with a two-photo basic unit for the case of minimum control. Use of the three-photo unit produces no significant improvement if redundant well placed ground

582 PHOTOGRAMMETRIC ENGINEERING

TABLE 4. ROOT-MEAN-SQUARE ERRORS IN DISCREPANCIES SEQUENTIAL ADJUSTMENTS

TABLE 5. SUMMARY OF COMPUTER STORAGE AND TIME REQUIREMENTS

(a) Direct Simultaneous Solutions

(b) Sequential Solutions

t Includes 10 separate runs of from 10 to 49 photo blocks.

tt Extra run for gross error detection.

control points are available and higher-degree adjustments are performed (Participant 5).

- significant difference between the Schlund and the US C&GS methods of strip adjustment (Participant 5).
- 9. The iterative block adjustment of Participant 6 converged rapidly (8 to 10 iterations) because each iteration was started with two outside strips containing planimetric control. Experiments with this same adjustment confirm that increasing the degree of the adjustment equations does not necessarily improve results and may cause increased discrepancies (Tests 34, 36 and 35, 37, Table 4). Under certain conditions, a higher-degree adjustment does improve results as illustrated by Tests 41 and 42 where a systematic third-degree variation in scale was corrected by introducing a thirddegree planimetric adjustment.
- 10. The simultaneous, three-dimensional, linear block adjustment (Participant 7_i Tests block adjustment (Participant 44, ..., 48) converged in ⁵ to ¹⁰ iterations using a Gauss-Seidel iterative solution of the normal equations. Note that results obtained with this linear adjustment are comparable to those produced by adjustments with polynomials.
- 11. A block height-adjustment by the method of linear adjustment of sections (section = 2 adjacent, 2-photo units), yielded results sufficiently accurate for contouring at a lO-meter interval with 1:60,000 photography in which lateral control is spaced at 45- to SO-kilometer intervals (Test SO, Table 4). Ground control at 10-model intervals in analytic strip triangulation provides results in height control equal in accuracy to that obtained by ^a com- plete block adjustment (Test 51, Table 4).

If the efforts of the Working Group are considered as a unit, several interesting conclusions can be drawn. First, readers should avoid making specific, direct comparisons between different procedures on the basis of the reported results-too many variables exist between the testing procedures to reach other than very general conclusions.

Secondly, if one selects the maximum rootmean-square errors of discrepancies from the entire series of tests, the height accuracy achieved is $1/8,000$ of the average flight elevation (11,000 meters) and planimetric accuracy is $1/40,000$ of the least dimension (60,000 meters) of the 100 photo block. Where the minimum values of root-mean squareerrors are considered, these ratios are 1/20,000 and 1/70,000, respectively. Jf one accepts the hypothesis that the simulated model does provide a reasonable facsimile for a real block of photography, then fully analytic triangulation systems are quite feasible for smallscale mapping projects where the photography and ground control are of high quality. In making this evaluation due regard should be given to evidence indicating inconsistencies in the simulated data or in the random

normal perturbations applied to plate coordinates (see Test Results).

Thirdly, as one would have expected, the method of simultaneous adjustment of photographs produced more accurate results at a generally higher cost. The exception to this cost differential was a simultaneous adjustment of photographs performed on a very large and fast computer (Part. 2, Table 5). As such computers become more common, this time differential will be of less consequence. In this respect, additional work should be devoted to iterative solutions of large systems of normal equations as applied to the block adjustment. Also, efforts should continue to be pursued in further development of the conjugate-gradient method of function minimization as applied by Participant 4 of the Working Group.

However, the bookkeeping problems and need for sequential methods to provide estimates still remain. If such sequential adjustments result in the same excellence as exhibited in the examples furnished by this \iVorking Group, then the need for further refinement in a simultaneous adjustment of photographs would not be warranted except for unusually large networks where extraordinary accuracy is required.

Finally, the simulated data block appears to be an excellent vehicle for experimental research in analytic triangulation and block adjustment. For example, a simulated block would be the ideal form of data for use in research directed toward establishing realistic specifications for the observed quantities used in analytic photogrammetry. This is an area not yet fully explored for fully analytic triangulation and adjustment systems and deserves further attention.

BIBLIOGRAPHY

- 1. Keller, M., and Tewinkel, G. c., "Block Analytic Aerotriangulation," C& *GS Technical Report* 35, November 1967.
- 2. Korhonen, U., "Horizon Controlled Analytical Block Triangulation for Extending Ground Control for Small-Scale Mapping," Depart-ment of Geodetic Science, Ohio State Uni-
- versity, 1966.

3. Kubik, K., "A Procedure for Analytical Block

Triangulation," Publ. of the International In stitute for Aerial Survey and Earth Sciences (ITC), Series A, No. 40. 4. Elassal, A. A., "Simultaneous Multiple Sta-
- tion Analytical Triangulation Program,' *Photogrammetria,* 21, 3 June 1966.
- 5. Schut, G. H., "Development of Programs for Strip and Block Adjustment at the National
Research Council of Canada," Рнотоскам-
меткіс Engineering, Vol. XXX, No. 2, March 1964.
- 6. Schut, G. H., "An Introduction to Analytical Strip Triangulation, with a Fortran Program," *Publication AP-PR* 34 of the Division of Applied Physics of the National Research Council
- of Canada (NRC 9396), December 1966. 7. Kratky, V., "Economical Indirect Solution of the Block Adjustment of Aerial Triangulation," *Geodeticky a kartograficky obzor,* Vol.
- 12 54), Special issue 1966.
8. Krá^{(tký, V., "On the Solution of Analytical} Aerotriangulation by Means of an Iterative Procedure," *Photogrammetria,* Vol. 22, No.5, 1967.
- 9. Bervoets, S. G., *Cartography,* Vol. 3, No.3, March 1960.
- 10. Smith, A. N., Miles, M. J., and Verrall, P., "Analytical Aerial Triangulation Block Adjustment; The Direct Height Solution Incor- porating Strips," *Photogramm.etric Record 5,*

29 April 1967.

- 11. Keller, M. and Tewinkel, G. C., "Aerotriangulation Strip Adjustment," C & *GS Technical Bulletin* No. 23, August 1964. 12. Schlund, R. A., Verlag, Leeman, *"A usgleichung*
- *raumlicher ^A erotriangulationen mit HilJ von electronischen Rechenautomated,"* Zu rich, ¹⁹⁶ I.
- 13. Weissman, S., 1967, "Horizon-Controlled Analytical Aerotriangulation," Civil Engineer-ing Studies, *Photogrammetry Series* #13, Uni-
- versityof Illinois. 14. Brown, Arthur S., "The Effects of Horizon Control on 20% Sidelap Photography for Analytical Aerotriangulation Block Adjustment, M.Sc. Thesis, Ohio State Universitv, March 1968.
- 15. Doyle, F. J., "Fictitious Data Generator for Analytical Aerotriangulation," *Photogrammetria,* Vol. 21, No.5, October 1966,179-196

Articles for Next Month

Peter Bock and J. G. *Barmby,* Survey effectiveness of spacecraft remote sensors.

J. *M. Burnham* and *P. R. Josephson*, Color plate stability.

Clifford J. *Crandall,* Radar mapping in Panama.

W. P. MacConnell and *Peter Stoll,* Evaluating recreational resources of the Connecticut River. *P. M. Merifield et al.,* Satellite imagery of the Earth.

R. K. Moore, Heights from simultaneous radar and infrared.

Paul E. Norman, Out-of-this-world photogrammetry.

J. *T. Parry et al.,* Color for coniferous forest species.

Semi-Annual Convention Portland, Oregon September 23 to 27, 1969

General Chairman: leonard H. Delano Program Chairman: Roger Chamard 1536 S.E. 11 th Avenue Portland, Oregon 97214