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Regional Slopes with Non-Stereo Radar

Geometric relationships permit images to be measured to derive reliable slope data applicable to surveys in geomorphology and hydrology.

Abstract: *Two monoscopic radar images of the same area of terrain can be used for measurement of regional slopes. By measuring the lengths of individual slopes on each of the images, and knowing the look angle at each point across the images the angle of each slope can be computed. The two pieces of imagery may be taken from opposite sides of an area or both from the same side. Accuracy for individual slopes is fair, but for regional mean slope the accuracy is high. Compensation can be made for apparent slope, and ground range or slant range display systems.*

Introduction

Geomorphic studies using radar imagery are often limited by an inability to develop slope data. It is, of course, possible to obtain radar images having stereoscopic capability, but to do so requires that flights be designed especially for stereo mapping with each successive pass of the aircraft having a particular orientation relative to other flight lines.[1,2] The great majority of radar imagery from which landforms may presently be studied does not have stereo capability. Therefore, measurement of terrain slopes from non-stereo radar images is desirable if radar images are to become a tool for research in geomorphology and hydrology. By vieweing slopes in a region from two different flight lines, which need not be oriented to produce stereo images, it is possible to obtain a mean value of slope angles which compare quite closely with measurements drawn from topographic maps.

Regional mean slope angles are useful for a variety of investigations. (1) The slope mean and variance could be used in many instances as a surrogate for channel gradient in a multiple correlation for estimating runoff. For example, Wong[3] found that average land slope had a higher correlation coefficient relative to mean annual flood in New England than did channel slope. (2) Variation of mean land slope values from head to mouth of a drainage basin provide an expression of the relationship between stream gradient and side slope and demonstrate the headward variation of slope-gradient relationships on a quantitative basis. (3) Regional geomorphologists may find mean slope measurements a useful and valid aid for delimitation of physiographic provinces and sub-provinces. Regional slope in combination with other terrain parameters can provide a method to regionalize physiographic features quantitatively. (4) Knowledge of slope values also provides a basis for correlations of slope angles, slope failure, lithology, and rainfall. (5) In locations where surface slopes are strongly controlled by geologic structures,

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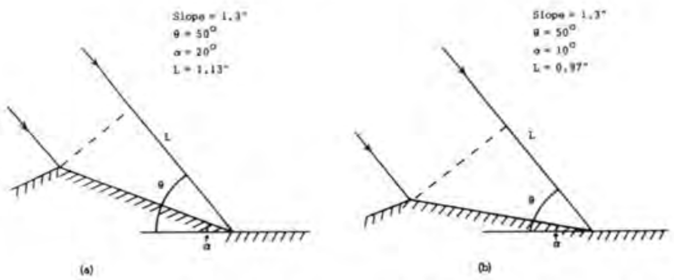

FIG. 1. The line segment L represents the slant range equivalent of the terrain slope. Where depression angle θ remains constant, and terrain slope α changes as in (b), L has a corresponding change.

(plateaus, homoclinal ridges) it is possible to obtain a measure of structural dips for geologic mapping.

The uses for mean slope measurements suggested above would be adaptable to the advantages of radar imagery for terrain studies. One major advantage of radar for landform analysis is its capability for large areal coverage with good detail of geomorphic features. Such broad coverage makes radar a suitable tool for regional generalization of terrain components, yet the information content of radar is very high relative to topographic maps of comparable scale (1:200,000 to 1:700,000 and greater). It has been demonstrated further that radar-derived terrain measurements, such as lengths, areas and numbers of streams, maintain a consistent relationship to measurements of the same features on topographic maps.[4] Previously it has been possible to measure only horizontal components of the land surface unless radar imagery of stereo capability was available. The following discussion outlines procedures for acquisition of regional slope data from paired monoscopic radar images.

THEORY

The slant range length measurement L of a hill slope on a radar image is a function of the depression angle (measured from horizontal) of the radar beam θ, and the angle of terrain slope α: $L=f(\theta, \alpha)$. Figure 1 demonstrates how slope angle affects slant range length. Note that the depression angle θ of the radar beam is the same in each instance, and that the magnitude of slant range length L varies with terrain slope α. If the same slope is next viewed from opposite directions along parallel flight paths, the angle of slope may be determined as a function of two depression angles and two slant range measurements, as seen in Figure 2:

$$\alpha = f(\theta_1, \theta_2 L_1, L_2).$$

In order to express the relationship of L_1 and L_2 it is convenient to make a ratio $R_L = L_2/L_1$ and thus reduce the length parameter to one value. The following steps lead to a solution of the slope angle α:

$$L_2/L_1 = R_L = [\cos(\alpha + \theta_2)]/[\cos(\alpha - \theta_1)]$$

$$R_L \cos(\alpha - \theta_1) = \cos(\alpha + \theta_2)$$

$$R_L(\cos\alpha\cos\theta_1 + \sin\alpha\sin\theta_1) = \cos\alpha\cos\theta_2 - \sin\alpha\sin\theta_2$$

$$R_L(\cos\theta_1 + \tan\alpha\sin\theta_1) = \cos\theta_2 - \tan\alpha\sin\theta_2$$

$$\tan\alpha = (\cos\theta_2 - R_L\cos\theta_1)/(\sin\theta_2 + R_L\sin\theta_1). \quad (1)$$

APPLICATION

Equation 1 expresses the relationship among the parameters marked in Figure 2. A nomogram for Equation 1 provides easy entry of known variables for solution of terrain slope (Figure 3). Application of this nomogram requires the following procedure:

1. The longer slant length measurement is labeled L_1 in order to maintain $R_L < 1$.
2. The depression angle corresponding to L_1 is called θ_1.
3. The R_L value is entered along the appropriate curved scale of the nomogram.

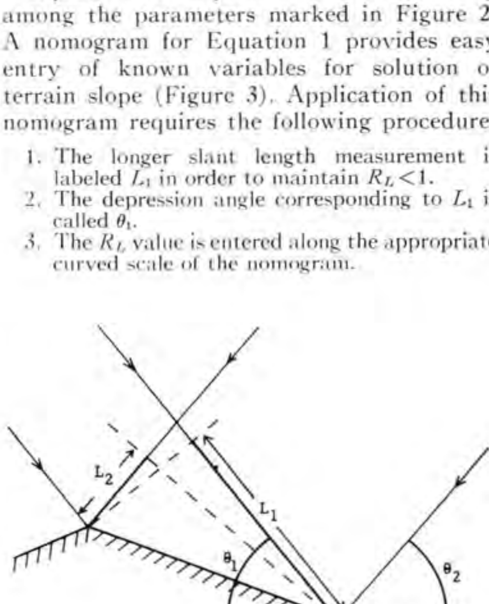

FIG. 2. By viewing the terrain slope from two directions there are two depression angles θ and two slant range lengths L. Using these measurements it is possible to compute the angle of terrain slope.

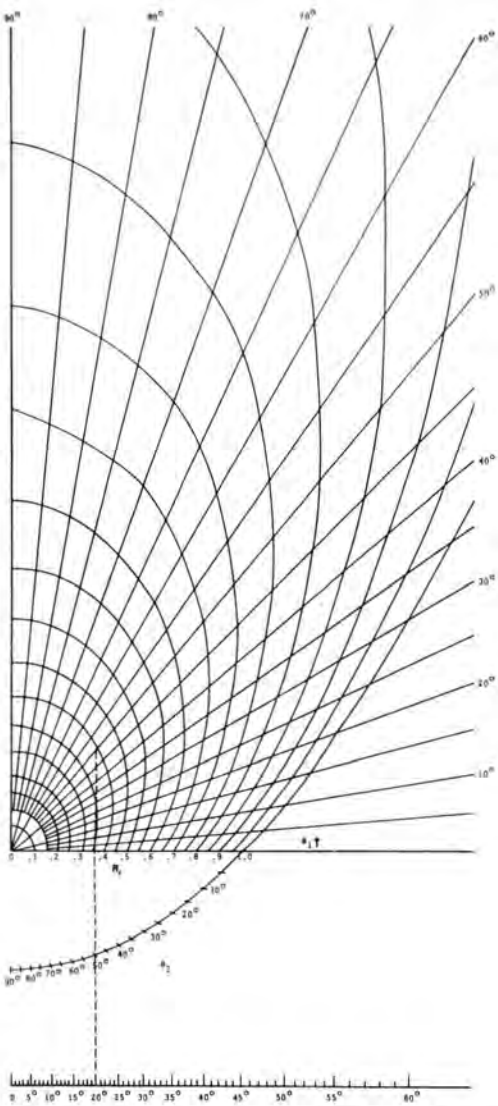

FIG. 3. A nomogram based on Equation 1 provides an easy solution of slope angle. The dashed vertical line is an example based on Figure 2 where $\theta_1=50°$, $\theta_2=50°$, $R_L=0.39$.

4. The θ_1 value is entered along the correct radial line.
5. The intersection of the R_L line and the θ_1 line creates Point 1.
6. θ_2 plotted on the appropriate curved scale gives Point 2.
7. Place a straight edge on Points 1 and 2 and read α along the linear scale at the bottom of the nomogram.

Two basic assumptions are made relative to L and θ.

1. Both images are either the same scale, or can be corrected to equivalent scales. This correction can easily be made by measuring the distance between any two objects situated essentially parallel to the flight path and appearing on both images. The ratio of the two measurements is the conversion factor by which the image scales may be equalized and slope length measurements made comparable.

It is not necessary to know the image scales, neither in the slant range nor parallel to the flight path. What must be determined is the relative difference in image scales taken along the flight path to be used as a correction factor for measurements taken along the slant range. In this way, differences in a pair of slope length measurements will express only the variation in length of signal travel paths L_1, L_2 needed for this method of slope determination, and scale differences resulting from changes in flight height are corrected. This method of determining slope angles does not require that any reference datum elevation on the terrain be known. This approach to slope computation depends on the difference in lengths of signal return paths at either end of a slope L_1 and L_2 on Figure 2, in combination with the angles of the signal paths from the horizontal θ_1 and θ_2.

2. The derivation of Equation 1 assumed equal depression angles at all point along the terrain slope. This condition will not exist if the slope is long, or if the distance to the airborne radar is small. If the range from maximum to minimum depression angle along a terrain slope is small, then an average value for θ can be entered in the nomogram. In the case of a large range of depression angles along a slope, both the extreme and the average values of θ should be used in order to estimate the magnitude of variation in computed slope angle. In most cases the variation will be small.

Because this method of slope determination

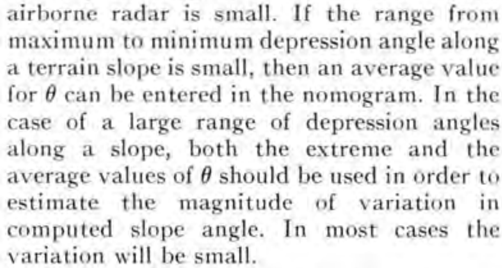

FIG. 4. The relation between slant range and ground range is a function of aircraft height and distance from the flight line. S is the slant range from radar to ground; G, ground range to Point 1; F, slant range between Points 1 and 2; g, ground range between Points 1 and 2; and H is the height of the aircraft.

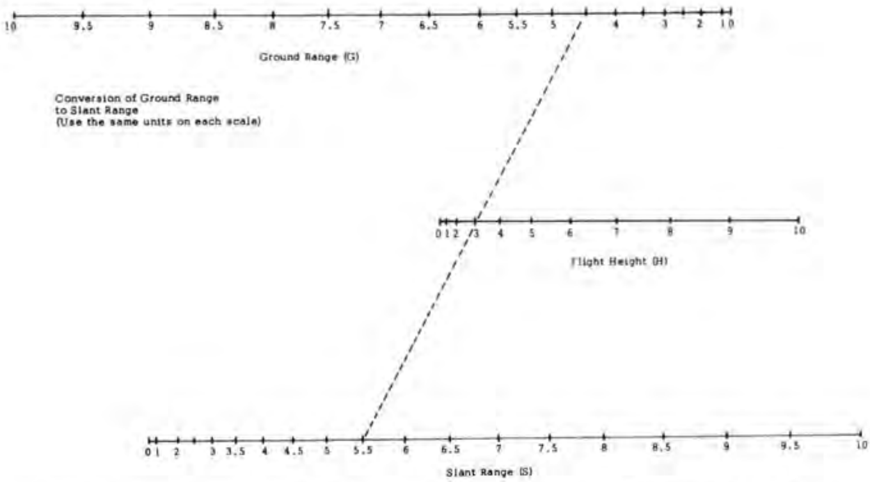

FIG. 5. A nomogram provides for conversion of either ground range or slant range. The same units must be used on each scale.

requires slant range lengths it is often necessary to convert measurements from a ground range display radar system to a slant range equivalent (Figure 4). Such a conversion is possible by removing in effect, the hyperbolic function of image restitution which is applied to produce a ground range display radar image. This conversion is made by the equations:

$$S(H, G) = (H^2 + G^2)^{1/2} \quad (2)$$

$$L = S(H, G + g) - S(H, G) \quad (3)$$

Ground range display images are produced by application of the above relationship assuming a flat terrain. It is therefore proper to correct ground range measurements to slant range using the same relationship and assumption. Errors introduced by assuming flat terrain decrease with increasing flight altitudes, and are not significant under usual operating conditions. Figure 5 is a nomogram

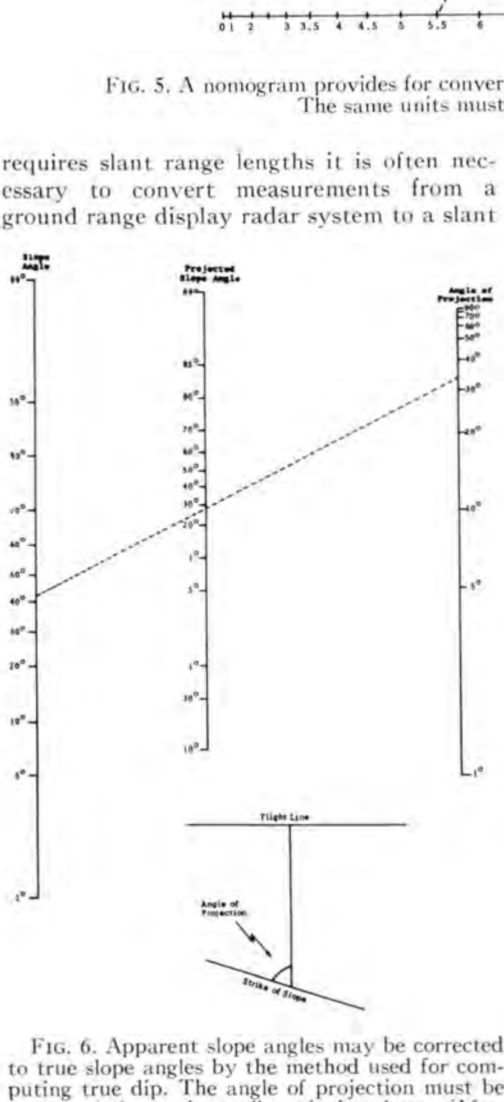

FIG. 6. Apparent slope angles may be corrected to true slope angles by the method used for computing true dip. The angle of projection must be measured from the strike of the slope. (After Palmer, 1919.)

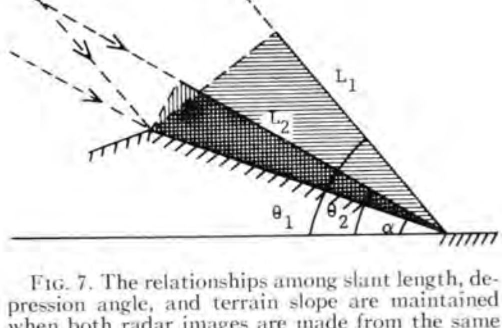

FIG. 7. The relationships among slant length, depression angle, and terrain slope are maintained when both radar images are made from the same direction, as well as from opposite directions.

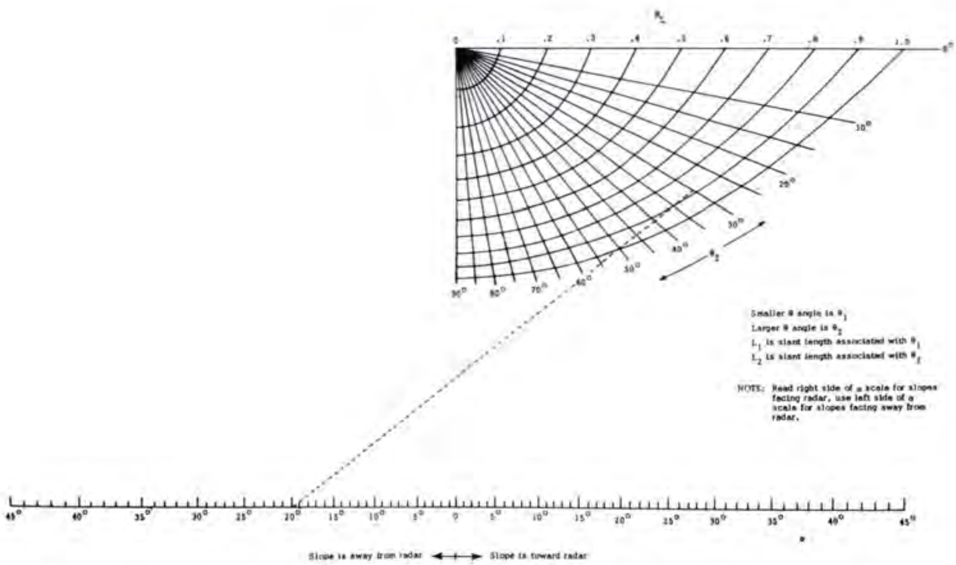

FIG. 8. When terrain slopes are imaged twice from the same direction, the above nomogram will yield slope angle. Data from Figure 7 is represented by the dashed line.

based on Equation 2. Application of this nomogram requires finding the two slant distances $S(H, G+g)$ and $S(H, G)$, and then calculation of L by Equation 3.

If the strike of a land slope is not parallel to the line of flight, the resulting slope angle α is an apparent slope rather than a true slope. The solution to this problem is similar to that described by Palmer[5] (1919) for apparent dips (Figure 6). Note that the projection angle, shown by the diagram in Figure 6, in the case of radar imagery is measured between the strike of the slope and a line perpendicular to the flight path.

The problem as shown in Figure 2 assumes that flight lines are on opposite sides of the object slope. A solution is equally feasible when flight lines are both on the same side of the slope as shown in Figure 7.

The nomogram in Figure 8 may be applied when flight lines are on the same side of a slope. The following procedure must be used:

1. The smaller depression angle is θ_1 and the larger angle is θ_2.
2. The longer slant length is called L_1 in order to maintain $R_L < 1$.
3. The R_L value is entered along the appropriate curved scale of the nomogram.
4. The θ_1 value is entered along the appropriate radial line.
5. The intersection of the R_L line and the θ_1 line creates Point 1.
6. θ_2 plotted along the outermost curved scale makes Point 2.
7. Place a straight edge on Points 1 and 2 and read α along the linear scale at the bottom of the nomogram.

If the land surface is sloping away from the radar as in Figure 7, the slope angle α is read on the left side of the scale. Conversely, slopes facing the radar source are read on the right side of the α scale.

ACCURACY

The methods outlined above have proved very accurate on graphically constructed experiments. Under practical conditions using radar imagery, several error sources tend to increase the variance of values. One of the greatest possibilities for error lies in the difficulty of selecting identical points on two different images. This problem is created by shadowing and highlighting on radar images, so that a point on a slope may be clearly visible on one image and obscured in shadow on the other. Further difficulty is encountered

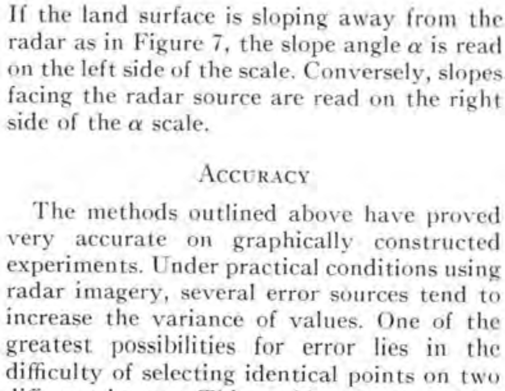

FIG. 9. By knowing the depression angle at each edge of the radar image it is simple to construct a scale which can be placed directly on the imagery to give depression angles at any point along the range of the image.

in obtaining accurate length measurements. Slight variations in measurement will change the value of R_L, and introduce a source of error in the computation of terrain slope α.

Another source of error, and an especially critical variable, is the determination of depression angles θ. This requires knowledge of pression angles at the edges of the radar image. Knowing these values, it is possible to interpolate the depression angle of any point along the range direction of the image. Because depression angles are measured from horizontal, and are independent of terrain slope, values for depression angles along the range direction can be interpreted accurately as shown in Figure 9. A simple scale of depression angles can then be devised to lay directly on the imagery and read the appropriate θ value for any point. Depression angles of the edges of the beam are controlled by the radar instruments and are entirely consistent in a given system. Therefore the geometric relationship illustrated in Figure 9 is independent of flight height. Changes in flight height cause variations in the width of ground coverage obtained at a given depression angle, but this is a matter of scale.

TABLE 1. COMPARISON OF SLOPE ANGLES MEASURED FROM RADAR IMAGES AND TOPOGRAPHIC MAPS

Slope Sample	*Slope Angles*		*Slope Sample*	*Slope Angles*	
	Radar	*Map*		*Radar*	*Map*
1	11°	14°	19	22°	24°
2	11°	13°	20	26°	27°
3	15°	19°	21	35°	35°
4	26°	23°	22	24°	22°
5	14°	18°	23	31°	28°
6	20°	24°	24	23°	20°
7	28°	34°	25	10°	11°
8	14°	12°	26	20°	14°
9	16°	18°	27	13°	14°
10	25°	21°	28	32°	29°
11	9°	10°	29	27°	27°
12	35°	33°	30	23°	18°
13	22°	25°	31	10°	11°
14	25°	27°	32	9°	11°
15	22°	22°	33	22°	19°
16	32°	34°	34	11°	11°
17	27°	26°	35	16°	13°
18	18°	16°	Mean	20.7	20.7

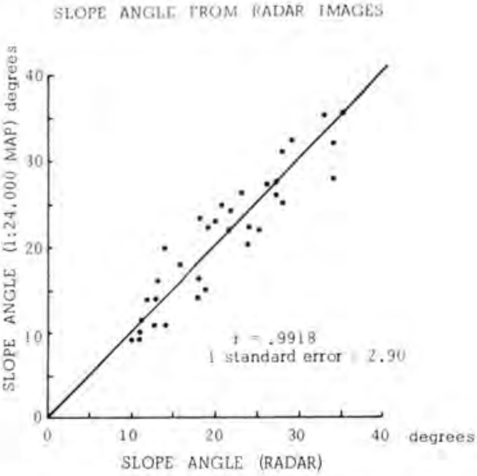

FIG. 10. Slope angles measured from radar are highly correlated with those derived from maps.

It becomes apparent from inspection of Table 1 and Figure 10 that the slope values measured from radar images are sufficiently accurate to permit analysis of individual slopes in certain types of studies, and that the mean value of regional slope angles attained by radar agrees closely with topographic map data. In an examination of the data in Table 1, one tends to assume that the map data correctly represents the slope angles in the area studies, and that differences in map and radar values represent errors in the radar measurements. This assumption is not valid in every instance because errors are also involved in the computation of slopes on topographic maps. The average difference between individual map and radar values on Table 1 is 2.5°, and a portion of this difference will be the result of errors in making slope measurements on the topographic map as well as errors in acquisition of the radar data.

Further, the difference in mean slope angle from map and radar decreases steadily to zero at 30 samples. This suggests that a sample of 25 to 30 measurements may perhaps be sufficient for a significant expression of regional slope. This supposition is, of course, subject to verification by additional experiments.

Experiments described here have shown that a geometric relationship exists among the length of terrain slope as portrayed on two monoscopic radar images, depression angle of the radar, and the terrain slope angle. Such a relationship will permit monoscopic radar images to be used to derive reliable slope data applicable to surveys in geomorphology and hydrology.

References

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Engineering Reports

Business–Equipment–Literature

Edited by Ivan R. Jarrett

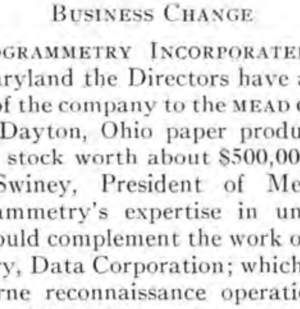
Business Change

Photogrammetry Incorporated, Rockville, Maryland the Directors have approved the sale of the company to the Mead Corporation, a Dayton, Ohio paper products company for stock worth about $500,000. Mr. J. W. McSwiney, President of Mead, said Photogrammetry's expertise in underwater optics would complement the work of a Mead subsidiary, Data Corporation; which is active in airborne reconnaissance operations. The company will continue to be operated by its present management headed by Mr. Gomer T. McNeil.

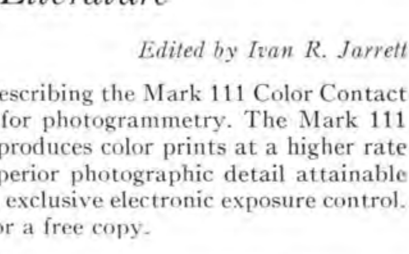
Technical Bulletin

LogEtronics, Inc., Photo-Optical Division, 7001 Loisdale Road, Springfield, Virginia 22150 have available a new Technical Bulletin SA-34 describing the Mark 111 Color Contact Printer for photogrammetry. The Mark 111 printer produces color prints at a higher rate with superior photographic detail attainable through exclusive electronic exposure control. Write for a free copy.

FUNCTIONAL SCHEMATIC



OPTOmechanisms, Inc., Engineers Hill, Plainview, L. I., N. Y. 11803 have introduced a new photogrammetric comparator for highly accurate measuring and inspection, particularly in the fields of micro-ciruitry, industrial and scientific photography, electronic and electromechanical components, and tool making. The Model 690–300 provides a 2×2-inch, ±2-micron measuring



stage as a precise base for high magnification viewing of the photographic imagery. Drum measuring increments, available in choice of the inch or metric system, are *direct reading*, minimizing the need for interpolation. A detailed brochure is available on request.