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Geometric Distortions in Television Imageries

As the distortions are highly systematic, appropriate measures can be taken to correct them.

(*Abstract on page 494*)

Introduction

Planetary explorations have opened new dimensions in photogrammetry. Besides introducing new applications for the photogrammetric principles, they also have introduced a new tool to the photogrammetrists, namely, television. Because of the impracticability of returning a physical photographic payload from outer space, television has become an essential data-collection device in all photographic missions to outer space.

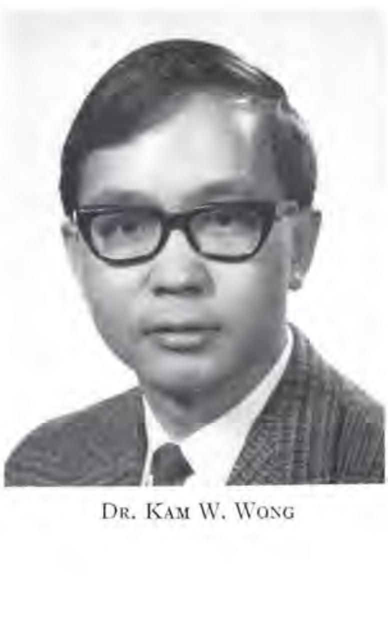
However, scientific applications of television is not limited to space explorations alone. Because of its ability to extend human vision to environments which are too hazardous and dangerous to the human observers, television has been used extensively in medical, biological, and ballistic researches and in the supervision of industrial processes. Because of its high sensitivity, television is also used in astronomical and underwater observations. If metric quality can be obtained from television pictures, application of the photogrammetric methods of measurement will provide valuable quantitative informations to those areas of scientific research that employ television as an observation device.

Elements of Television

A photographic television system consists of four major components: (1) a television pickup system which includes a TV camera and a transmitter, (2) a communication channel, (3) a receiver system which includes a cathode ray tube, and (4) a photographic camera which is used to photograph the display on the CRT screen.

Figure 1 illustrates the photographic processes involved. A visual image of the object is focused by an optical lens system onto the target of the camera tube. The target is made of either photoconductive or photoemissive materials, and the amount of electron charge on an elemental area of the target is proportional to the intensity of the incident light. Thus the image on the target is defined by the distribution of electron charges. This electron charge image is scanned in an orderly manner and converted to video signal. The video signal is then converted to radio signal and transmitted through a communication channel.

At the TV receiver, the radio signal is reconverted into a video signal which is used to modulate the intensity of the electron beam in the cathode ray tube. This electron beam at the CRT must move in the identical manner as that in the camera tube. As the beam



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ABSTRACT: *This paper describes the various sources and forms of geometric distortions in a TV system. Equations are formulated to describe the composite effect of the systematic distortions in TV pictures. It is evident from this study that geometric distortions in a TV system are highly systematic in nature. Through careful prephotography calibration and an efficient use of a reseau grid, relatively high-quality geometric measurements can be made from TV pictures.*

strikes the phosphor screen, it causes phosphorescence. The intensity of phosphorescence depends on the intensity of the electron beam. Consequently, a visual image of the original object is reproduced on the screen of the CRT. Finally, a photographic camera is used to photograph the display, giving a photographic image of the object.

Analysis of Geometric Distortions

By far the most serious source of geometric distortions is the scanning process. In order to attain perfect scanning geometry, the movement of the electron beam at both the camera tube and the CRT must strictly follow the following pattern:

1. The beam moves with constant velocity along each scan line.
2. The lines are straight.
3. The lines are equally spaced.
4. The beginnings and ends of the lines fall on lines forming the borders of the image field.
5. The lines all have the same inclination with the horizontal.
6. There must be perfect synchronization between the movement of the two scanning beams: for example, the beginning and end of each line on the CRT must correspond directly to the beginning and end of the same line at the camera tube.

One obvious solution to the above problem is to use identical deflection circuits at the camera tube and the CRT. However, this is not possible due to the difference in the design of these two types of tubes. Hence, the geometric fidelity of a TV system depends largely on how well the actual scanning patterns can satisfy the above conditions.

DISTORTIONS AT THE CRT

Proper deflection of the electron beam at the CRT is effected by two mutually perpendicular magnetic fields. Two pairs of magnetic coils are mounted on the neck of the CRT. The top and bottom coils produce a vertical field which deflects the beam in a horizontal direction, whereas the side coils produce a horizontal field which causes vertical deflection. The amount of deflections in the horizontal and vertical directions depend on the amount of current flowing through the corresponding pair of coils.

Figure 2 is a vertical cross-section of a CRT showing the principle of magnetic deflection. H is a horizontal magnetic field produced by the side coils. Let it be assumed that the following idealized conditions exist:

1. The magnetic field H is uniform over the length of the coil d.
2. The field H drops abruptly to zero outside of the region bounded by the coils.
3. The CRT screen has a radius of curvature equal to y, which is the distance from the center of the field H to the screen.
4. The maximum deflection of the beam is small compared to the distance y.

Under the above idealized conditions, the magnitude of deflection is directly proportional to the current in the deflection coil and can be expressed by the following equation:

$$D = aI \tag{1}$$

where D = deflection distance, a = system constant, and I = current in deflection coil.

Such a linear relationship between D and I is the main objective of all deflection coil design, because simple sawtooth current can then be used to produce the line scanning pattern.

In practice, a perfectly uniform magnetic field is difficult to attain. The magnetic flux generally forms a pincushion or barrel-shaped field. In a pincushion field, the field intensity decreases outward from the center. Consequently, the beam is deflected less at the outer part of the field than at the center resulting in a barrel-shaped distortion pattern. In a barrel-shaped field, the field strength increases outward from the center. The beam is now deflected more at the outer part of the field and a pincushion distortion results.

Non-uniform field also causes distortion of the shape of the electron beam.

These effects can be minimized by shaping the pole pieces properly and distributing the windings on the deflection coil so that a

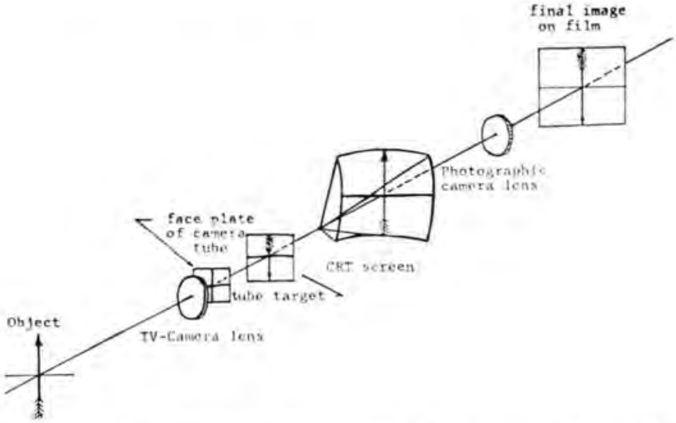

FIG. 1. Photographic process of a TV system for photogrammetric applications.

nearly uniform field is produced. However, a non-uniform field is sometimes intentionally set up to compensate for the distortions introduced by other sources.

It is also not possible to produce a field which drops abruptly to zero outside of the deflection region. The small fringe field which does exist both in front of and behind the deflection region forms a circular magnetic lens and causes aberration in the electron beam as well as distortions in the scanning pattern. The behavior of the distortions is similar to that caused by non-uniform field.

The small-fringe field behind the deflection region may also interact with the focusing field which is normally placed in front of the electron gun to focus the scanning beam. The effects of this interaction are complicated and not too well understood. At least two effects due to focusing-deflection interaction have been recognized. They are: (1) a rotation of the image about the axis; and (2) line splitting which results in bowed scan lines. The rotational effect can be corrected by rotating the deflection coils by the same amount but in the opposite direction. The line-splitting effect must be minimized by proper isolation of the focusing and deflection fields.

Another assumption made in the derivation of Equation 1 is that the CRT has a radius of curvature equal to the distance from the screen to the center of the deflection field. In actual cathode ray tubes, the curvature is made much flatter to avoid excessive image distortion due to curvature. If the screen is made perfectly flat, it can be easily seen from Figure 3 that the deflection distance D_1 is proportional to the tangent of the deflection angle θ; and

$$D_1 = D + AD^3 \qquad (2)$$

where A is a constant. Because of the cubic term, deflection increases more rapidly than the current. The result is the familiar pin-

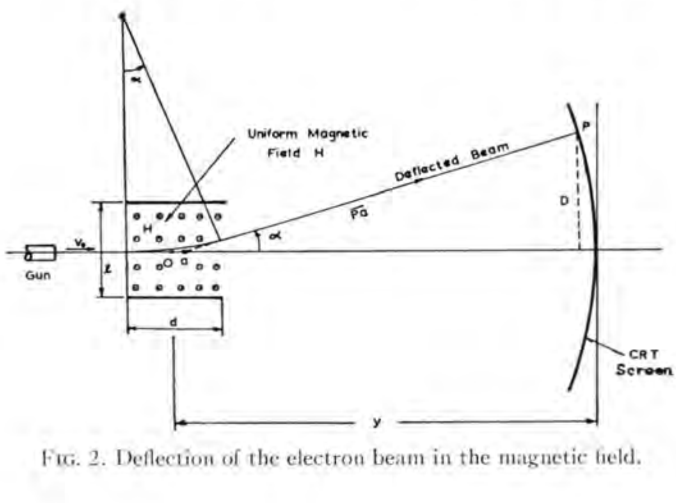

FIG. 2. Deflection of the electron beam in the magnetic field.

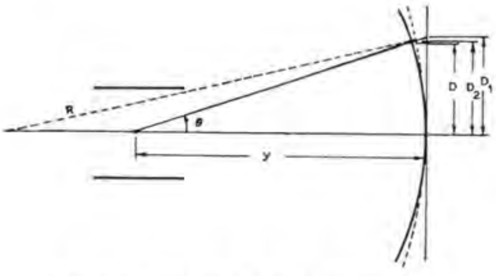

FIG. 3. Distortion due to *flat*-faced tubes.

cushion distortion in which a straight line $y=y_0$ becomes distorted to a parabola. A circle with center at the origin remains a circle, but its radius is enlarged from $r=r_0$ to

$$r = r_0(1 + Ar_0^2).$$

In actuality, the CRT screens are not perfectly flat but have a rather large radius of curvature. The deflection D_2 is then proportional to a function whose value lies between the sine and the tangent of the deflection angle. This function results in a smaller value for A in Equation 2.

This pincushion-distortion effect can be corrected by using a pincushion deflection field. However, a non-uniform deflection field still has the undesirable effect of causing defocusing of the electron beam.

The deflection of the electron beam is also affected by any external magnetic or electromagnetic fields may be present in the area between the deflection field and the screen. One magnetic field is always present in the atmosphere, namely, that of the earth magnetic field.

The amount of deflection of the beam caused by the earth's magnetic field depends on the following factors:

1. The strength of the earth's magnetic field at that particular location.
2. The distance the electrons have to travel to reach the screen.
3. The velocity of the electrons.
4. The angle between the path along which electrons are traveling and the direction of the earth's field.

For a vertical field strength of 0.434 oersted and a 21-inch CRT operating at 16 kilovolts, distortion at the image corners amounts to about 1/32 of an inch.

The effect of the earth's magnetic field can be eliminated by providing a compensating field at the deflection coil. However, because the strength of the earth's magnetic field differs from one location to another on the earth's surface, and because other external fields may also be present, the best solution for eliminating external interference is to provide adequate magnetic shielding around the cathode ray tube.

A distortion called *hook* is caused by insufficient flyback time at the end of each line sweep. Under such conditions, the deflected beam does not have sufficient time to return to its zero position, and a residual deflection remains at the beginning of the next line sweep resulting in a displacement of the beginning of that line. In practice it is found that the vertical return time (for the horizontal deflection coil) of the scanning pattern cannot be less than 7 percent of the total scan period, nor the horizontal return time less than 15 percent of the line-scan period.

Line *jitters*, or random starting position of the scan lines, is caused by a non-uniform scanning rate, and line wiggles can be caused by oscillations in the individual windings of the deflection coil or cross-talk between windings. Oscillations in the deflection coil become more serious at high-frequency scanning and must be adequately insulated by proper tube design.

DISTORTIONS AT THE CAMERA TUBE

Magnetic deflection is also used in all modern camera tubes, such as the image orthicon and the vidicon. However, the principle of operation is different from that used by the CRT. Both the image orthicon and the vidicon tubes require orthogonal scanning, that is, the electron beam must always approach the target at a right angle. Moreover, the electrons in the beam must approach the target at a near-zero velocity.

Because of the use of magnetic scanning, such problems as non-uniform field, fringe field, and oscillations in the windings, are also present at the camera tube. However, with the presence of the axial focus field, their effects on the scanning pattern are much more difficult to analyse and no discussions on this subject can be found in the literature.

One source of geometrical distortion inherent in the present image orthicon and vidicon tubes is the non-uniform electric field set up between the target and the wall-anode. If this field is perpendicular to the target, it will not have any effect on the electron beam. But this condition is satisfied only in the region close to the tube axis. At the outer edges of the target, the field makes a small angle with the tube axis and thus has a component radiating from the center of the target. As the electron beam approaches the target, this radial field component deflects the beam away from the center of the target.

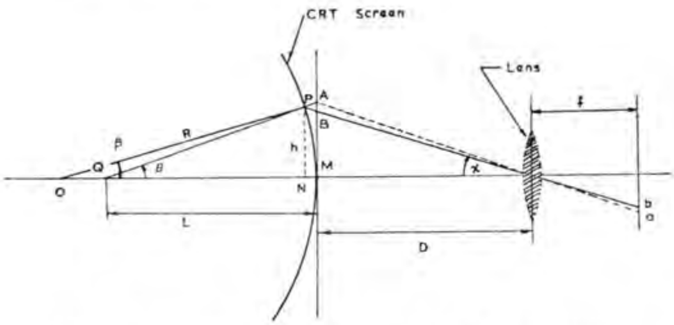

FIG. 4. Distortion due to the curvature of the CRT screen.

This motion of the beam in turn cuts the axial magnetic field which then deflects it in a direction tangential to its radial motion. As a result of this, the scanning pattern is rotated through a small angle. The angular rotation is not constant for all points on the target. Because of the larger radial component of electric field at the edges of the target, the angular rotation is larger at the edges than at the region near the center. At the center region, the rotation is zero. Thus straight lines passing through the center of the scanning pattern are distorted into the shape of an elongated letter *S*.

SYNCHRONIZATION DISTORTION

Two forms of synchronization are required by the scanning systems to produce a distortionless pattern: (1) the horizontal and vertical components of deflection at each scanning system must be rigidly synchronized in both frequency and phase, and (2) the instantaneous position of the scanning beam at the CRT must be synchronized with that of the beam at the camera tube.

Failure in the first condition will result in the rotation of each scan line about the horizontal, and failure in the second condition will cause a linear shift of the scan lines with respect to each other. In either case, image distortion results.

DISTORTIONS AT THE TRANSMISSION CHANNEL

Geometric distortions are also caused by non-linear phase-shifts at the amplifiers and at the transmission channel. Due to non-linear phase-shift, signals of high frequencies arrive at the receiver ahead of low-frequency signals thus resulting in a shifting of the fine picture details with respect to the coarser elements. This kind of distortion cannot be corrected after the picture has been reproduced, but can be minimized by the use of equalisers to provide a nearly constant frequency response within the transmission channel. When significant non-linear phase-shifts occur within the useful range of a TV system, the relative displacement of image details will result in such tonal distortions that it can render the picture entirely useless.

DISTORTION DUE TO CURVATURE OF THE CRT SCREEN

As the target of the camera tube is a flat surface, if its charge image is scanned and finally displayed on the curved surface of the CRT screen as a visual image, image distortion is caused by the curvature of the screen. This effect is further aggravated if the display is photographed on film which is again a flat surface.

In Figure 4, P represents a point on the screen with a deflection angle θ and b is its image on the film. If the screen is a flat plane, at the same deflection angle the point P should be at point A with a corresponding image a. Thus the total curvature distortion is the distance ab. It can be derived from Figure 4 that

$$\Delta r = ab = \frac{f \cdot h}{D}\left[\frac{L}{L - R + \sqrt{R^2 - h^2}} - \frac{D}{D + R - \sqrt{R^2 - h^2}}\right]. \tag{3}$$

The equation shows that radial distortion due to curvature is approximately linearly proportional to h.

DISTORTIONS DUE TO THE ORIENTATION OF THE PHOTOGRAPHIC CAMERA

Finally, image distortions are also caused by imperfect alignment of the optical axis of the camera with the axis of the CRT. If the image coordinates are referred to fiducial marks that are on the photographic camera itself, then image distortion due to misalignment of the camera axis can be expressed as

$$(\Delta x)_0 = d_0 + d_1 xy - d_3 y + d_2 x^2$$
$$(\Delta y)_0 = d_4 + d_3 x + d_2 xy + d_1 y^2 \qquad (4)$$

where x, y are the measured image coordinates and $(\Delta x)_0$ and $(\Delta y)_0$ are the distortions in the x and y direction respectively.

In the case where fiducial marks are etched on the target plate of the TV camera tube, then translational movements of the exposure center as well as the rotation about the optical axis produce negligible distortions in image coordinates. Equation (4) can then be simplified to

$$(\Delta x_0)^1 = d_1 xy + d_2 x^2$$
$$(\Delta y_0)^1 = d_2 xy + d_1 y^2. \qquad (5)$$

Formulation of Composite Distortion Equations

Symmetric Radial Distortions

Symmetric radial distortions are caused by the optical lens systems, non-uniform magnetic field, fringe field effect, and curvature of the CRT screen. These distortions are functions of only the radial distance of the image point from the principal point and may be described by an odd-order polynomial, such as

$$(\Delta r)_{s,r} = k_1 r + k_2 r^3 + k_3 r^5 + k_4 r^7 + \cdots$$

$(\Delta r)_{s,r}$ may be decomposed into x- and y-components to give

$$(\Delta x)_{s,r} = x(l_0 + l_1 r^2 + l_2 r^4 + l_3 r^6 + \cdots) \qquad (6)$$

and

$$(\Delta y)_{s,r} = y(l_0 + l_1 r^2 + l_2 r^4 + l_3 r^6 + \cdots)$$

where $(\Delta x)_{s,r}$ and $(\Delta y)_{s,r}$ are the x- and y-components of symmetrical radial distortions respectively, and x and y are the measured image coordinates.

Symmetric Tangential Distortions

Symmetric tangential distortions are caused by the interactions between the focusing and deflection fields and by the electric deceleration field at the camera tube. This form of distortion is zero at the principal point and increases with the radial distance. It can be expressed by an even-order polynomial

$$(\Delta t)_{s,t} = q_1 r^2 + q_2 r^4 + q_3 r^6 + q_4 r^8 \cdots$$

If the expression is decomposed into its x- and y-components, $(\Delta t)_{s,t}$ becomes

$$(\Delta x)_{s,t} = -y(q_1 r + q_2 r^3 + q_3 r^5 + q_4 r^7 + \cdots) \qquad (7)$$

and

$$(\Delta y)_{s,t} = x(q_1 r + q_2 r^3 + q_3 r^5 + q_4 r^7 + \cdots).$$

Asymmetric Tangential and Radial Distortions

The principal source of asymmetric tangential and radial distortions is the decentering of the lens elements from the optical axis. Where they are decomposed into their x- and y-components, the combined effects of decentering distortions are expressed as

$$(\Delta x)_d = -(a_1 r^2 + a_2 r^4 + a_3 r^6 + \cdots) \sin\theta_0 \qquad (8)$$

and

$$(\Delta y)_d = (a_1 r^2 + a_2 r^4 + a_3 r^6 + \cdots) \cos\theta_0$$

where θ_0 is the angle which the axis of maximum tangential distortion makes with the x-axis of the photo.

Composite Distortion Equations

The composite distortion pattern of the entire TV system can then be obtained from the direct summation of Equations 4, 6, 7 and 8:

$$\begin{aligned}\Delta x &= (\Delta x)_{s,r} + (\Delta x)_{s,t} + (\Delta x)_d + (\Delta x)_0\\ &= x(l_0 + l_1 r^2 + l_2 r^4 + l_3 r^6 + \cdots)\\ &\quad - y(q_1 r + q_2 r^3 + q_3 r^5 + \cdots)\\ &\quad - (a_1 r^2 + a_2 r^4 + a_3 r^6 + \cdots)\sin\theta_0\\ &\quad + (d_0 + d_1 xy + d_2 x^2 - d_3 y)\end{aligned} \qquad (9)$$

$$\begin{aligned}(\Delta Y) &= (\Delta y)_{s,r} + (\Delta y)_{s,t} + (\Delta y)_d + (\Delta y)_0\\ &= y(l_0 + l_1 r^2 + l_2 r^4 + l_3 r^6 + \cdots)\\ &\quad + x(q_1 r + q_2 r^3 + q_3 r^5 + \cdots)\\ &\quad + (a_1 r^2 + a_2 r^4 + a_3 r^6 + \cdots)\cos\theta_0\\ &\quad + (d_4 + d_3 x + d_2 xy + d_1 y^2).\end{aligned} \qquad (10)$$

Equations (9) and (10) take into consideration all the major sources of systematic distortions in a TV system. However, they do not include the distortion effects caused by external magnetic and electromagnetic fields, imperfect synchronization, insufficient fly back time for the scanning beam, line-bowing due to focusing-deflection interaction, nor by the non-linear frequency response of the transmission channels. The occurrence of these distortions follow a random or semi-random nature. They must be minimized by efficient system design and system control during operation.

Geometric Calibration of Television System

One of the chief objectives of calibration is to determine the distortion characteristics of each TV picture so that the images can be restored to their true positions as accurately as possible. Although a major portion of the distortions follows some systematic pattern—

because of the inconsistency in the performance of the electronic components—the actual magnitudes of the systematic distortions do vary with time. Moreover, the random components of distortion are too large to be negligible by the photogrammetric standards. Therefore, in order to achieve maximum geometric accuracy from TV pictures, some sort of geometric control must be provided for each individual picture.

The best solution to the above problem is the use of a reseau grid. Such a reseau grid can be etched onto the target of the camera tube. This grid becomes an integral part of the original picture and goes through the various stages of the TV system together. By comparing the coordinates of the grid intersections on the reproduced picture with those on the target plate, it is then possible to determine the image distortion pattern caused by the entire TV system.

Prephotography calibration involves the determination of three parameters: (1) the equivalent focal length, (2) the positions of the principal point, and (3) the distortion characteristics of the entire TV system. If a reseau grid is available, the third parameter is reduced to the distortion characteristics of the optical lens in front of the TV camera. However, one must also determine the scale factor λ between the reproduced photograph and the image on the target plate. As this scale factor is directly related to the focal length, the same value should be used in all subsequent image corrections using the reseau grid.

Procedures developed for the calibration of a photogrammetric camera can be adopted with little changes to calibrate a TV system. An object field consisting of a number of known control points are televised and the CRT display is photographed under normal operation condition. The unknown calibration parameters can then be determined using observation equations.

RESEAU GRID AVAILABLE

If a reseau grid is available, the observation equations describing the projective relationship between the reproduced photograph and the object field are as follows:

$$\begin{bmatrix} x_a' + (\Delta x)_l - x_p \\ y_a' + (\Delta y)_l - y_p \\ -f \end{bmatrix} = k_a \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} X_A - X^c \\ Y_A - Y^c \\ Z_A - Z^c \end{bmatrix} \tag{11}$$

where

x_a', y_a' = photo coordinates of image point a after corrected for distortions using Equations 9 and 10

x_p, y_p = photo coordinates of principal point

$-f$ = focal length

X_A, Y_A, Z_A = spatial coordinates of ground point A

X^c, Y^c, Z^c = spatial coordinates of exposure center

$m_{ij} = F(\omega, \phi, \kappa)$ where ω, ϕ, and κ are camera rotations

$(\Delta x)_l$, $(\Delta y)_l$ = corrections for image distortions due to the optical lens in front of the TV camera.

The expressions for $(\Delta x)_l$ and $(\Delta y)_l$ can be obtained from Equations 6 and 8.

NO RESEAU GRID AVAILABLE

If no reseau grid is available, then the observation equations to be used for prephotography calibration should be as follows:

$$\begin{bmatrix} x_a + \Delta x - x_p \\ y_a + \Delta y - y_p \\ -f \end{bmatrix} = k_a \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} X_A - X^c \\ Y_A - Y^c \\ Z_A - Z^c \end{bmatrix} \tag{12}$$

where

x_a, y_a = measured image coordinates of image point a

Δx, Δy = corrections for image distortions as expressed by Equations 9 and 10.

CONCLUSIONS

It is evident from this study that geometric distortions introduced in a television system are highly systematic in nature. It is believed that through a careful prephotographic calibration and an efficient use of a reseau grid, relatively high-quality geometric measurements can be made from TV pictures.

Unfortunately, no result of precise measurements of the geometric fidelity of TV pictures can be found in the literature. At the present time, TV manufacturers calibrate geometric fidelity by comparing a televised electron grid with a ball-type linearity chart on the surface of the CRT. From this type of measurement, television systems with image distortion less than 1 to $\frac{1}{2}$ percent of the height of the CRT screen have been reported.

Before the usefulness of television pictures for photogrammetric measurements can be fully evaluated, laboratory experiments must be conducted to establish: (1) the present state of the art of television systems in terms of geometric fidelity; (2) the effectiveness of image correction using a reseau grid; and (3) the effect of television line structure on stereoscopic viewing. Plans are being made to initiate these kinds of experiments by the Department of Civil Engineering at the University of Illinois.

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Errata

In the February 1969 issue, in the article "Tree Heights and Upper Stem Diameters" by Messrs. Ashley and Roger, (1) on page 141, column 2, line 6, in the formula for M_i a minus sign should appear in front of the first parenthesis: $M_i = -((\ldots$ And (2) on page 144, column 1, paragraph 2, line 8, change $\frac{1}{2}$ percent to read "1 to 2 percent".

Mr. Howard Jarmy is the correct spelling of the author's name shown on page 297 of the March 1969 issue, for the article "Prototype Film Transport".
