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Triangulation Test for Topographic Mapping

C&GS and USGS conduct a cooperative operational test of fully analytic aerotriangulation.

(*Abstract on next page*)

INTRODUCTION

THE TEST PROJECT described in this paper is the direct outgrowth of earlier tests conducted by the Coast and Geodetic Survey in the Parsons, Kansas, area[1]. These earlier tests demonstrated that the Coast and Geodetic Survey block method of fully analytical aerotriangulation, when used with small-scale photographs, was capable of determining horizontal positions of points with a root-mean-square error† of about 2 feet for large peripherally-controlled areas. They also demonstrated that the method had potential for establishing elevations of high accuracy.

These results made it desirable to determine whether the same kind of accuracies could be obtained if the method were applied to the somewhat different problem encountered in topographic mapping in the Geological Survey. The essential difference is that the aerotriangulation with the small-scale photographs would be required to provide positions and elevations for pass points of larger-scale photographs that would be used to compile the maps.

To evaluate this capability, an area near Tucumcari, New Mexico, was chosen for a test to be conducted jointly by the Coast and Geodetic Survey and the Geological Survey. Specifically, the project had the objectives (1) of determining whether or not the block aerotriangulation with the small-scale photographs can provide sufficiently accurate horizontal and vertical model-orientation control for the larger-scale mapping photographs, and (2) of providing an insight into the technical and logistic problems associated with the method.

THE TEST PROCEDURE

Briefly, the plan for the test was that a comparatively large area, peripherally defined by existing control, would be aerotriangulated by the Coast and Geodetic Survey, using their own methods and small-scale super-wide-angle photographs, to provide horizontal and vertical model-orientation control for larger-scale mapping photographs. The aerotriangulation was to be constrained to targeted horizontal and vertical control on the perimeter of the project and also to a few vertical control points in the interior of the area.

The solution was to be evaluated in part by comparing photogrammetrically-derived positions and elevations for a pattern of test points with their field-determined values from precise surveys, and in part by compiling and field testing two selected quadrangles. As of this date, the aerotriangulation has been completed, and the compilation of the test quadrangles is underway.

PARAMETERS OF THE TEST PROJECT

CHOICE OF TEST SITE

The site chosen for the test is a 1,200-square-mile rectangular area lying astride the Texas-New Mexico boundary northeast of Tucumcari, New Mexico. This site is almost ideally suited to the purpose, being bounded

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† In this paper *error* is defined as the difference between the field-determined value (considered as the true value) and the photogrammetrically-determined value.

by existing control of nearly adequate density, with terrain and culture compatible with targeting of the stations, and conducive to easy access for field surveys and targeting work. The area is large enough to provide a definitive test of the aerotriangulation method under map-production conditions. This part of the country is favored with good weather from the standpoint of both photography and field work. An additional consideration was the fact that the map production schedule for the area was comparatively low priority. Figure 1 shows the sixteen 7.5-minute quadrangles presently scheduled for mapping and the existing control. The maps will be compiled and published at 1:24,000 scale with a 10-foot contour interval.

ABSTRACT: *The Coast and Geodetic Survey and the Geological Survey were engaged in a cooperative test of the feasibility of using the Coast and Geodetic Survey fully analytical block aerotriangulation system with small-scale photographs to provide horizontal and vertical stereomodel control directly for larger-scale map-compilation photographs. This approach would have application whenever topographic mapping is contemplated for large uncontrolled areas. The test involves the aerotriangulation of a 30-minute-square area defined by existing horizontal and vertical control, using 160,000-scale super-wide-angle photographs that have 60 percent sidelap. The aerotriangulation solution has been completed. Evaluation by use of test points established for the purpose in the interior of the project area shows that root-mean-square errors of 2.1 feet, horizontal, and 1.7 feet, vertical, were achieved.*

CONTROL AND TEST POINTS

The existing high-order control consisted of 12 second-order horizontal stations, 11 of which had known elevations, and one vertical station, bounding the project area except across the south edge. In the interior of the area were four vertical stations. Seven new horizontal stations were established by second-order methods, and their elevations were determined by third-order leveling (see Figure 1). All control points, both the existing and the new, were targeted.

A network of 18 horizontal-and-vertical test points was established in the interior of the project area, two near each interior quadrangle corner. One of each pair was targeted; the other was a carefully selected, discrete, easily-identifiable natural image. The field surveys for the control and the targeting were done by a Geological Survey crew with an observer from the Coast and Geodetic Survey.

TARGETS

All targets were made of unbleached white muslin attached to wooden frames (Figure 2).

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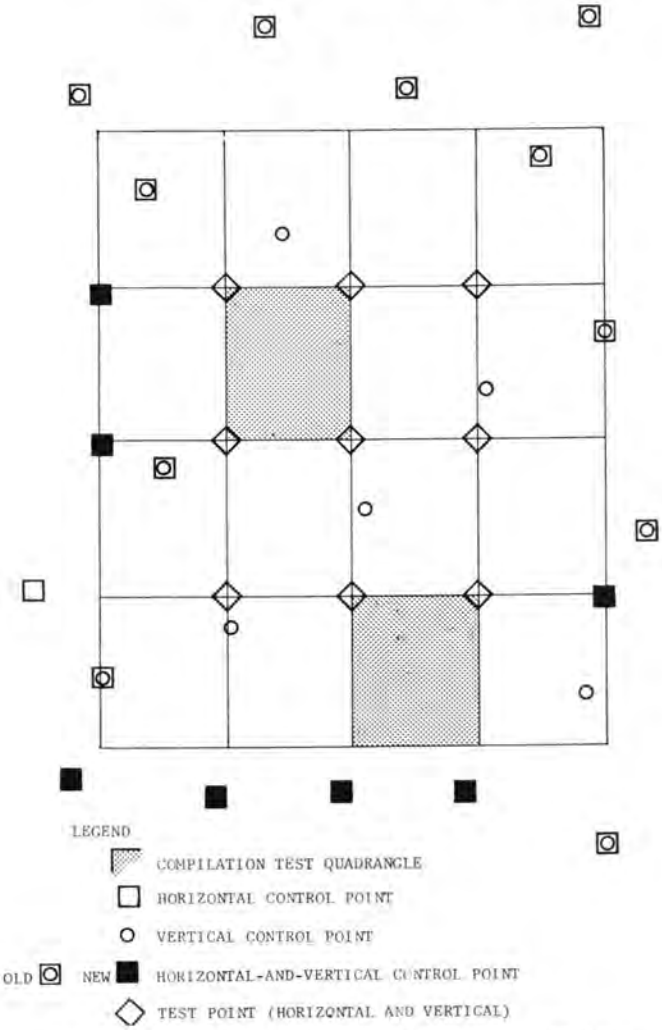

FIG. 1. Control and test points for the Tucumcari Project

They had the form of either an equilateral triangle with three detached wing panels, or a square with four detached wing panels, or, when ground obstructions made it necessary, a triangle with only two collinear wing panels. The central triangles were 18 feet on a side and were centered on the station mark. The central squares were 10 feet on a side. Because they were expected to define the elevations of the stations as well as their horizontal positions, the central panels were required to be level within 0.3 foot. The wing panels were 6-by-30-foot rectangles arranged radially and symmetrically about the central panels and separated from them a distance of 25 feet. The targets were laid just prior to the taking of the photographs.

PHOTOGRAPHIC INPUT

The photographic operations were carried out by a Coast and Geodetic Survey crew, using their own aircraft and cameras. Two sets of photographs were obtained—a small-scale set for use in the aerotriangulation, and a larger-scale set for use in compilation of the topographic maps.

The aerotriangulation set consisted of about 150 super-wide-angle photographs, at a scale of 1:60,000, in 9 north-south flight strips. The photographs have 60-percent side-lap across the strips and the conventional 60-percent overlap along the strips.

The set of compilation photographs consisted of about 520 wide-angle photographs, taken with a 6-inch, 90° camera at 1:20,000 scale, in 14 north-south flight strips having the usual 20-percent sidelap.

It is important to note that the camera used for the aerotriangulation photography for this test as well as for the Kansas test was a standard super-wide-angle, 120-degree camera containing four fiducial marks located in the corners of the format. (Some of the cameras used by the Coast and Geodetic Survey are equipped with eight fiducial marks for the purpose of better correcting for film distortion.) It is also significant that, although research is currently being conducted at the Coast and Geodetic Survey on stellar methods of camera calibration, the corrections for symmetric and asymmetric lens distortion were computed from the manufacturer's calibration certificate only.

TEST QUADRANGLES

It was planned that the results of the aerotriangulation would be evaluated in the usual manner—by determining the errors at control points and test points. It was foreseen, however, that this determination would not provide an altogether conclusive evaluation, partly because of the relatively small number of test points and their comparatively wide spacing, and partly because the errors at such points cannot be directly related to the accuracy of the map that will be compiled using the model control determined by the solution. The accuracy of the map is, after all, the final determinant as to the acceptability of any of the processes used in its creation.

Therefore, two of the sixteen 7.5-minute quadrangles of the project were selected in advance for a compilation test. As shown in Figure 1, the two quadrangles chosen were those farthest removed from control, particularly vertical control, one lying in the comparatively flat northern half of the project and the other lying in the rougher southern half. These two quadrangles are to be compiled by conventional methods and the compilation carefully checked by field methods.

Aerotriangulation Operations

POINT SELECTION AND TRANSFER

Six pass points were chosen and marked on each aerotriangulation plate (glass diapositive having a thickness of one-fourth inch). They were chosen in pairs, one pair near the center of the plate and one pair near the midpoint of each lateral edge. Because the coordinates of the points were to be measured in a stereocomparator, using the plates in pairs, it was not necessary to transfer the points along the flight strips, but they were carefully chosen

Fig. 2. Part of a target array, showing a triangular central target and one of the wing panels.

under stereoscopic observation so that they could be transferred to the adjacent plates. They were transferred to one plate on each adjoining flight strip. With the sidelap of 60 percent, a point may thus occur on as many as 9 plates.

On the mapping photographs for the two test quadrangles, four pass points were selected in each model area. Each of these was marked on only one plate of the strip, and none was transferred to the adjoining strips. However, each pass point was transferred to one small-scale aerotriangulation plate of each strip on which it appeared so that its position and elevation could be determined.

Although it was obviously not necessary to mark or transfer any of the *targeted* control points or test points, the *natural-target* test points were identified and marked on the large-scale plates and transferred to the aerotriangulation plates.

A stereoscope equipped with a 60-micron drill was used for the point transfers. To permit transfers from the 1:20,000-scale mapping photographs to the 1:60,000-scale aerotriangulation photographs, a zoom microscope, adapted for mounting on the transfer device, was used to permit stereoscopic viewing of the two different scales of photography.

COORDINATE MEASUREMENTS

The photocoordinates of the four corner fiducial marks, the pass points, the control points, and both types of test points were measured on the aerotriangulation plates, using a two-plate stereocomparator equipped with a 1-micron readout. Multiple readings were used for all types of points and for the fiducial marks.

PRELIMINARY STRIP SOLUTIONS

As the measurement of coordinates of each strip was completed, the data were processed through a series of preliminary computer programs. These programs have the following functions:

1. Average the coordinate readings and transform the averaged readings to an origin at the principal point.
2. Correct the coordinates for systematic errors, including atmospheric refraction, film distortion, and radial lens distortion.[2]
3. Generate strips, in the coordinate system of the first photo of each strip, by performing a series of three-photo orientations on successive overlapping triplets.[3]
4. Fit the strips individually to ground control or to an adjacent already-adjusted strip, using polynomial equations.[4]
5. Transform the photocoordinates of all points to a local secant-plane coordinate system for the block solution.[5]

The end product of this processing consisted of approximate values of the pass point ground positions and elevations needed as input for the block adjustment.

ERROR DETECTION AND ELIMINATION

Before entering a block solution, it is important to detect and eliminate errors or blunders which will impair the accuracy of the solution. Coordinate readings which differ from the average for a point by more than a specified amount are rejected. The three-photo relative orientation program reveals points with errors larger than are considered acceptable. A comparison of positions of common points, after the strip adjustments, enables detection of poorly transferred or misnumbered strip tie points, as well as errors in control point coordinates, elevations, or identifications. A block edit program is used to detect any remaining errors in the input for the block solution. It is interesting to note that, despite the fact that approximately 8,000 photo points were handled in this project, only about 25 such errors were detected.

WEIGHTING CONSIDERATIONS

The Coast and Geodetic Survey block adjustment program provides for weighting of three types—a resolution weight, a collinearity weight, and a position weight. The *resolution* weight, which depends on the location of the point on the photographic plate, has not been used because the accuracy of image pointing relative to radial distance has not yet been determined. The two remaining types of weighting were used.

The *collinearity* weight is used to enforce the collinearity condition more strongly for horizontal and/or vertical ground control points than for pass points during the least-squares orientation solution. The weighting is applied by multiplying the observation equations for the control points by the selected weight factor. For points so weighted, the plate residuals will tend to be smaller.

The *position* weight is used to constrain the solution to the ground control stations. The larger the weight on a point, the more closely a control point is held. The weight is applied to the appropriate diagonal term of the normal-equation system.

The selection of suitable weights is somewhat arbitrary and is the result of some experimentation. A collinearity weight of 3 and a position weight of 5 gave very acceptable results with the super-wide-angle camera in this test area. These weights were selected on the basis of an indicated need to hold rather rigidly to the framework of ground control inasmuch as the test points were located by traverse from these control points.

In a solution using the minimum of 2 horizontal and 3 vertical control points, the root-mean-square error of the photo residuals was 5.9 microns, which is considered as approaching the least value that can be obtained from the photogrammetric solution under the existing conditions. As a result of the weighting, this increased only to 7.2 microns, which represented 1.4 feet on the ground. The root-mean-square error of fit to horizontal ground control was 1.7 feet and the root-mean-square error of fit to the vertical control was 0.8 foot. In view of this good balance between the fit to control and the internal fit, it did not seem realistic to use weighting that would have held the ground control more closely.

BLOCK SOLUTION

All control available, as shown in Figure 1, was used in the block solutions,[6,7] which were run on the ESSA computer facility. Two runs were made, an erroneous point being detected in the first run. The execution time was just under 25 minutes per run; for the two runs, the total cost was $400.

AEROTRIANGULATION RESULTS

Figures 3 and 4 are graphic representations of the results of the block solution. Shown are the errors at the targeted horizontal and vertical control points, the nine targeted test points, and the nine natural-target test points. A summary of the results is shown in Table 1.

EVALUATION OF AEROTRIANGULATION RESULTS

Considering the conditions of the test—the project size, the density and distribution of control, the photographic scale, and the

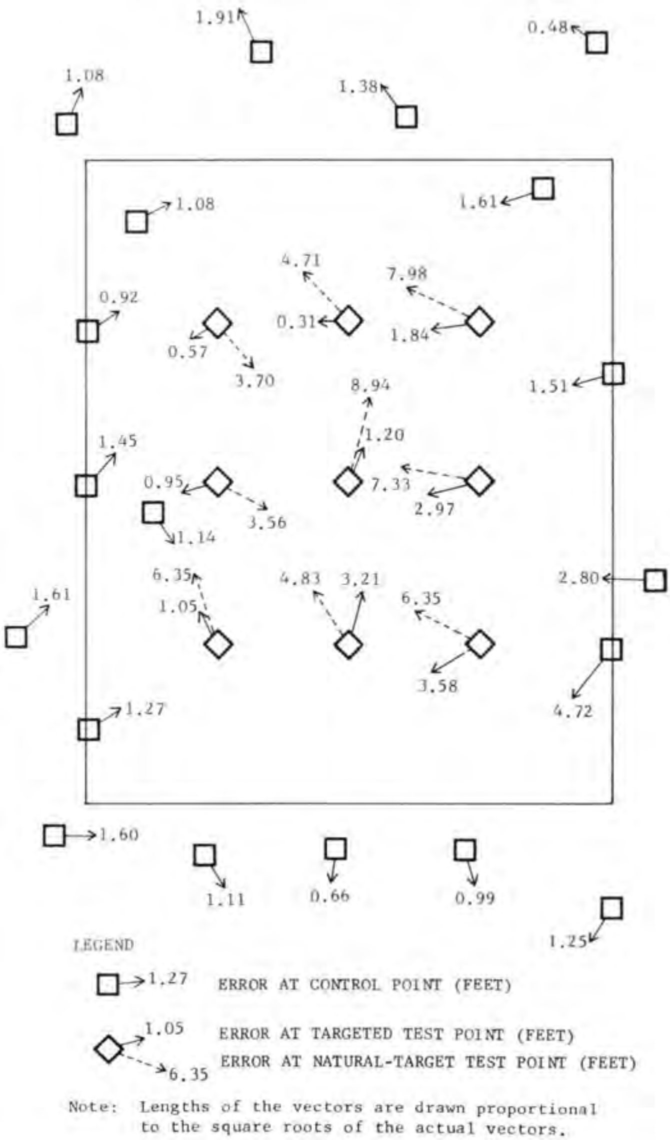

FIG. 3. Horizontal error distribution.

TABLE 1. SUMMARY OF BLOCK RESULTS FOR TUCUMCARI PROJECT

	Errors at Control (Feet)		*Errors at Test Points (Feet)*			
			Targeted Points		*Natural Targets*	
	Horiz.	*Vert.*	*Horiz.*	*Vert.*	*Horiz.*	*Vert.*
RMS Error	1.7	0.8	2.1	1.7	6.2	2.4
Max. Error	4.7	2.2	3.5	3.1	8.9	4.7

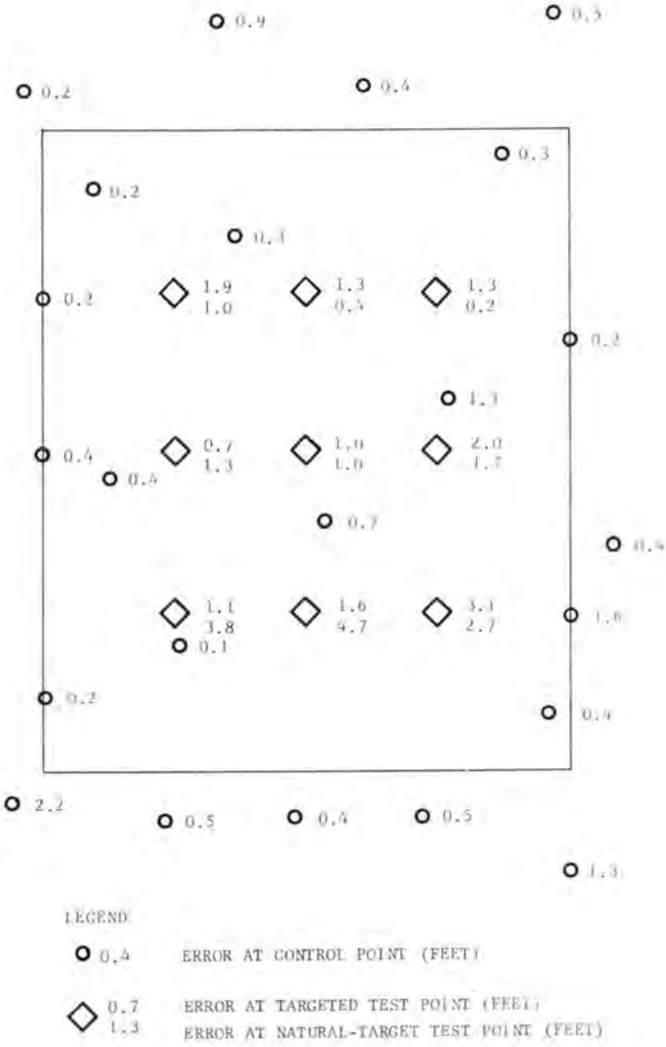

FIG. 4. Vertical error distribution.

fact that super-wide-angle photographs with only four fiducial marks for determining film distortion were used—these seem to be excellent results. Certainly they compare favorably with results that have been reported in the literature for other tests with other methods.

In a practical sense, however, the quality of an aerotriangulation solution can be evaluated only in terms of the requirements of the use to which the resulting data will be put. In this instance, the data will be used in the preparation of maps of the National Topographic Series, the accuracy requirements for which are specified in the National Map Accuracy Standards. Although these standards specify the required accuracies of the maps as finally published, they can be used as a basis for determining, in turn, the accuracies required for the networks of points upon which the maps will be constructed. Quite obviously, these points must have a higher accuracy than the map itself to allow for errors that will be introduced in the compilation operations.

Without going into the details of the derivation, it can be stated that the specifications of the National Map Accuracy Standards, as applied to these particular maps (1:24,000 scale, 10-foot contour interval), are equivalent to a requirement that the root-mean-square horizontal error of well defined, point-size map features shall not exceed 24 feet and that the root-mean-square vertical error of such points shall not exceed 3 feet.

Tests have shown that very little inaccuracy is introduced in horizontal position during compiling operations, so it seems quite safe to say that the horizontal accuracy achieved (2.1 feet, root-mean-square error, at targeted test points and 6.2 feet at natural-target test points) far exceeded requirements. On the other hand, because of the much tighter vertical tolerance, the outcome in this dimension was, as expected much closer to the maximum limit Preliminary results from tests still underway indicate that standard vertical-accuracy requirements can be met when the root-mean-square error at model-orientation points is perhaps as large as 0.25 contour interval, which is equivalent to 2.5 feet in this instance. On this basis, the achieved values of 1.7 feet for the targeted test points and 2.4 feet for the natural targets indicate adequate vertical accuracy. Although, as indicated earlier, this apparent success is subject to confirmation through compilation and field tests yet to be performed, there is every reason to be optimistic about the ultimate results.

Comparison of the horizontal errors at the natural-target test points with those at the nearby targeted test points (Figure 2) confirms the expected superiority of artificial targets.

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2. *Two* copies (the original and first carbon) of the complete manuscript and two sets of illustrations should be submitted. The second set of illustrations need not be prime quality.
3. Each article should include an abstract, which is a *digest* of the article. An abstract should be 100 to 150 words in length.
4. Tables should be designed to fit into a width no more than five inches.
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