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Holography as Viewed by a Photogrammetrist

Holography cannot replace well established techniques of photogrammetry but it can well supplement some of them.

(*Abstract on next page*)

INTRODUCTION

HOLOGRAPHY MAY BE defined as the technique by which electromagnetic wave-fronts (light), scattered from an object, are recorded on a photographic plate, called a *hologram* and, on subsequent replay of the recorded hologram, wave-fronts are reconstructed to reproduce the three dimensional image of the recorded object. The technique of holography is also known as the *wave-front reconstruction photography*. The word hologram has its origin from the Greek words for *whole* and *writing*.

The essential elements of the theory of holography were worked out in 1948 by Dennis Gabor[4] of the Imperial College of Science and Technology, London. The invention of the laser in 1960 made holography practicable and since then many new advances and applications have been reported.

A wave-front scattered from an object is completely recorded if both its *phase* and *amplitude* information are obtained. Amplitude is recorded as a variation in the transmittance of the photographic emulsion; but the phase information is not easily recorded. The phase information can be recorded by standard techniques of Interferometry.

INTERFEROMETRY IN HOLOGRAPHY

The basic requirement for interference of electro-magnetic disturbances is *coherence*, i.e., constant phase relationship. Incoherent disturbances do not interfere.

Now consider a coherent-light beam divided into two parts (Figure 1). The first part illuminates the subject to be recorded, and on reflection gets modulated both in amplitude and phase. Thus, this part of the coherent beam acts as a carrier for the signal from the object. The second part acts as a reference for comparison of phase and amplitude of the

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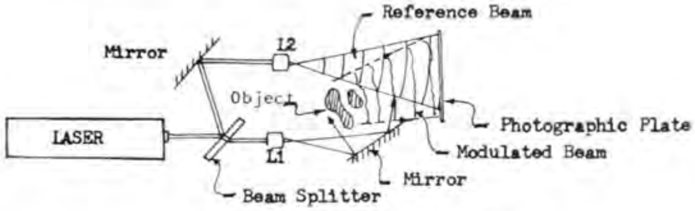

FIG. 1. Construction arrangement.

modulated beam. The superimposition of the two beams results in an interference pattern, which is capable of being recorded on the photographic emulsion.

COHERENT LIGHT SOURCE

One of the requirements of coherence is monochromaticity. No light source emits purely monochromatic light. The frequency of light emitted fluctuates with time. Also, the source emits light intermittently, sending out trains of waves, which do not have fixed phase relationship with each other. A wave-train may be considered coherent over its average length, which is, therefore, called *coherence length.*

ABSTRACT: *In this paper an attempt is made to present holography from the point of view of a photogrammetrist. First, the underlying theory, practical limitations, typical features, and general applications are dealt with; and, secondly, some of the important properties of holography and the directions for future research, which may be of interest to a photogrammetrist, are brought out.*

Further, each source consists of an infinite number of point sources, each of which emits trains of waves, which do not maintain a fixed phase relationship with each other or with themselves over a period of time. It is, therefore, essential to have a point source for coherent light.

Today, of course, there are *lasers* capable of producing coherent light beams which are highly monochromatic and spatially coherent. Many lasers are coherent over 10^{10} wavelengths and monochromatic to within 2 cycles per second.

THEORETICAL FOUNDATIONS OF CONSTRUCTION AND RECONSTRUCTION STAGES

Strictly speaking, a rigorous electro-magnetic theory of scattering, diffraction and polarization is required for an exact treatment of holography. However, approximations in physical and geometrical optics are generally found to be sufficient.[9]

RECORDING PROCESS (Construction)

In the construction stage an hologram is obtained by super-imposition of a coherent reference beam and a signal-bearing wave, scattered from an object, onto a photographic plate (Figure 1).

MATHEMATICAL TREATMENT OF THE RECORDING PROCESS

The failure to record phase only by a single electro-magnetic field (scattered from the object) can be explained rather simply. For simplicity only one transverse dimension X is considered on the recording plane (Figure 2). The electric field $E(x)$ at any point, distant x from O is:

$$E(x) = A(x) \cdot \cos(\omega t + \phi(x)) \tag{1}$$

where,

$A(x)$—amplitude,
ω—angular velocity,
t—time,
$\phi(x)$—phase of the electro-magnetic field.

The time-averaged intensity is given by:

$$\begin{aligned}\langle E^2(x)\rangle &= \langle A^2(x)\cos^2(\omega t + \phi(x))\rangle \\ &= (1/2)A^2(x).\end{aligned} \tag{2}$$

In Equation 2 the phase information $\phi(x)$ at the point on the recording plane is completely lost.

The keynote of holography is interference between two electromagnetic fields: $E_o(x)$, the field scattered from the object, and $E_f(x)$, the field from the reference beam. The time-averaged intensity is, therefore

$$\begin{aligned}\langle E^2_{(\text{tot})}\rangle &= \langle [E_0(x) + E_f(x)]^2\rangle \\ &= A_0^2(x)/2 + A_f^2(x)/2 + [A_0(x) \times A_f(x) \times \cos(\phi_0(x) - \phi_f(x))].\end{aligned} \tag{3}$$

If $A_f(x) \gg A_o(x)$, then:

$$\langle E^2_{(\text{tot})}\rangle = 1/2 A_f^2(x)[1 + \{2A_0(x)/A_f(x)\}\cos(\phi_0(x) - \phi_f(x))]$$

or

$$\langle E^2_{(\text{tot})}\rangle = \langle E_f^2\rangle[1 + \epsilon] \tag{4}$$

where

$$\epsilon = \{2\cdot A_0(x)/A_f(x)\}\cos(\phi_0(x) - \phi_f(x)). \tag{5}$$

Now, the exposure

$$\begin{aligned}\text{EXP} &= \text{Power} \times \text{Time} \\ &= \text{const.} \times \langle E^2_{(\text{tot})}\rangle\end{aligned} \tag{6}$$

$$\begin{aligned}&= k\langle E_f^2\rangle[1 + \epsilon] \\ &= \text{EXP}_f[1 + \epsilon].\end{aligned} \tag{7}$$

or,

$$\begin{aligned}\log(\text{EXP}) &= \log(\text{EXP}_f) + \log(1 + \epsilon) \\ &\cong \log(\text{EXP}_f) + \epsilon.\end{aligned} \tag{8}$$

From the characteristic curve (Figure 3) of the photographic emulsion, we see that ϵ is the modulation of exposure and ΔD is the corresponding density modulation at the point under consideration. The amount of amplitude modulation is $2A_o(x)/A_f(x)$ and phase modulation is $[\phi_o(x) - \phi_f(x)]$. If $A_o(x) = 0$, there is no modulation at all. The strength of modulation depends on $A_o(x)$.

Thus, by the principle of interferometry, both the phase and amplitude information of the scattered wave-front is captured.

RECONSTRUCTION PROCESS

To obtain an exact reconstruction of the original wave-fronts that were scattered from the recorded object, we take the hologram and put it back in its original place

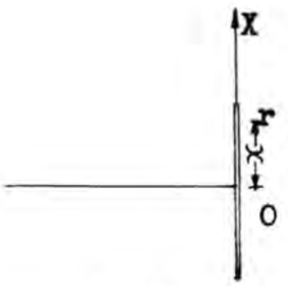

FIG. 2. Recording plane.

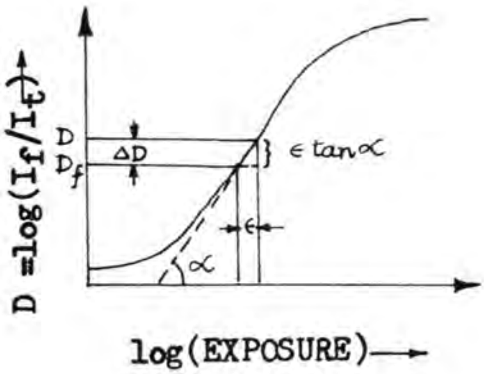

FIG. 3. Characteristic curve.

in the arrangement for recording (Figure 1) and illuminate it with the original reference beam. In other words, we keep the original geometry but eliminate the object.

MATHEMATICAL TREATMENT OF THE RECONSTRUCTION PROCESS

Considering again the transverse X-direction on the recording plane. Let E_f be the field at a point due to the reconstruction beam (Figure 4) and D the density at that point. Now,

$$D = \log (I_f/I_t) \tag{9}$$

where,

I_f—intensity of reconstruction beam,
I_t—intensity of transmitted beam.

Therefore,

$$I_t = I_f \times e^{-D} \tag{10}$$

$$E_t = E_f \times e^{-D/2}. \tag{11}$$

From the characteristic curve (Figure 3) of the photographic emulsion,

$$\begin{aligned} D &= D_f + \Delta D \\ &= D_f + \gamma\epsilon \end{aligned} \tag{12}$$

where,

D—density at a particular exposure,
D_f—density at reference exposure,
γ—tan α, where α is the slope of the characteristic curve.

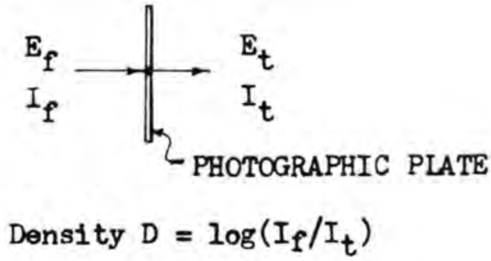

FIG. 4. Density defined.

Substituting the value of D from Equation 12 into Equation 11,

$$E_t \cong e^{-D_f/2} E_f (1 - \epsilon\gamma/2) \tag{13}$$

$$E_t = E_f \tau \tag{14}$$

where

$$\begin{aligned} \tau &= e^{-D_f/2}(1 - \epsilon\gamma/2) \\ &= \tau_0(1 - \epsilon\gamma/2) \end{aligned} \tag{15}$$

and where

$$\tau_0 = e^{-D_f/2}$$

τ = amplitude transmission.

After some substitutions and manipulations,

$$\begin{aligned} E_t = \tau_0 E_f &- [\gamma\tau_0 A_0(x)/2] \cos(\omega t + \phi_0(x)) \\ &- [\gamma\tau_0 A_0(x)/2] \cos(\omega t + 2\phi_f(x) - \phi_0(x)) \end{aligned} \tag{16}$$

Equation 16 is known as the *Hologram Equation* after Gabor.[4] The first term is the attenuated reference beam, transmitted through the plate. The second term is proportional to the original electromagnetic field $E_o(x)$, scattered from the object, recorded on the hologram. And the third term gives rise to a false image of the original object, as will be shown in the next section.

Mathematical Treatment of Holography Considering General Arrangement of Construction and Reconstruction

In this generalized treatment the following is considered: the object to be recorded is a point object P_o and the point source for reconstruction P_c is located at a different place than the reference point source for construction P_f (Figure 5). Further, the coherent light sources used in construction and reconstruction have different wavelengths, λ and λ', respectively. Also, from Figure 6, the phase difference between the fields at point O and at a point, distant x from O, due to an off-axial point source P is given by:

$$\phi(x) - \phi(o) = [-2\pi/\lambda](x^2/2D + \alpha x) \tag{17}$$

where $|Xs/D| \ll 1$ and $\alpha = \sin\theta$.

If at any stage we can show that the phase difference at a point r and reference

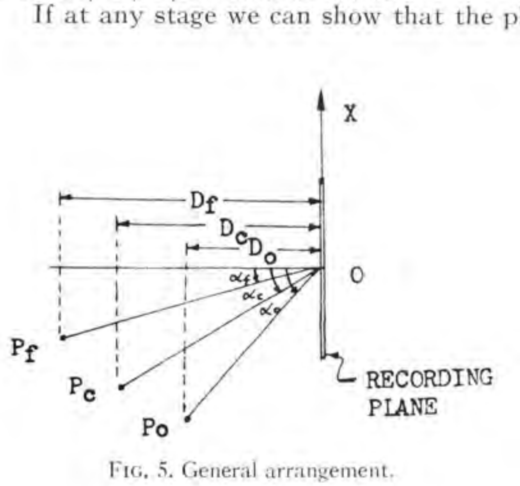

FIG. 5. General arrangement.

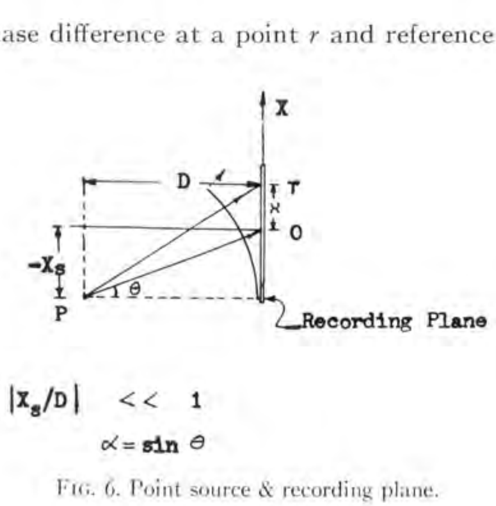

FIG. 6. Point source & recording plane.

point O is given by an expression similar to Equation 17, then this will imply that the reconstructed point is at a distance D and at an angle α from the reference point and the axis (Figure 5). It is assumed that the hologram has already been constructed. In the reconstruction let the incident field of reconstruction E_c be incident at a point on the hologram. Thus, from Equation 14, the transmitted field is:

$$E_t = \tau E_c.$$

Substituting the value of τ from Equation 15 in the above equation and, after some trigonometric manipulations, we get:

$$E_t = \tau_0 E_c - \left[\frac{\gamma}{2} A_0(x)/A_f(x)\right] \times A_c(x) \times \cos(\omega' t + \phi'(x)) - \left[\frac{\gamma}{2} A_0(x)/A_f(x)\right] \times A_c(x) \times \cos(\omega' t + \phi''(x)) \tag{18}$$

where

$$\phi'(x) = \phi_0(x) - \phi_f(x) + \phi_c(x) \tag{18a}$$

$$\phi''(x) = \phi_c(x) - \phi_0(x) + \phi_f(x). \tag{18b}$$

Let us consider Equation 18. The first term is the attenuated field of the incident field. The second term has phase $\phi'(x)$ which can be written as in Equation 18a. Using Equation 17 and simplifying, we get:

$$\phi'(x) = [-2\pi/\lambda'](x^2/2D + \alpha' x) + \text{const.} \tag{19}$$

where,

$$1/D' = (\lambda'/\lambda)(1/D_0 - 1/D_f) + 1/D_c \tag{19a}$$

$$1/\alpha' = (\lambda'/\lambda)(\alpha_0 - \alpha_f) + \alpha_c. \tag{19b}$$

From Equation 19 it can be deduced that an image P' is formed at an angle α' and at a distance D' from the reference point O. (Figure 7).

It is interesting to note that if the object point P_o is moved laterally a distance $\Delta x_o \approx \Delta\alpha_o D_o$ (Figure 7), then there is a corresponding movement of the image P' by a distance $\Delta x' \approx \Delta\alpha' D'$. This gives an expression for transverse magnification to first order in α and x/D:

$$m_1' = \Delta x'/\Delta x_0 = (\Delta\alpha'/\Delta\alpha_0)(D'/D_0) \tag{20a}$$

From Equation 19b, on differentiation, we get $\Delta\alpha'/\Delta\alpha_o = \lambda'/\lambda$ so that

$$m_1' = [\lambda'/\lambda](D'/D_0). \tag{20b}$$

Similarly, the longitudinal magnification is given by

$$m_2' = \Delta D'/\Delta D_0 = [\lambda'/\lambda](D'^2/D_0^2) \tag{21}$$

A similar discussion holds good for the third term of Equation 18 in which the phase

$$\phi'' = \text{const.} - [2\pi/\lambda'](x^2/(2D'') + \alpha'' x) \tag{22}$$

where,

$$1/D'' = 1/D_c + [\lambda'/\lambda](1/D_f - 1/D_0) \tag{22a}$$

$$\alpha'' = \alpha_c + [\lambda'/\lambda](\alpha_f - \alpha_0). \tag{22b}$$

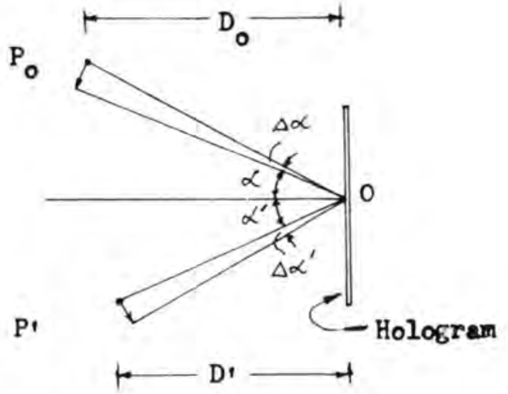

FIG. 7. Lateral magnification.

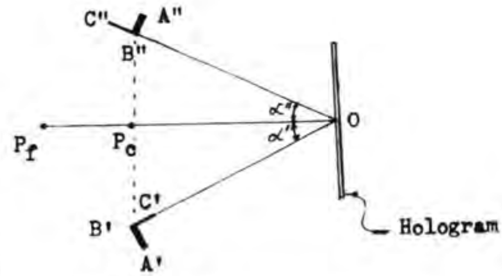

FIG. 8. Special arrangement: Fourier transform holography.

From Equation 22 it can be deduced that an image P'' is formed at an angle α'' and distance D'' from the reference point O. In this case the angular orientation tends to be opposite to that of P'' because, if both $\alpha_f = \alpha_c = 0$, we have $\alpha'' = -\alpha_o[\lambda'/\lambda]$, and from the second term for the image P', $\alpha' = \alpha_o[\lambda'/\lambda]$.

Also the transverse magnification is given by

$$m_1'' = -(\lambda'/\lambda)(D''/D_0) \tag{23}$$

and longitudinal magnification by

$$m_2'' = -(\lambda'/\lambda)(D''^2/D_0^2). \tag{24}$$

PARTICULAR GEOMETRICS

Consider an important special case of Fourier Transform holograph (Figure 8). The point object and the reference point source are equidistant from the recording plane, so that $D_o = D_f$. From Equations 19a and 22a we get $D' = D'' = D_c$. Further, if $\lambda = \lambda'$ and $\alpha_f = \alpha_c = 0$, then from Equations 19b and 22b we get $\alpha' = \alpha_o$ and $\alpha'' = -\alpha_o$. Thus, if we consider the images of three points A, B, C (Figure 8), the double primed image has an incorrect perspective. This is the characteristic of the image from the third term in Equation 18, which image is also called *false image*.

SOME PRACTICAL DIFFICULTIES AND LIMITATIONS

In the recording process besides the requirement of a coherent source of light the whole of the equipment and the object should remain stationary during the period of exposure. This requires a firm foundation (preferably a large stone optical bench) for the apparatus and the object in order to achieve seismically quiet surroundings. Even changes in humidity and temperature during the exposure time can destroy the fringe pattern being photographed. To obtain satisfactory conditions it is desirable to reduce the time of exposure. This can be done by high intensity pulsed lasers. Stronger pulsed laser-beams and high speed emulsion, having high resolution, would therefore be desirable. At present Kodak 649 plates are standard for photographing fringe pattern but the emulsion is of very low speed (ASA 0.5).

. . . because of these restrictions, it is uncertain at the present time how practical it will be to use holography to record images of common objects.[8]

SOME TYPICAL FEATURES OF HOLOGRAPHY

The most striking general characteristic of holography is the three-dimensional presentation without any focusing optical system or viewers for the observer. Also, the observer sees an image as though he is looking at the real object. He perceives all the parallax that is present in the original scene. That is, he can move around and change his viewpoint to see objects that would otherwise be hidden from view. Thus, there is very little masking effect in the image. The observer has to refocus his eyes for observing far and near points of the image. Thus, there is unlimited depth of focus in the reconstruction process. For all practical purposes the light rays that strike the eyes of the observer are the same as those that come from the original scene.

One of the most exploited features of holography is its tremendous capability of giving very high magnification. This magnification can be as high as 10^6 utilizing both the wavelength and geometrical factors.*

Each fragment of the hologram, no matter how small, can produce the entire image. This phenomena is explained by the fact that each point on the hologram receives light from all the parts of the object and each image point is affected by the whole of the hologram.

The reconstructed image has very nearly the same contrast as the original object regardless of the contrast properties of the photographic emulsion. This property is not easily explained,[5] but it is related to the use of reference beam and also the fact that each point of the object is recorded on the entire hologram.

Another curious property of the hologram is that it does not produce negatives. If the hologram were copied by contact printing, the hologram would be reversed in the sense that opaque areas would become transparent and vice-versa. Physically, it can be viewed by recalling that the information on the hologram is embodied by the fringe pattern, viz., fringe contrast and fringe spacings; neither of these is altered by contact printing.

Still another interesting property of the hologram is that several images can be superimposed on a single plate on successive exposures. Each image can be recovered without being affected by the other images.

Some Applications of Holography

REPORTED APPLICATIONS

Microscopy. One area where holography has been successfully applied is microscopy. A magnification of 10^6 can be attained. However, the limiting factor is the emulsion grain size. The advantage of unlimited depth of focus and the ability to look around obstructing object-images makes it specially suited to the study of biological objects in their natural surroundings without first having to make samples.

X-ray microscopy. The added advantage of not requiring any elaborate optical system for construction and reconstruction stages has been used in x-ray microscopy because no optics have thus far been designed to focus completely the x-rays without loss of resolution. The use of monochromatic light in the visible spectrum for reconstruction gives magnification in a simple way.

Interferometry. There are three areas of interferometry where holography has been reported to have many applications: Real-time interferometry, time-lapse interferometry and time-averaged interferometry.

In real-time interferometry changes are observed in the subject as they occur. If the hologram and the subject are returned to precisely the same relative position as they occupied during the original exposure and both of these illuminated by the same arrangement of direct and reference beams, the wave-front from the subject will superimpose the wave-front from the hologram. These interfere and give rise to interference fringe pattern, which is a measure of alterations that have taken place in the subject.

In time-lapse interferometry the situation before and after a change is recorded on

* See Equations 20, 21, 23 and 24.

the same hologram. On reconstruction the wavefronts for the two images interfere and give rise to fringe pattern, which indicates the changes in the subject.

In time-averaged interferometry the object's motion is recorded by long exposures. It is primarily used to study a subject in rapid vibrations. The time-averaged hologram is equivalent to a double exposed hologram in which the subject is recorded at the two extreme positions of its vibrations. The interference pattern of reconstruction provides a measure of the amplitude of vibration of the subject.

The rigorous analysis of all the above techniques of interferometry is somewhat complex, but the applications and the general conclusions are relatively simple.[7]

APPLICATIONS UNDER INVESTIGATION

Holographic contouring. Another important application of holography is contouring. In the *Metalworks News* of January 15, 1968, it was reported that the Auto Big 3 were increasing the study of uses for laser holography for body design and production. By using a two-wavelength coherent source (laser), contour lines can be implanted on the holographic image of a clay model or any other object.

Volume holograms and reconstruction by white light. If during the construction of a hologram the angle between the reference and modulated (scattered from an object) wave-fronts is made large, the finest details of the interference pattern become so small that they have dimensions close to the wavelength of light. In most instances this is far smaller than the thickness of the photographic emulsion. The interference pattern is thus recorded in what we call a *volume hologram*, which in effect is a three-dimensional grating. These volume holograms are both angle-selective and color-selective, determined by the Bragg's reflection condition. The thicker the volume hologram, the narrower the spectral band of light that is diffracted at a given angle of illumination. A volume hologram is, therefore, capable of selecting a narrow band of wavelengths from a white light beam and yielding a good reconstruction of the image. This property is being made use of in investigations with respect to Holosigns[3] for traffic control.

Discussions and Concluding Remarks

Some salient points of difference between holography and conventional photography (including stereo-pairs) are compared:

Holography	Photography
• Phase and amplitude of the scattered wavefront from an object are recorded.	• Light intensity is recorded as a variation in density of emulsion.
• No lens or any other image forming apparatus is used between the object and the photographic plate. Each point on the photographic plate receives light from the entire object.	• Lenses and other image forming apparatus are used for photographing the object.
• No stereo-viewers are used to view the 3 dimensional image reconstructed.	• Stereo-viewers of some sort are needed to view the 3 dimensional image produced by stereo-pairs.
• Fringe pattern is imaged which has no resemblance to the original scene.	• Images of objects can be interpreted. This is a big advantage.
• Each point on the Hologram during recording is affected by all the object points; and each reconstructed image point is affected by every point on the Hologram. This makes the analysis by analytical methods very complex.	• There is a one to one correspondence between the object and its image. Thus analysis by analytical methods is easily made possible.

• Coherent light source is necessary for construction and reconstruction processes.	• There is no such requirement.
• Conditions of stability of the object and the recording apparatus during the time of exposure is a must.	• There is no such stringent requirement, except that image motion effects should not exceed a certain limit.
• The reconstructed image has feature of realism. There is very little masking effect.	• A three dimensional reproduction is possible, but there is masking effect, depending on the topography.
• Unlimited depth of focus.	• Limited depth of focus.
• Extreme magnification is possible.	• Limited magnification.
• Theoretically speaking, the reconstructed image has high metrical qualities, provided the original Hologram is used and under similar conditions as at the time of construction. Unevenness of emulsion and plate are accounted for on the basis similar to the Porro-Koppe principle. However, the metrical quality of image from duplicates is very doubtful.*	• The lens distortion and other factors causing systematic error can be accounted for while forming a model, analogically or analytically.
• The use of Holography is restricted to the laboratory for some time to come.	• The use of photography is more universal.

DIRECTIONS FOR FUTURE INVESTIGATIONS

The likely direction for future research is obviously making holography more universal in its applications. At present there are several practical limitations and restraints. The coherence and stability requirements are very stringent. To off-set these requirements, to some extent, the exposure time should be made as short as possible. This necessitates high-speed emulsions and high-intensity pulsed Lasers. High-speed emulsions have low resolution. The combination of high speed and high resolution needed for superior holograms is not yet attained to the degree which would make holography universal in its applications. Much research is needed in this direction.

Another important aspect for research is to investigate the metrical properties of the reconstructed image, especially if one plans to use contact prints of holograms.

Instrumentation for obtaining contoured maps of objects by means of holography is yet to be designed, manufactured, and tested.

CONCLUDING REMARKS

The various aspects of the technique of holography have not yet been fully investigated. However, there are very definite advantages and limitations which will restrict the use of holography to specific problems. Further, it can be said that holography cannot replace other well established techniques of photogrammetry, but rather supplement them.

A final remark of interest to the photogrammetrist:

. . . it seems safe to predict that most future applications will centre on the three dimensional, highly realistic imagery that the method produces.[5]

BIBLIOGRAPHY

1. Born and Wolf, *Principles of Optics*, Macmillan, 1959.

* In the opinion of the author, during contact printing of a Hologram, a fringe pattern caused by phenomena similar to Newton's rings would distort the fringe pattern on the duplicate contact-print of the Hologram.

2. DeVelis, J. B. and Reynold, G., *Theory and Applications of Holography*, Addison-Wesley Publishing Co., 1967.
3. Foster, H., "Holosigns", *Traffic Engineering*, April 1968.
4. Gabor, D., "A New Microscopic Principle", *Nature*, Vol. #161, May 1948.
5. Leith, E. N. and Upatnieks, J., "Photography by Laser", *Scientific American*, Vol. #212, June 1965.
6. Leith, E. N. and Upatnieks, J., "Microscopy by Wavefront Reconstruction", *Journal of Optical Society of America*, Vol. #55, No. 8, August 1965.
7. Pennington, K. S., "Advances in Holography", *Scientific American*, Vol. #218, February 1968.
8. Stroke, G. W., "Lensless Photography", *International Science and Technology*, May 1965.
9. Stroke, G. W., *An Introduction to Coherent Optics and Holography*, Academic Press, New York, London, 1966.