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"Lincap" for Length Measurement

A novel linear capacitor system has been developed to measure length over a range of 25 cm with an accuracy of better than one micrometer

(*Abstract on next page*)

INTRODUCTION

IN MOST PHOTOGRAMMETRIC instruments, the coordinates of a point on the photographic plate are measured by means of precise screws. The screw, in addition to having a limited accuracy, is subject to wear and is slow in operation.

In the search for ways which would overcome the limitations of the screw, consideration has been given to electrical methods based on the measurement of capacitance, where the latter is a linear function of displacement.[1] Developments in this direction published in the literature did not lead to accurate measuring systems. This was caused by the fact that the capacitance to be measured was a first-order function of all three dimensions of the capacitor, which were not stable enough to permit precise repeatability at all times when one dimension is varied in the process of measurement.

In the *Lincap*, a special capacitor is used[2] whose capacitance is a first-order function of the dimension which is measured, but is invariant to the first-order with the remaining two dimensions. In addition, the system has been designed to measure the ratio of capacitance and not the absolute value, thus eliminating the influence of electrical disturbances and humidity changes. It is also possible to match the temperature coefficient of the device to that of the object to be measured. The *Lincap* is noncontacting and therefore a fast system. It measures the absolute values of the coordinates and the reading is recoverable after power discontinuity.

* Presented at the Annual Convention of the American Society of Photogrammetry, Washington D. C., March 1968.

DESCRIPTION

PROPERTIES OF THE FOUR-CYLINDER SYSTEM AS A LENGTH MEASURING DEVICE

The device consists of two sets of four cylinders, insulated electrically, and a movable plate which shields a part of the cylinder cross-capacitance of the first set and a complementary part of the second as shown in Figure 1. The vertical plate or shield is connected to the carriage on which the object to be measured, such as a photographic transparency, is mounted. In the practical realization of the device, four quartz cylinders of twice the length have been used and a metallic coating has been deposited on them to form two electrical sets of four cylinders.

The first set forms one arm of a bridge (see

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Figure 2) and the second set the other arm. Two cross-capacitance measurements are taken, by balancing the bridge with a precision ratio transformer; the first with the cylinder pairs a–b active in both sets and the cylinder pairs c–d at ground potential and the second measurement with the potentials interchanged. The mean value of the readings at bridge balance is not only invariant to the first-order with the cross-sectional dimensions of the two cylinder sets, but it is also a precise linear function of shield displacement.

The above will be demonstrated briefly. Assume a small dimensional error $\delta \ll 1$ from ideal symmetry of the two cylinder pairs a–b and c–d of the first set, and an error $\epsilon \ll 1$ in the second set. It can be shown that the two cross-capacitance values of the first cylinder set are then given by

$$C_{1a-b} = X(C - a\delta + b\delta^2 + \cdots) \tag{1}$$

$$C_{1c-d} = X(C + a\delta + c\delta^2 + \cdots), \tag{2}$$

and the two values of the second cylinder pair by

$$C_{2a-b} = (1 - X)(C - a\epsilon + b\epsilon^2 + \cdots) \tag{3}$$

$$C_{2c-d} = (1 - X)(C + a\epsilon + c\epsilon^2 + \cdots) \tag{4}$$

where X is the displacement of the shield, C is the cylinder cross-capacitance per unit length when $\epsilon = \delta = 0$

$$\left(C = \frac{\ln 2}{4\pi^2}\ \text{cm/cm or } 19.54 \cdot 10^{-15}\ \text{farad/cm}\right)$$

a, b, c are constants.

The readings at bridge balance for the two measurements are given by

$$R_{a-b} = \frac{C_{1a-b}}{C_{1a-b} + C_{2a-b}} \tag{5}$$

$$R_{c-d} = \frac{C_{1c-d}}{C_{1c-d} + C_{2c-d}}. \tag{6}$$

The expressions given by Equations 1, 2, 3 and 4 are substituted in Equations 5 and 6. When the mean value of the latter is taken, it is

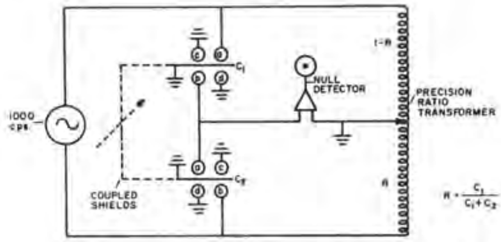

FIG. 2. A bridge circuit in which the ratio of the cylinder crosscapacitances C_1 and C_2 can be determined with a precision ratio transformer.

ABSTRACT: *A novel linear capacitor system has been developed to measure length or displacement over a range of 25 cm with a nominal accuracy of better than one micron. The reading of position can be obtained either with an automatic bridge which gives rapid digital display or by manual balance. The readings are recoverable after power failure. Applications of the* Lincap *are expected in photogrammetry, machine tools, and many research areas such as spectroscopy, microscopy, astronomy, nuclear physics, etc.*

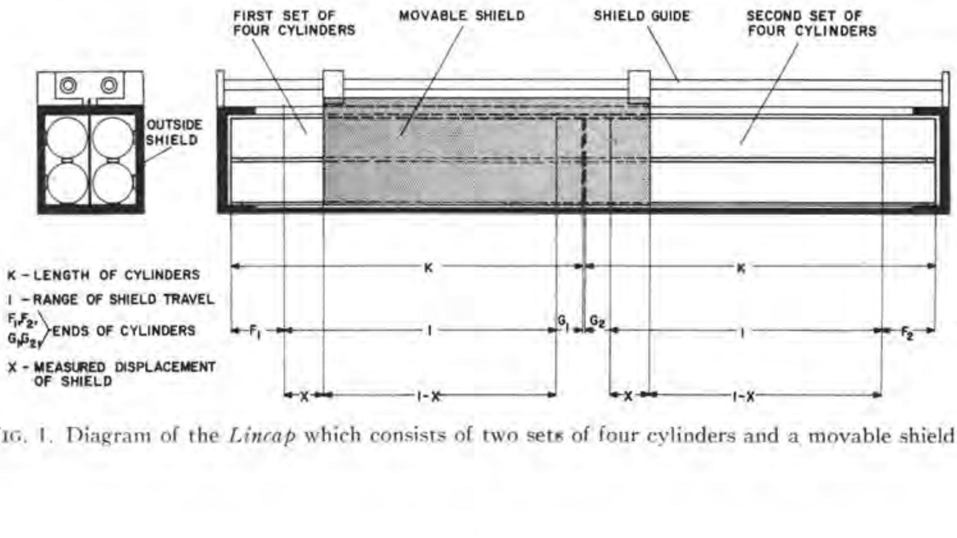

FIG. 1. Diagram of the *Lincap* which consists of two sets of four cylinders and a movable shield.

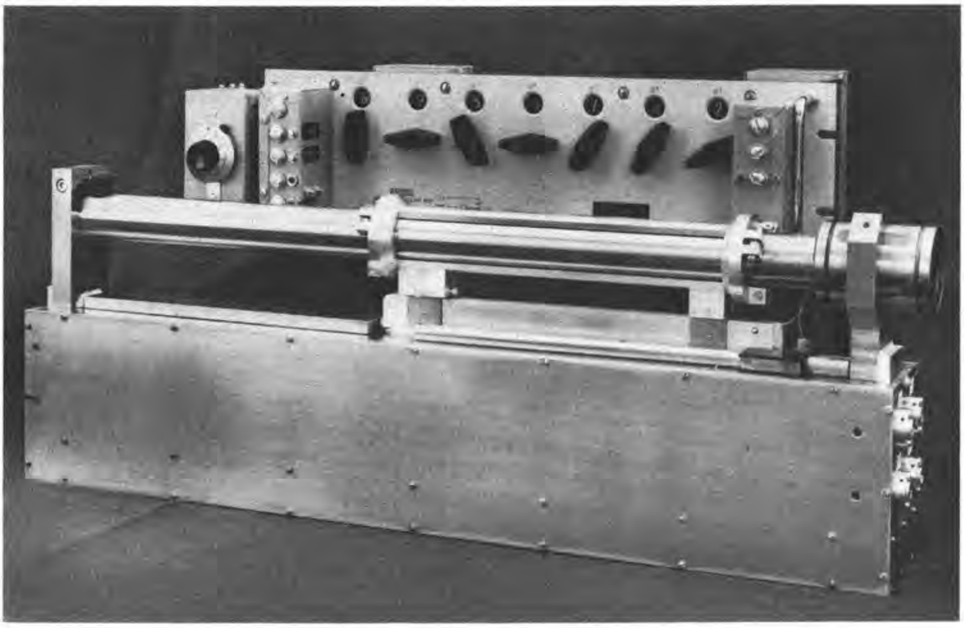

FIG. 3. A photograph of the experimental unit and the manually balanced bridge with a seven-digit display.

found that all first-order terms cancel out and the mean value is equal to the shield displacement X, influenced only by higher order terms:

$$\frac{R_{a-b} + R_{c-d}}{2} = X(1 + p\delta^2 + q\epsilon^2 + \cdots), \quad (7)$$

$p, q =$ constants.

The fact that the mean value of the reading is not affected by first-order errors has an important significance. For example, dimensional errors introduced by limited machining precision, imperfect assembly of the device, subsequent relaxation of stresses and external disturbances, have little influence on the accuracy of measurement. To take advantage of this property two measurements are taken, the second with the potentials on the cylinders interchanged. The mean value of the two measurements is then a precise measure of position.

These measurements can be performed and displayed in less than one second in an automatic bridge which has an accuracy of balance of one part in 10^6. For a 25-cm system, this represents an accuracy of 0.25 μm. In the manually balanced system used, the accuracy is three parts in 10^7, which is less than 0.1 μm.

A prototype unit of the *Lincap* with a manually balanced bridge is shown in Figure 3.

THE MOVABLE SHIELD

In order to take full advantage of the properties of the four cylinder capacitor, described above it is important that the second element of the *Lincap*—the movable shield—does not introduce errors. It has been found that lateral deviations of the shield with respect to the cylinders have little influence on the reading, but rotational deviations in the plane of the shield have a first order effect. Such errors can be introduced, for example, by imperfections in straightness and surface quality of the guides on which the shield is suspended. The relationship between the vertical error δ and the resulting error E in measurement is approximately given by

$$E = \frac{b}{2a}\delta \quad (8)$$

where

a = shield length
b = effective shield height, about equal to the spacing between the opposite cylinders.

For example, for $\delta = 10^{-3}$ cm and

$a = 25$ cm,
$b = 2$ cm
$E = 0.4{:}10^{-4}$ cm
$= 0.4$ μm

A considerable improvement can be achieved by suspending the shield on two external guides which are placed at the level of the middle of the shield. It can be shown that in such a case, the error E is not a first-order, but a higher-order, function of δ and is equal to approximately

$$E = \frac{b}{2a}\delta^2 \qquad (9)$$

The corresponding error E for $\delta = 10^{-3}$cm is then 4.10^{-8} cm or .0004 μm. When using such a design, the effect of the vertical error δ will then be eliminated.

THE TEMPERATURE COEFFICIENT OF THE "LINCAP"

In making precise measurements, it is always desirable to have the temperature coefficient of the instrument matched to that of the object to be measured. If this is not the case, the temperature at which the measurement was taken must be known precisely in order to make a valid correction.

The *Lincap* provides a simple facility for matching the temperature coefficient. This is done by replacing the shield by another one made of different material. It can be shown that for a typical system with a ratio of cylinder length T to shield length S of 2.2, the following relation holds for the equality of the temperature coefficients of the instrument and object:

$$\sigma = 2.2\tau - 1.2\epsilon \qquad (10)$$

where

σ = temperature coefficient of the shield,
τ = temperature coefficient of the cylinders,
ϵ = temperature coefficient of the object to be measured.

There is a wide range of materials for which matching can be accomplished by simply substituting another shield. For example, Equation 10 will be nearly satisfied for the combinations of materials shown in Table 1 for the shield and object when steel cylinders are used ($\tau \sim 13$ppm/°C).

TABLE 1. THERMAL COEFFICIENTS OF EXPANSION OF SEVERAL MATERIALS

ϵ		σ	
Glasses	2 ppm/°C	Aluminum	26 ppm/°C
Glass, titanium	6 ppm/°C	Brass	21 ppm/°C
Steel	10 ppm/°C	Steel alloys	16.5 ppm/°C
Brass	21 ppm/°C	Glasses	4.5 ppm/°C
Aluminum	23 ppm/°C	Vycor	1. ppm/°C

An interesting special case is obtained for $\epsilon = 0$:

$$\frac{\sigma}{\tau} = \frac{T}{S} = 2.2.$$

The above condition, which can easily be fulfilled, represents the situation when the temperature coefficient of the *Lincap* is zero.

EXPERIMENTAL RESULTS

Measurements completed to date on the present prototype unit indicate a standard error in repeatability of reading over a period of a few weeks of ± 0.2 microns and linearity of ± 0.5 micron. The linearity error is a smooth function of displacement and can therefore be largely eliminated by calibration.

ACKNOWLEDGMENT

The author wishes to express his appreciation to Mr. T. J. Blachut who posed the problem and who suggested a linear transducer manufactured by Sogenique as a possible avenue of investigation. From this initial study, the concept of the *Lincap* has eventually emerged. Thanks are also extended to Mr. A. Zuidhof who has conscientiously carried out the tests and measurements and contributed to the development of the bridge circuit.

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