Discussion Article

Remarks on Semi-Analytical Triangulation

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Dr. Weissman's paper "Semi-Analytical Aerotriangulation" in the August 1969 issue of PHOTOGRAMMETRIC ENGINEERING prompts me to make the following remarks.

i. Dr. Weissman mentions that it is a shortcoming of my program for semi-analytical triangulation that it does not provide means for obtaining the coordinates of the projection centers. This is indeed correct. I presumed that every user would know how to determine these coordinates in one of the two following ways:

- By placing a grid plate in a camera, leveling the grid, and measuring coordinates of grid points at one given height in the model space.
- By placing either a grid plate or a photograph in the camera and, with an arbitrary orientation of the camera, measuring coordinates of points at two different heights.

However, many quests for the NRC programer have been followed by requests for a method of determining the coordinates of the projection centres and for even more basic information. Obviously, a program like this often serves as a first introduction to analytical photogrammetry and, therefore, it must be as complete and lay-proof as possible.

ii. For electronic calculation, one can program these two procedures in a more sophisticated form than the one used by a photogrammetric operator. The first procedure becomes here an ordinary resection in space and it can be used also if the projection camera is tilted. It is the only one mentioned by Dr. Weissman.

The choice between the two procedures should be based on their accuracy and expediency. A rough comparison of the accuracy can be obtained by considering the determination of the height of a projection centre from measurements in a vertical plane through this centre. According to Figure 1, measurement of two grid points at one height gives for the height H of the projection centre above the reference height:

$$H = (f/d)s_1. \qquad (1)$$

According to Figure 2, measurement of two points at two heights gives

$$H = hs_1/(s_1 - s_2). \qquad (2)$$

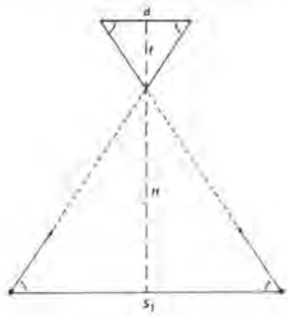

FIGURE 1

For the first procedure, with $f = 150$ mm. and $d = 200$ mm, it follows from Equation 1 that the standard error in the determination of H is about equal to that in pointing at a grid point. For the second procedure, the standard error depends on the ratio h/H. In the second-order plotters with optical viewing the maximum value of this ratio varies from 1:2 to 1:3. In anaglyph instruments, using a tracing table with an extension tube, it is possible also to obtain values between 1:2.5 and 1:3. With the same position of points as in the first procedure, it follows from Equation 2 that the standard error in H is larger than the standard error in pointing by a factor of 2.5 for the ratio 1:2 and by a factor 4 for the ratio 1:3.

Therefore, the first procedure is more accurate. If the plate holders of the plotter contain a glass plate with marks which can be used as grid points, the first procedure should be used. If this is not the case, one can use the second procedure and one should then use perhaps 9 to 16 well spaced points to obtain a good accuracy.

iii. Dr. Weissman characterizes my program by the feature that an exact connection between models is made at the common projection center. However, this is an inessential feature of the program. In view of the uncertainty about the coordinates of the projection center, the connection could be made instead at the central tie point between models. This does not require any modification of the program. Alternatively, by adding a few instruc-

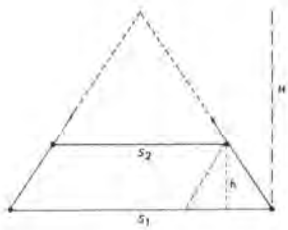

FIGURE 2

tions, the connection can be made at the center of gravity of common points.

The distinguishing feature of the program is the fact that the rotation which is part of the transformation of each model is written as a rotation in four dimensions. This makes it possible to compute the parameters of the rotation from exact and very simple linear equations. This computation is much shorter and more elegant than the conventional one where the 3×3 orthogonal matrix of the rotation is linearized and the parameters are computed in an iterative procedure.

iv. Dr. Weissman gives the results obtained with only one strip, triangulated in different ways. This has no statistical value. In addition, his results are rounded off to the first decimal place. Thus, in Table 2, the reader is uncertain whether the actual differences before round-off were almost as large as 0.2 or as small as 0.01.

However, the z-differences between "Shut's method" in Table 2 and the first two methods in Table 1 are too large to be explained away so easily. Usually, the difference is rather small. One must suspect that in Dr. Weissman's data something is wrong with either the determination or the stability of the projection centers.