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Principles for Selecting Equipment

A least-squares analysis of the four parts of a photogrammetric system can yield a unique number which is an unquestionable, rigorous measurement of the quality of performance

INTRODUCTION

IN THIS PAPER, the term *photogrammetric equipment* will be used in a broad sense. It is not restricted to the purpose of mapping the ground only, but refers to photogrammetry in the general sense of this word, i.e., a method for the measurement of geometrical data (size, position, shape) of an object with the aid of photographs.

In practice the photogrammetric procedure usually has to perform its measurements to a certain degree of geometrical quality and completeness within the shortest possible time and at lowest possible cost. The requirements of quality and completeness must of course be stated by the users of the results of the procedure. These requirements may vary from one instance to another, but they need to be given in clear and unique terms. The main problem of the photogrammetrist is then to apply the material and equipment that will fulfill the requirements within minimum of time and cost. Consequently, some method needs to be available to define the qualities of material and equipment in order that the geometrical quality, the time consumption, and the costs to be expected of the entire work, can be estimated with good reliability in the planning stage. In addition, because photogrammetric equipment in general is rather expensive and has to be used for a considerable time, the criteria to be used in connection with the acquisition of such equipment are of great practical importance for the photogrammetrist.

ABSTRACT: *Generally accepted, rigorous, uniform criteria are essentially nonexistent for measuring and judging the quality of performance of photogrammetric cameras, plotters, comparators, and other precision equipment. For this analysis, the field can be subdivided into four distinct categories: (1) the resolution of the image-forming function of the camera and subsequent laboratory processes; (2) the degree of conformity to the principle of the central perspective, including the measuring instrument; (3) the relative accuracy of the re-intersection of pairs of rays (relative orientation); and (4) the absolute orientation of the photogrammetric model. A least-squares analysis can be applied in each instance in a manner which will yield a valid numerical quantity of performance. The general adoption of such techniques would be worthwhile economically not only to the user in selecting and appraising equipment, but also to the manufacturer in designing and testing his equipment before delivery.*

In the individual case, certainly many factors need to be taken into account. First of all, the type of measurements to be made are of course very important. For instance, if the measurements are to result in photo maps of the ground at a certain scale, the corresponding equipment must be of quite another type as compared to satellite photogrammetry or x-ray photogrammetry. Further, available facilities concerning buildings and personnel (operators), aircraft, and general meteorological conditions, may have considerable influence on the general conditions for the acquisition of equipment. In this con-

* Submitted under the title "Principles for the Selection of Certain Photogrammetric Equipment."

nection all such possible conditions cannot be taken into account.

THE DISCUSSION below is concentrated on the fundamental operations of the photogrammetric procedure that is common to all three-dimensional photogrammetry. The fundamental operations are usually defined as follows[1]:

1. The imaging of an object on photographs through central projection.
2. The reconstruction of the ideal bundles of rays at the moment of exposure with the aid of image coordinates and the elements of the interior orientation.
3. The relative orientation of reconstructed, overlapping pairs of bundles of rays.
4. The absolute orientation of the three-dimensional point group (model) created through the relative orientation.

The first operation obviously refers to the camera, the photographic negative material, and possible copies of this material to be used in the measuring instrument (generally contact or projection prints on glass diapositives).

The second operation refers to the measuring instrument. In analytical photogrammetry the instrument is usually a mono-or stereo-comparator, and in analogue photogrammetry the instrument is of the projection type (optical, mechanical or optical-mechanical projection).

The third operation in analytical photogrammetry is of a purely numerical character, i.e., the condition that corresponding pairs of rays shall be brought to intersect is fulfilled by an (iterative) computation of the elements of the relative orientation. This operation is generally performed according to the method of least squares because the relative orientation cannot be performed without discrepancies in redundant pairs of rays. In analogue photogrammetry, the relative orientation is performed in the actual instrument with optical-mechanical methods, i.e., frequently by successive approximation, until the operator is satisfied with the magnitude of residual y-parallaxes. Sometimes the residual y-parallaxes are measured at a sufficient number of model points (at least nine) and then the method of least squares is applied to determine possible additional corrections to the preliminary elements of the relative orientation. The significance of the corrections can be determined from the least squares adjustment and elementary principles from mathematical statistics[2].

The fourth operation is performed in analytical photogrammetry through the computation of coordinate transformations only, and in analogue photogrammetry again through optical-mechanical methods, sometimes in combination with numerical computations of the corrections to be inserted into the instrument. In both instances the method of least squares is sometimes applied for the adjustment of discrepancies of redundant control points.

Aerial triangulation includes a repetition of the fundamental operations and additional coordinate transformations, including interpolations or adjustment procedures.

IN THIS PAPER the basic terminology for errors of measurement has been chosen according to the following principles which were presented[8] and accepted at the I.S.P. Congress in Lausanne in 1968. In the infinite population of errors of measurement are the three well-known classes of errors: gross errors or blunders, systematic errors, and accidental or random errors. Large gross errors can usually be detected and eliminated but small blunders cannot generally be distinguished from random errors.

Through repeated realistic calibration procedures, systematic errors should be distinguished as well as possible from random errors. A sample of the error population is drawn under operational conditions for realistic estimation of systematic errors. The method of least squares is the most effective and convenient procedure for this purpose. Estimated, significant systematic errors are termed *regular errors* and can for the most part be corrected for. However, these corrections can never be exact because there are always residual discrepancies in the calibration procedure. The estimated regular errors are, in other words, always affected with uncertainties from the calibration procedure.

The residual variance from the calibration is uniquely obtained from the least squares solution. The square root of this variance (the accuracy variance) is termed the *standard error of unit weight*. This is an expression for the undetermined mixture of small undetected blunders, small undetected systematic errors and the random errors of the sample remaining after the calibration as residuals. This mixture of basic errors is termed *irregular errors* and should be carefully distinguished from the term *accidental* errors. Although the residuals are not entirely independent, experience has proved that they usually form a normal distribution,

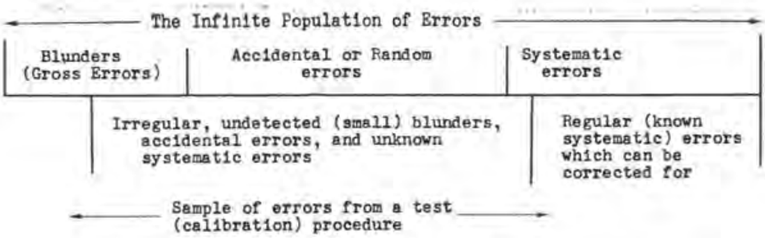

FIG. 1. Schematic divisions of errors of measurement.

which may be explained by the central limit theorem. A summary of these definitions and basic quality expressions is shown in the Figure 1 and Table 1.

HARMONIC RELATIONS BETWEEN THE FUNDAMENTAL OPERATIONS

Because the fundamental operations are closely related to each other, the errors of the operations will propagate together. Systematic errors of the individual operations can accumulate or can compensate each other. Certain systematic errors, estimated as regular errors in particular radial distortion in fundamental operation No. 1, can intentionally be compensated for in operation No. 2 provided, of course, that the radial distortion of operation No. 1 is known under the actual operational conditions.

The irregular errors, statistically expressed as the standard error of unit weight of the fundamental operations, cannot in principle compensate each other provided that they are sufficiently independent and sufficiently well normally distributed. The propagation of irregular errors through the operations can be determined under these circumstances with the aid of the laws of error propagation.

TABLE 1. BASIC TERMS FOR GEOMETRICAL QUALITY

1. *Precision—Closeness together* of results of measurements and observations.
   Terms for precision:
   1.1 Standard deviation of a single measurement or observation (S).
   1.2 Standard deviation of the mean or of other functions of measurements or observations ($S_{\bar{x}}$).
2. *Accuracy—Closeness to the "truth"* of measurements and observations.
   Terms for accuracy:
   2.1 Root-mean-square value of discrepancies (*rmse*, $\bar{S}$).
   2.2 Standard error of unit weight of observations (S_o).
   2.2 Standard error of functions of observations (S_f).

Sometimes, however, the theoretical error propagation can in practice be influenced by correlation effects which act as certain compensations.

In general, because of the obvious summation of the irregular errors, certain relations are desirable between the geometrical quality of the fundamental operations in the final results of the photogrammetric procedure. If fundamental operation No. 1 results in photographs of low geometrical quality (large standard error of unit weight of image coordinates), there is little reason to use a restitution instrument of high geometrical quality (and high cost) in an attempt to increase the final geometrical quality above the level given by the photographs. On the other hand, in order to derive as much of the available accuracy of the photographs as possible, the measuring instrument must be of such a geometrical quality that the standard error of unit weight of the image coordinates is maintained without any appreciable increase in the standard error of the reconstruction of the bundles of rays. In other words, to "strain at a gnat and swallow a camel" does not improve accuracy.

Up to now, the geometrical quality of the photographs (image coordinates) has generally been regarded as limiting the accuracy of the entire photogrammetric procedure. But so far only a few investigations have been devoted to the numerical determination of the basic geometrical quality of image coordinates under real operational conditions. It is evidently very desirable to calibrate aerial photographs under such conditions in order to distinguish as well as possible between systematic errors of the image coordinates (in particular such regular errors which can be corrected in the subsequent fundamental operations) and irregular errors, and to estimate them in clear statistical terms. Considerable investigation of this problem has been performed[3,4,5] and will be summarized briefly here. The principles used for these tests[3] can be applied directly to tests of the reconstruction of bundles of rays in analogue plotting instruments, and with some modifications also to tests of the geometrical quality of comparators for the measurement of image coordinates[6]. In all such tests the basic geometrical quality of the performance of the fundamental operations No. 1 and No. 2 are uniquely determined according to least squares as a standard error of unit weight in full agreement with Resolution 6, Comm. II ISP, London 1960. Such data shall in principle also be furnished by the manufacturers of photogrammetric instruments according to Resolution 6, Comm. II ISP, Lisbon 1964.

If such information is available, the accuracy relations between operations No. 1 and No. 2 can be investigated, among other things, for the selection of instruments in order to arrive at a harmonic relationship between the two fundamental operations mentioned. The geometrical quality of the operations No. 3 and No. 4 can then be judged relative to operations Nos. 1 and 2 because the accuracies of the relative and absolute orientation procedures are limited by the geometrical quality of the reconstruction of the bundles of rays from photographs of given geometrical quality; moreover, see Resolution 1b, Comm. III ISP, Lisbon 1964.

The Geometrical Quality of the Camera (Photographs)

In connection with the laboratory calibration of cameras, the elements of the interior orientation of the camera are determined under laboratory conditions. Seldom, however, are the geometrical qualities of the image coordinates and of the elements of the interior orientation determined. Under all circumstances these qualities would be of limited importance because the bundles of rays are to be reconstructed from photographs taken under operational conditions, and the corresponding accuracy should therefore be determined under such conditions. Consequently, if the choice of a camera should be made with respect to the geometrical quality of the image coordinates of the photographs, the geometrical quality should pertain to operational conditions. Such information is so far not available from the manufacturers. Comprehensive tests of aerial and terrestrial photographs have been made in Sweden to determine the actual basic geometrical quality of image coordinates. Special test fields on the ground for aerial photographs, have been constructed where regularly distributed test points have been marked on rocks and located geodetically with the highest possible geometrical quality (standard errors of the coordinates less than 20 mm). The geometrical quality of the image coordinates have been determined for aerial photographs taken from different flying altitudes after measurement with a stereocomparator having a geometrical quality such that the errors of the coordinate measurement can be neglected in comparison with the irregular errors of the image coordinates. Results of these tests have been published[4,5] and will be briefly summarized here.

The wide angle cameras $C=153$ mm used with film on an acetate base have proved to give a root-mean-square value of the standard error of unit weight of image coordinates of 7.5 μm. The standard errors of unit weight with the radius was found to vary according to the expression

$$s_0' = 2.1 + 0.053r' + 0.00023(r')^2\ \mu\text{m}$$

where r' is the radius from the principal point in mm. After a numerical correction of the image coordinates according to the measured discrepancies of the fiducial marks (primarily due to shrinkages), the root-mean-square value of the standard error of unit weight was reduced to 4.6 μm and the expression for the variation of the standard errors of unit weight was found to be

$$s_0' = 1.1 + 0.02r' + 0.00017(r')^2\ \mu\text{m}.$$

Full use of the indicated increase in accuracy can be made only in analytical photogrammetry where corrections to the image coordinates can be introduced into the computing program.

Few tests have been made for super-wide angle cameras ($C=88$ mm) under operational conditions, and only preliminary values can be given, indicating a root-mean-square value of the standard error of unit weight of the order of magnitude 8 μm.

Recent tests, the results of which have been published,[8] indicate a certain increase in accuracy of the image coordinates if polyester film base is used. For wide-angle cameras the following results were obtained according to Morén and Kaasila: root-mean-square values of the standard error of unit weight $S_0=5.5$ μm and 4.5 μm, respectively, for analytical methods.

The weight variation was still pronounced. No significant improvement was found in a reseau camera.

From such information on the basic accuracy to be expected from image coordinates in aerial photographs, specifications for the pur-

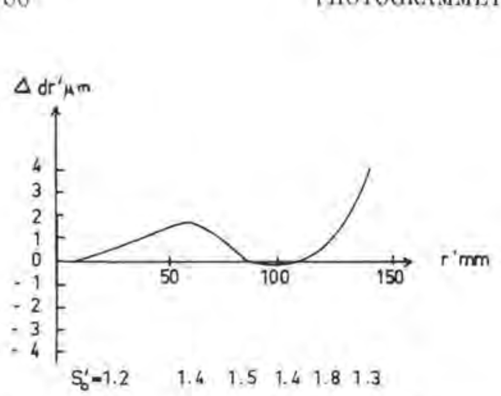

FIG. 2. Deviations between the radial distortion effects of four projector-correction glass plates and the average distortion to be compensated.

chase of aerial cameras can be made according to the principles of mathematical statistics. Provided that the individual residual errors of the image coordinates are normally distributed sufficiently well (which always should be tested in connection with calibration), the chi-square, *F*- and *t*-tests can be used as indicated.[2] This means that tests of aerial photographs with similar methods as indicated above, should give results not exceeding certain limits which can be calculated for the actual number of redundant observations (degrees of freedom) and on a certain confidence level. According to Resolution 6, Comm. II ISP from the Lisbon Congress in 1964, this level should be chosen as 5 percent.

Consequently, for the choice of an aerial camera for the basic photography, information from the manufacturer concerning the basic accuracy of the image coordinates to be expected should be stated and checked in practice at the delivery and also at regular intervals during the practical use of the camera. Similar tolerances should be set up for the photographic quality of the images according to Resolution 4, Comm. I ISP at the Lisbon Congress. Much work remains to be done concerning tolerances for the photographic quality. This became evident at the I.S.P. Comm. I conference in London, Sept. 1966.

Similar tolerances for terrestrial cameras for geometrical and photographical quality should be set up according to information from the manufacturer. So far, however, no information is available on the performance of terrestrial cameras. In Sweden and Finland, some detailed tests[3] have been made on the basic accuracy of image coordinates from modern terrestrial wide-angle cameras, $c=60$ mm. The root-mean-square value of the standard error of unit weight of image coordinates was found to be 5 μm. A considerable weight variation with the radius from the principal point was found, probably due mainly to the lack of flatness of the glass plate negatives.

MEASURING INSTRUMENTS

After the camera has been chosen according to the desired geometrical quality of the image coordinates, the selection of the plotting instrument can be made so that a harmonic relation can be established between the geometrical quality of the image coordinates on one hand and the quality of the instrument on the other.

First of all, the regular errors of the image coordinates of the photographs to be used in the restitution procedure shall be compensated for (corrected) as far as possible, i.e., at least within the confidence limits of the regular errors. In analytical photogrammetry this correction can be made in the computing program. Also, possible systematic errors of the comparator (the lack of orthogonality of the coordinate axes, affinity,* etc.) can be corrected similarly, provided of course that these errors of the instrument have been determined as regular errors, together with the corresponding confidence limits.

In analogue instruments, certain systematic errors of image coordinates, primarily radial distortion, can generally be corrected. The corrections can be introduced with optical or mechanical means. The corrections should obviously refer to the regular errors of the actual photographs.

It is of course most important that the performance of the correction devices be checked through test measurements in the instrument, and that the possible deviations between the regular errors of the photographs, and of the correction devices, be tested for significance. Exact agreement can never be expected, but limits for the discrepancies should be determined according to mathematical statistics and the method of least squares (Figure 2).

Systematic errors other than radial distortion usually cannot be corrected in the analogue measuring instruments. Affinities, however, may be corrected in connection with numerical coordinate transformation of model coordinates.

The basic geometrical quality of projection plotting instruments should be expressed as the standard error of unit weight of projector coordinates after adjustment of test measure-

* Difference between the *x*- and *y*-scales.

ments (calibration) according to the method of least squares[4,6].

Further, a distinction should be made between the application of the instrument to the determination of coordinates of single points of the photographed object, and the continuous graphical plotting of lines and contour lines in planimetry. In the first instance the *precision* of the setting of the measuring mark on the points can be arbitrarily increased from repeated settings, and using the average of the observed instrument coordinates. The influence of backlash and contrast differences between the two photographs can be reduced and practically eliminated with well-known methods. For continuous drawing, however, only one setting can be made on each detail, and the direction of the settings can change from one detail to another. This means that the backlash of the instrument can influence the geometrical quality of the plotting and consequently that tolerances for errors of this type must be established.

Further, the Fertsch-effect, caused by different contrasts and illuminations in the two photographs, can cause considerable error in continuous plotting, particularly in elevation, but also in planimetry. Systematic changes with the time in stereoscopic settings of the measuring mark, which can be different from one operator to another, should especially be tested and kept under control. It is possible that such systematic effects can be different from one instrument to another.

For each type of measuring instrument information should be available concerning the basic accuracy to be expected. This is clearly in the Resolution 6, Comm. II ISP, Lisbon 1964. The most suitable information would be possible systematic errors of the instrument under different, well-defined conditions (radial distortion, Figure 3), and further a standard error of unit weight representing the statistical value of the irregular errors after the determination of the regular errors. So far only limited information from manufacturers seems to be available. Up to now only VEB Carl Zeiss, Jena and Carl Zeiss, Oberkochen have published standard errors of unit weight of stereocomparators. From comprehensive series of tests of instruments in practical use[7,8], certain information is available, but so far it cannot be regarded as sufficient for general information about the geometrical quality of the actual instruments. Therefore it is difficult up to now to base the acquisition of instruments on such information, although it would be most desirable.

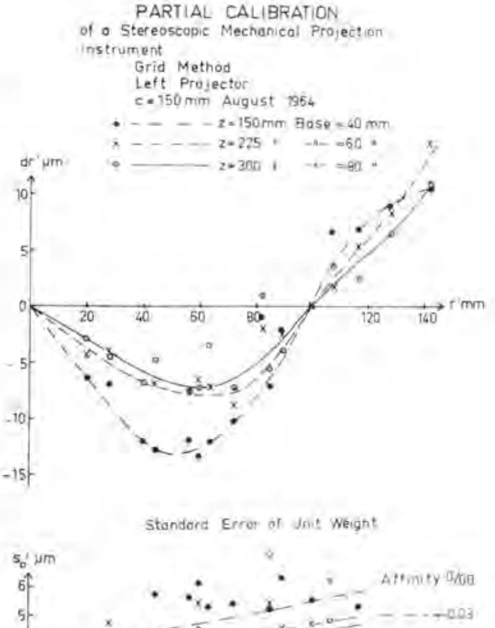

FIG. 3. Geometric quality of a projector of an instrument with mechanical projection.

One can, of course, expect that instruments of high geometrical quality will be more expensive than instruments of lower quality. The last micrometers of the standard error of unit weight are usually very expensive, which is evident from modern stereocomparators. The price of measuring instruments will doubtless have a very important influence on the choice.

For the judgment of suitable relation between the standard error of unit weight of image coordinates s_{oi} (fundamental operation No. 1) and of projector coordinates s_{op} (fundamental operation No. 2) the following procedure can be used.

According to well-known principles (the special law of error propagation), the combined effect can be written

$$s_r = (s_{oi}^2 + s_{op}^2)^{1/2}$$
$$s_r = s_{oi}[1 + (s_{op}^2/s_{oi}^2)]^{1/2}$$

In order to keep the increase from s_{oi} to s_r within 10 percent, the ratio $s_{op}:s_{oi}$ should be less than about 0.45.

The standard error of unit weight of projector coordinates should be about the half of the standard error of image coordinates; this can be used as an important criterion for the acquisition of photogrammetric measuring instruments. In each individual instance, however, a considerable number of other factors should undoubtedly be taken into account.

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Book Reviews

Photogrammetrie (in German), by Prof. Richard Finsterwalder and Prof. Dr. Walther Hofmann, $6\frac{1}{2}$ by $9\frac{1}{2}$ inches, 455 pages, 64 photographs and 125 figures. Walter De Gruyter & Co., Berlin, Germany, 1968, DM 48. (about $12.).

This well-known textbook by Prof. Finsterwalder, who passed away in 1963, has been completely revised and edited by his former associate, Prof. Hofmann, with scientific contributions by Prof. Dr. H. Frieser and Dr. E. Schmidt-Kraeplin.

This new edition not only takes into consideration the most recent developments in photogrammetric acquisition and evaluation techniques, but also takes account of the profound changes and developments that this most modern technique of surveying has experienced in the last decade. Most remarkable is, therefore, the preference given to mathematical presentations of the inter-relations between object space and its corresponding image which now play such an important role in analytical photogrammetry.

The first chapter on fundamentals of optics with emphasis on photogrammetric application deals with distortion correction and quality control through the use of transfer functions. Completely new is the section on fundamentals of photography. It is followed by a section on stereoscope viewing and mensuration.

The second chapter is devoted to terrestrial photogrammetry. Recent developments in cameras and plotting instruments are presented and evaluation procedures as well. Satellite triangulation does not enjoy mention because of its small significance and novelty but, even in this short presentation, the development of the Wild BC-4 camera system should not have been left out.

The third chapter, which occupies almost half of the book, is devoted to aerial photogrammetry and is divided into several subdivisions: flight planning, aerial cameras and auxiliary equipment, and the actual photo flight, rectification procedures and equipment, the orientation of single photographs and pairs of photographs, photogrammetric methods and procedures, stereoplotters (including the B8 as a representative of automatic plotting instruments), radial and aerial triangulation procedures and the accuracies obtainable by these methods. The fourth chapter presents photo interpretation techniques and applications to various areas of interest; however, photo interpretators in the U.S.A. may want to fall back on the Manual of Photographic Interpretation because of its vastness of information.

—*Heinz Poetzschke*

Reference Guide Outline: Specifications for Aerial Surveys and Mapping by Photogrammetric Means. Rev. 1968. Prepared by the Photogrammetry for Highways Committee of the American Society of Photogrammetry. Published by the U. S. Department of