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Radiographic Measurements

Photogrammetric techniques can be effectively applied to the precise three-dimensional measurement of X rays.

(Abstract on next page)

Introduction

It has been observed that many important fields for research occur at the boundaries of different subjects, where the two apparently different subjects seem to unite, to give birth to a new field of knowledge.

Work in the field of stereoradiography was mostly centered around an empirical use not based on any specific measurements or quantitative approach, but on the identification of shadows or the impression in the third dimension.

The technique of photogrammetry can be affectively applied for precise measurements in radiography. Attempts have been made in this direction by photogrammetrists[9,17–26,29] but, as most of the literature is still in the formative stage and doubts exist about its real usefulness to the medical man, a vigorous attempt needs to be made to project this new technique to the radiologists.

Brief History

The year 1895 marked the beginning of a new era when Prof. Roentgen discovered X-rays. When Elihu Thomson[1] (1896) suggested *X*-ray stereography or stereoradiography, the *theory of stereoscopy* had already been propounded by Sir Charles Wheatstone (1832) and the first optical instrument (1838), the Wheatstone Mirror Stereoscope was already developed (Figure 1). Later James Mackenzie Davidson of London further developed the idea, and it has been employed in the medical field by radiologists since that time.

Medical literature reports some other stereoscopes as shown in Figures 2, 3, 4 and 5, but these were mainly utilized for stereoscopic viewing.

* Based on this paper, under the title "Precision measurements in Radiography by Photogrammetry", Mr. Singh received the Bausch & Lomb Honorable Mention Award in the Graduate Student Division.

Technique of Obtaining Stereoradiographs

The stereoradiographs are made at angles of the same degree but on opposite sides of the object radiographed. Consequently the *X*-ray tube is shifted from approximately 1¼ inch from one side of the center to the same distance on the opposite side of the center, a total shift of 2½ inches, which is the mean interpupillary distance (Figure 6.) This tube shift is variable for different focus-film distances, and for different object thicknesses.

The patient needs to be immobilized, because no movement should take place during the interval when the first film is removed and the second film is inserted and exposed. The tube is centered usually at a distance of 25 inches from the *X*-ray film (but this distance may vary according to the requirements). This distance is called the focus-film distance and is being adopted as the most convenient distance for use between films and mirrors in the stereoscope. Some radiologists

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ABSTRACT: *Precision measurements in radiography, or X-rays, have always been a problem to radiologists. Ever since the discovery of X-rays in 1895 by Prof. Roentgen, radiologists sought a means to measure in the third dimension from radiographs. The "theory of stereoscopy" had already been propounded in 1832 by Sir Charles Wheatstone, when Elihu Thomson (1896) suggested X-ray Stereography or Stereoradiography. Several attempts were made, using different designs of stereoscopes, for stereoscopic viewing and planimetric measurements in the stereoscopic model space by including a measuring scale. Using simple geometry and parallax measurements with a scale, depths were also calculated. But the photogrammetric technique of depth measurements, using "floating mark" has not been mentioned in radiology literature.*

Several other techniques, such as Kymography, Tomography, and Seriescopy, have been developed for the study of interior sections of human body, but these involve a much greater dose of X-rays to the patient and still provide no means for precision measurements in depth. A parallax formula is developed for precise depth determination in radiography using stereoradiographs, a mirror stereoscope and a parallax bar.

Neurosurgery, in addition to some other areas of radiological examination, demand very accurate measurements, and photogrammetry can certainly be very useful in such applications. Topics studied here include: the technique of obtaining stereoradiographs, the geometry of stereoradiography, and experimental investigations.

prefer that the first film be made in the conventional manner (keeping the X-ray tube vertically above the center of object), and the entire shift be made for the second film[2,5,6].

It is important that each pair of films should be treated as a unit, and both films, therefore, taken and processed under identical conditions.

It has been suggested[7] that two X-ray tubes with the same focal positioning may be provided for taking stereoradiographs. With this arrangement it is only necessary to change the cassette (X-ray film holder) between exposures.

AMOUNT OF TUBE-SHIFT

It is generally agreed by radiologists that the amount of tubeshift (the distance through which X-ray tube is moved for taking stereoradiograms), may be increased along with the increase in focus-film distance, such that the ratio is maintained at 1:10, Dr. Joseph Selman has suggested the following inter-relationship[6].

$$\frac{\text{Tube-shift}}{\text{focus-film-distance}} = \frac{\text{Interpupillary distance}}{\text{Viewing distance}}.$$

It was observed that for a focus-film distance of 30 inches a total tube shift of 3 inches produced some exaggeration of depth. If one exceeds this 1:10 ratio, resolved stereoscopy is difficult, if not impossible.

For the usual 25-inches focus-film distance it has been found that a 1:15 ratio of tube shift will produce a more accurate perspective of the actual depth of the subject. Another desirable advantage is that the 1:15 ratio requires far less optical accommodation than 1:10 or 1:9 ratio. However, with tube-film

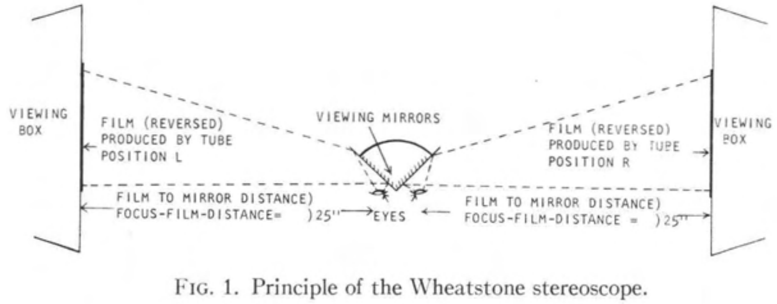

FIG. 1. Principle of the Wheatstone stereoscope.

distance of 6 feet, a 1:10 ratio may be used, lest the result will be too shallow due to increased tube-film distance.[2]

Scope of Stereoradiographic Studies

Examples of the application of stereoradiographic studies include the following:

- *Localization* involves radiographic investigation to determine the presence and position in the body of any foreign object, termed as a *foreign body*, such as broken glass, needles, pins, coins, bullets, metal fragments, trapped gases in tissues, etc.
- *Study of Tumors*[30]
- *Orthopaedics*[15]
- *Dentistry*[21,27,28]
- *Pregnancy*—finding the diameter of foetus head[5]
- *Angiography*, which indicates the radiographic examination of the circulatory system. As the system is largely transradient, it is necessary to introduce an opaque medium into the blood stream to enable the blood vessels to become visible for radiological investigation. Angiography embraces both the arterial sys-

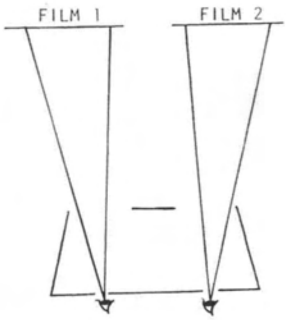

FIG. 2. Simple box stereoscope.[2]

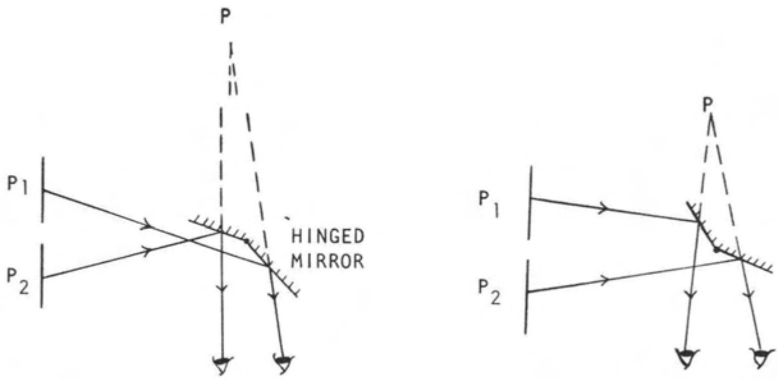

FIG. 3. Pair of hinged mirrors developed by Ove Matterson[3a] (Karolinska Hospital, Stockholm) for stereoscopic viewing of stereoradiographs.

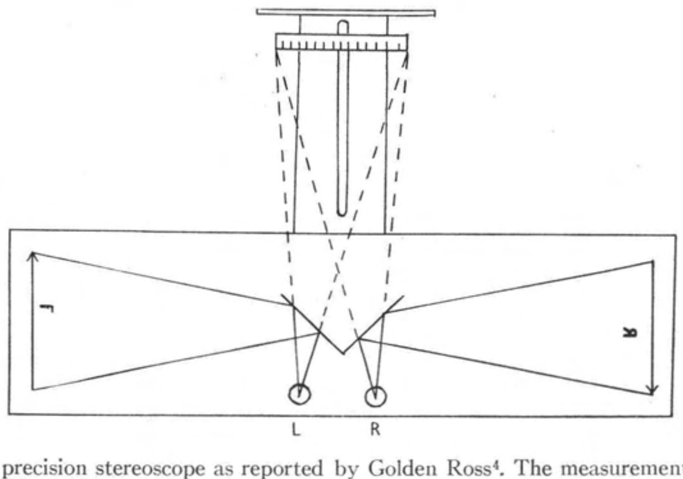

FIG. 4. The precision stereoscope as reported by Golden Ross[4]. The measurements in a horizontal plane (not in depth) could be made by placing a scale under stereoscopic vision into the stereoimage and reading directly.

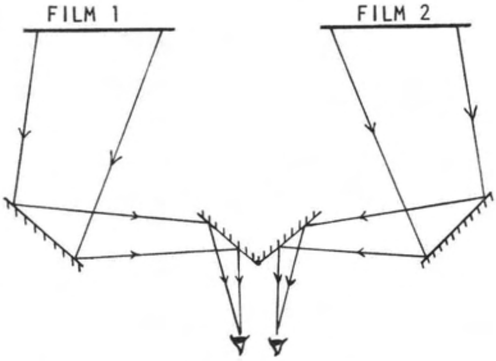

FIG. 5. Simple four-mirror stereoscope.

> tem (arteriography) and the venous system (venography, or phlebography). The closely allied lymphatic system (small vessels in the body containing lymph or alkaline fluid, watery in appearance) is also included. Descriptive titles for the technique as applied to the various regions concerned include: abdominal aortography, cardiac angiography, cerebral angiography, lymphangiography, peripheral angiography, and portophlebography.

It is believed by Herbert M. Stauffer, Frederick Murtagh, John F. Mokrohisky and Robert E. Paul Jr. that stereoscopy is eminently successful in cerebral angiography.[3]

Stereolymphangiogram of right leg (Figure 7) was obtained by the author at Safdarjung Hospital, New Delhi in 1963 at 30 inches tube-film distance and $2\frac{1}{2}$ inches tube-shift.[11] Stereoradiograph of Human Artery System (Figure 8) was issued by J. F. Bergmann, of Wiesbaden in 1911.[8]

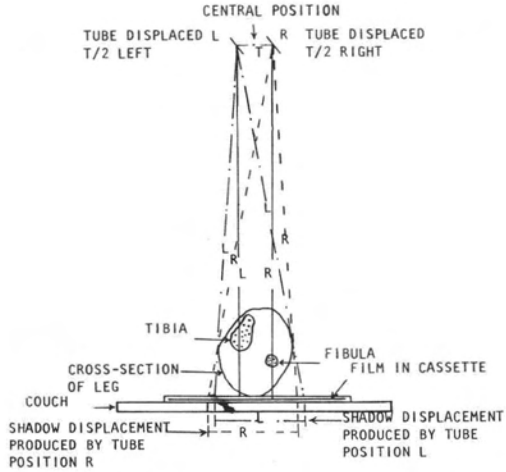

FIG. 6. Principle of stereoradiographic projection

GEOMETRY OF STEREORADIOGRAPHY FOR THE NORMAL CASE

The geometry of most kinds of photogrammetry is based on the concept of central projection. With the development of the design of X-ray tubes, the source of X-rays (focus) has been reduced to the effective size of 0.3 mm diameter, which may be considered as a point source. Thus the radiographic shadow image could be considered as the intersection between the bundle of rays from the tube-focus and the image plane. In developing the geometry it has been assumed that all rays emanate from a mathematical point and that all individual rays are mathematically straight

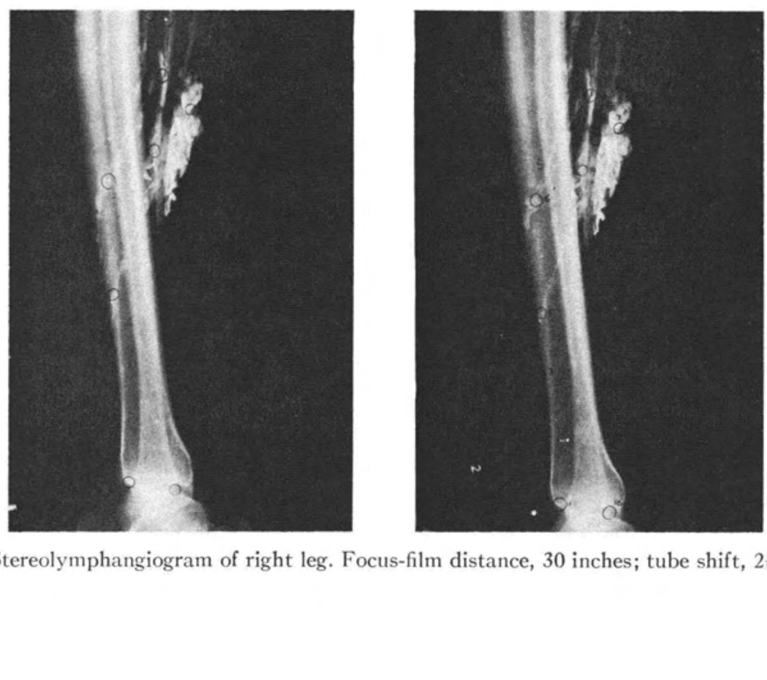

FIG. 7. Stereolymphangiogram of right leg. Focus-film distance, 30 inches; tube shift, $2\frac{1}{2}$ inches.

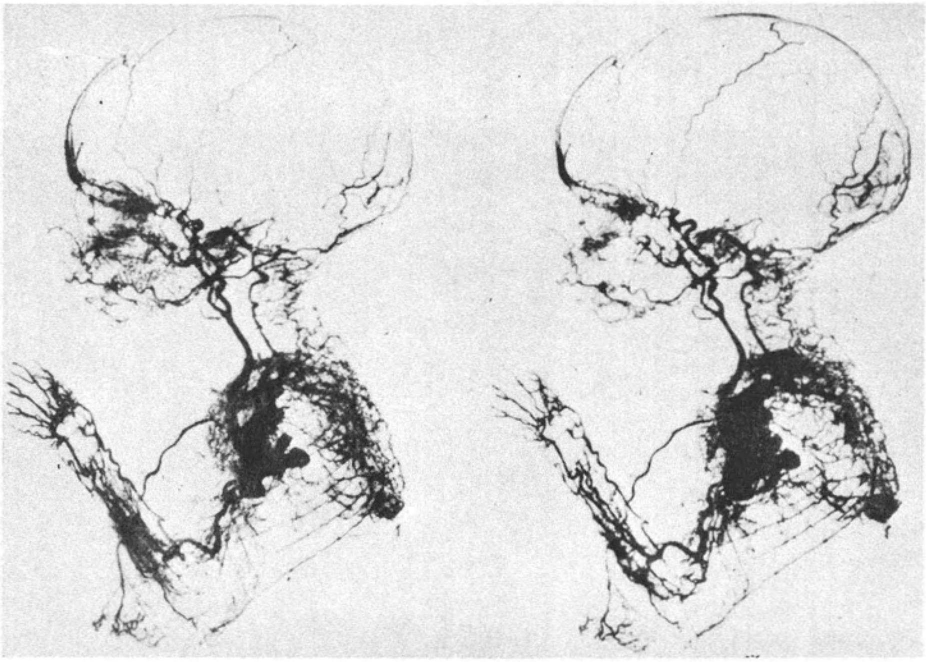

FIG. 8. Stereoradiograph of human artery system.[8]

lines. It is also assumed that the image plane is strictly mathematical plane. The X-ray tube is assumed to be such that the central rays are perpendicular to the image plane.

THE MARKER

For stereoradiographic measurements by photogrammetry, a metal piece, either triangular, rectangular, circular or arrow-head shaped has to be secured centrally on top of the area to be radiographed, at a known height from the X-ray film at the image plane. This piece, hereafter called *the marker*, serves the same purpose as the ground control points in photogrammetry. The shadow of the marker on the pair of stereoradiographs help in finding the parallax difference. This parallax difference could be used in determining heights of other points in the image by comparing the parallax differences. For higher accuracy more than one marker could be used.

HALLERT'S APPROACH

Dr. B. Hallert has developed a formula for the normal case in stereoradiography in which (Figure 9) a point p is imaged from two X-ray tube positions, o_1 and o_2 separated by the distance b, into the points p_1 and p_2, respectively. He assumed H_1 and H_2 as the principal points of o_1 and o_2 and assumed these to be the origins of the image coordinate systems x_1, y_1 and x_2, y_2. These points H_1 and H_2 are *assumed to be marked in the images in some way, for instance by the aid of fiducial marks.*[9]

Projecting the oblique triangles o_1o_2p into the vertical plane through the base o_1o_2 (Figure 10), the following relationship is obtained by similarity:

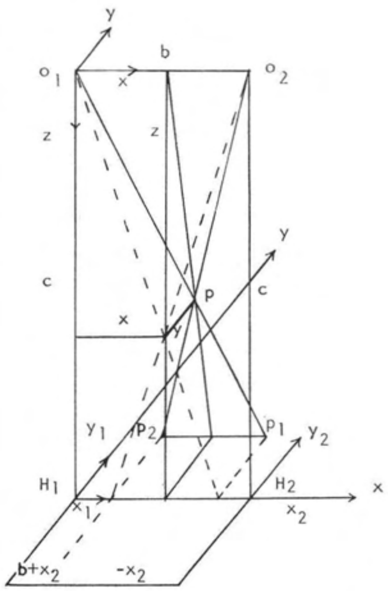

FIG. 9. Stereoradiography of a point p in the image plane xy.

$$z = \frac{bc}{x_1 - x_2} = \frac{bc}{p_x}$$

$$x = \frac{z}{c} x_1 = \frac{b}{p_x} x_1$$

$$y = \frac{z}{c} y_1 = \frac{z}{c} y_2 = \frac{b}{p_x} y_1 = \frac{b}{p_x} y_2$$

where $x_1 - x_2 = p_x$ is denoted as the horizontal or x-parallax.

Disadvantage of Hallert's Approach

The main disadvantage of the above approach is in the marking of the principal points H_1 and H_2. As stated by Hallert,[9] *these points are assumed to be marked in the images in some way, for instance by the aid of fiducial marks.* At the time of taking stereoradiographs, the cassette containing the X-ray film, is placed beneath the couch on which the patient lies and is exposed from tube position o_1. In as short a time as practicable, the X-ray tube is shifted through the distance b to position o_2 and cassette removed and replaced by a second film. Even if fiducial marks are present in the cassette it will not be possible to obtain the exact position of principal points H_1 and H_2.

To overcome the above disadvantage the author suggests the use of a marker.

The Parallax Formula Normal Case in Stereoradiography as Developed by the Author. For[11]

Figure 11 represents geometrical conditions for stereoradiography. The marker M and a foreign body N inside the body cast their shadows onto X-ray film RR at m_1 and n_1 from tube positions T_1, and at m_2 and n_2 from

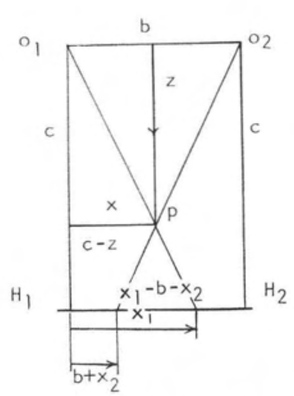

FIG. 10. Diagram used in the derivation of Hallert's parallax equation.

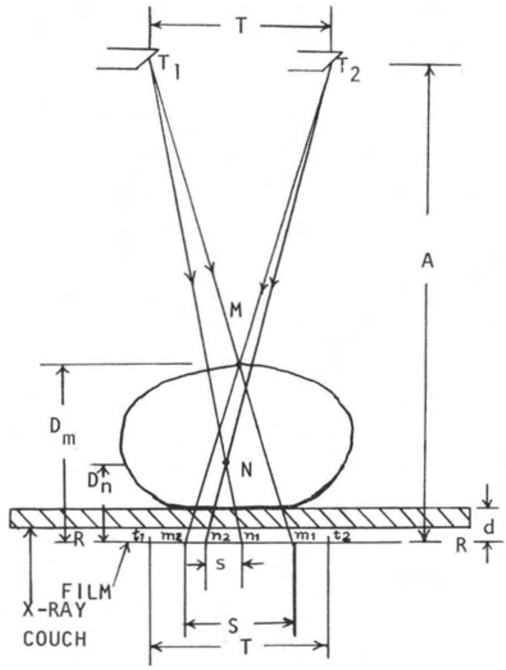

FIG. 11. Geometry of radiography.

tube position T_2. The focus-film distance is A, and D_m and D_n are the heights of M and N from film.

Figure 12 represents geometrical conditions for normal case in aerial photogrammetry. Here T_1' and T_2' are the positions of aerial camera in space, separated through the base distance T'; p_1 and p_2 the principal points on

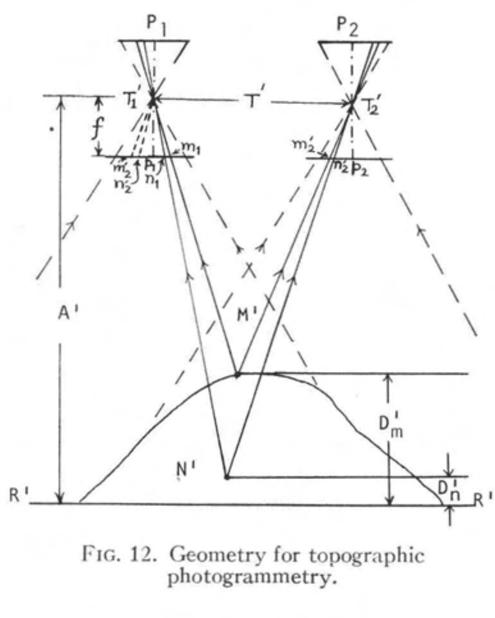

FIG. 12. Geometry for topographic photogrammetry.

diapositives; f the principal distance of the camera. A' is the flying height above mean sea level; D_m' (known) and D_n' (unknown) the heights of two points M' and N' in a terrain measured above sea level.

Draw $T_1'm_2'$ parallel to $T_2'm_2'$ and $T_1'n_2'$ parallel to $T_2'n_2'$.

For developing the parallax formula, the pair of stereoradiographs could be compared with the two aerial photographs p_1 and p_2.

Referring to Figure 12, let the parallax for point N' be $p_n = n_1n_2'$ and the parallax for point M' be $p_m = m_1m_2'$. From similar triangles,

$$p_n = \frac{fT'}{A' - D_n'} \tag{1}$$

$$p_m = \frac{fT'}{A' - D_m'}. \tag{2}$$

Let

$$\Delta p_{mn} = p_m - p_n \tag{3}$$

$$\Delta D_{mn}' = D_m' - D_n'. \tag{4}$$

$$D_n' = D_m' - \Delta D_{mn}'. \tag{5}$$

By subtracting Equation 1 from 2 and using 3, 4, and 5, we obtain

$$\Delta p_{mn} = p_m \frac{\Delta D_{mn}'}{A' - (D_m' - \Delta D_{mn}')}$$

or

$$\frac{p_m}{\Delta p_{mn}} = \frac{A' - D_m' + \Delta D_{mn}'}{\Delta D_{mn}'}.$$

Subtracting Equation 1 from both sides,

$$\frac{p_m - \Delta p_{mn}}{\Delta p_{mn}} = \frac{A' - D_m'}{\Delta D_{mn}'}.$$

$$\therefore \Delta D_{mn}' = \frac{\Delta p_{mn}(A' - D_m')}{p_m - \Delta p_{mn}}. \tag{6}$$

This is the fundamental formula used in photogrammetry for the determination of heights by parallax measurements.

Proceeding on similar lines, in case of stereoradiography, we have from Figure 11:

$$\text{Parallax for } M: \quad S = \frac{D_mT}{A - D_m} \tag{7}$$

$$\text{Parallax for } N: \quad s = \frac{D_nT}{A - D_n}. \tag{8}$$

Let

$$\Delta s_{mn} = S - s \tag{9}$$

$$\Delta D_{mn} = D_m - D_n. \tag{10}$$

$$D_n = D_m - \Delta D_{mn}. \tag{11}$$

Subtracting Equation 8 from 7 and using 9, 10, and 11 we obtain:

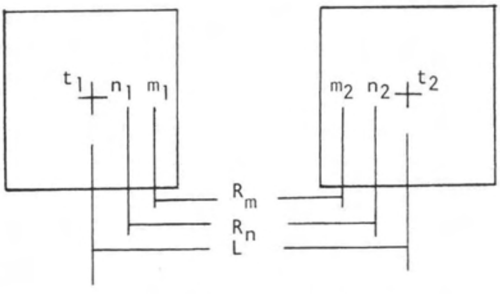

FIG. 13. Radiographs oriented for stereoviewing.

$$\Delta s_{mn} = \frac{T}{(A - D_m)} \cdot A \cdot \frac{\Delta D_{mn}}{A - (D_m - \Delta D_{mn})}.$$

From Equation 7,

$$\Delta s_{mn} = \frac{S}{D_m} \cdot A \cdot \frac{\Delta D_{mn}}{A - D_m + \Delta D_{mn}}$$

or

$$\frac{AS}{\Delta s_{mn} D_m} = \frac{A - D_m + \Delta D_{mn}}{\Delta D_{mn}}.$$

Subtracting Equation 1 from each side,

$$\frac{AS - \Delta s_{mn} D_m}{\Delta s_{mn} D_m} = \frac{A - D_m}{\Delta D_{mn}}$$

$$\boxed{\Delta D_{mn} = \frac{\Delta s_{mn} D_m (A - D_m)}{AS - \Delta s_{mn} D_m}} \tag{12}$$

This is the formula developed by the author. In this the values of A, D_m and T are directly measured; that of S is calculated from Formula 7, and parallax difference Δs_{mn} is obtained from parallax bar measurements using a mirror stereoscope as shown below in Formula 13.

Let the two radiographs of a stereopair be mounted side by side (Figure 13) on an illuminating box (in horizontal plane) and adjusted for stereoview under a mirror stereoscope (Figure 14). If t_1 and t_2 be the principal points of T_1 and T_2 (Figure 11) separated through a distance L (Figure 13), and R_m and R_n the distances between the corresponding shadows m_1m_2 and n_1n_2, which are measured with a parallax bar, then, since

$$S = L - R_m$$

and

$$s = L - R_n$$

$$\therefore \Delta s_{mn} = S - s = R_n - R_m$$

But the scale graduations of the parallax bar

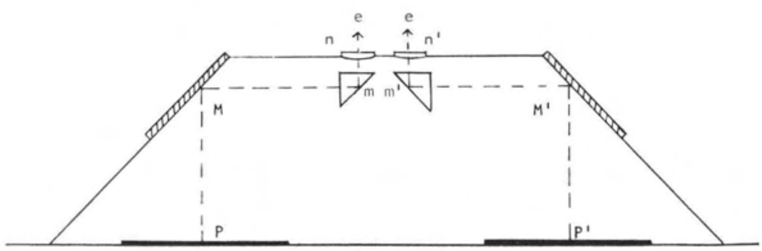

FIG. 14. Principle of the mirror stereoscope.

are such that with the increase in distance the readings decrease, and as such

$$\Delta s_{mn} = R_m - R_n \quad (13)$$

Thus height difference ΔD_{mn} is calculated from Equation 12 and, by subtracting from D_m, the height of the foreign body N is obtained.

Experimental Investigation and Discussion

Experiments were conducted at Safdarjung Hospital, New Delhi, and the University of Roorkee in obtaining stereoradiographs of skull (Figure 15), chest, colon, lymphatics (Figure 7), etc., using the described technique above. Measurements for random selected points were obtained using a mirror stereoscope and parallax bar and their depths calculated using Formula 12. In a few instances depths were also calculated using Hallert's method.

To ascertain the reliability of results, stereoradiographs of wooden test blocks (Figures 16 and 17) with nails driven at known heights were obtained.

Table 1 shows the difference in heights from actual values as well as the maximum difference in height in the two test-block experiments. The reason for greater discrepancy by Hallert's method is not because the procedure is incorrect but because it is very difficult to mark the principal points accurately.

It is possible to obtain a higher accuracy by measuring distances T, A, D_m with higher precision and using more accurate photogrammetric instruments. In most radiographic work; including neurosurgery, a much higher accuracy than what has been obtained by the author may not be necessary. Thus the method of photogrammetry can be very useful in radiological precision measurements.

Conclusion

The technique of photogrammetry could be effectively applied to precise radiographic measurements in depth. Stereoradiography is less hazardous in X-ray doses than other conventional methods such as Kymography,[5,11,12,13] Tomography,[5,11,12,14,15] Seriescopy,[5,11] which need 4 to 16 times greater X-ray doses than for normal radiographs. Stereoradiography also gives perception of the three dimensional space which brings about

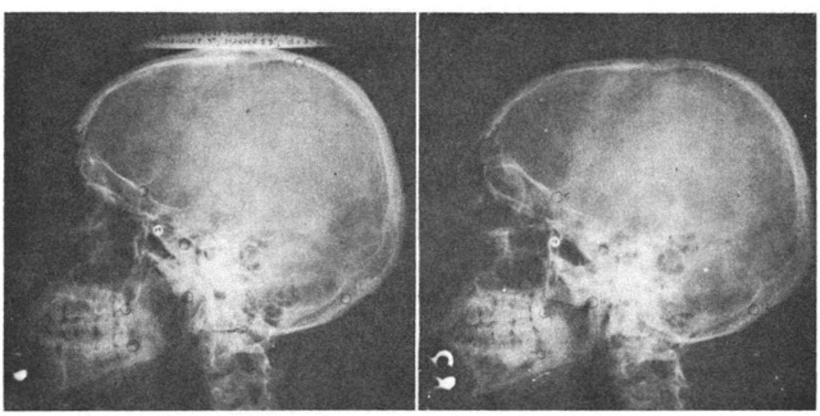

FIG. 15. Stereoradiograph of human skull. Focal-film distance, 35.4 inches; tube shift, 3 inches; height of marker, 5.9 inches.

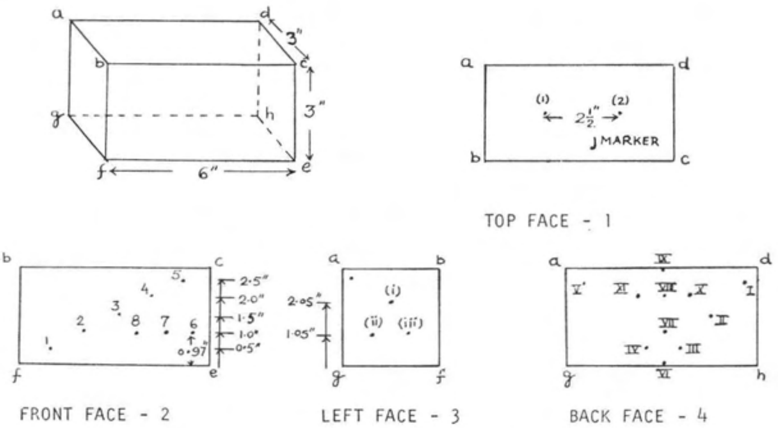

FIG. 16. Wooden test block No. 1.

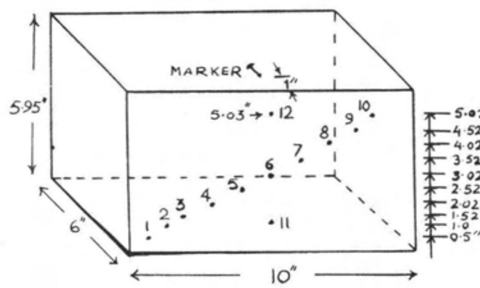

FIG. 17. Wooden test block No. 2.

a splendid effect as if X-rays could be sensed by normal vision, which is limited to the spectrum of visible light.[16]

The use of marker and the parallax Formula 12 could be used for precise measurements in radiography by photogrammetric method.

The field of stereoradiography needs further research in:

- Developing the relationship between tube-shift and focus-film, and the object-thickness relationship.
- Developing suitable instruments for radiographic measurements by photogrammetry.
- Developing an analytical approach.

An earnest effort should be made by photogrammetrists to convince the men of the medical profession of the usefulness and reliability of the photogrammetric technique. Too much mathematics is bound to be less popular with the doctors, and as such, the photogrammetric approach in its simplest form should be projected.

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TABLE 1 COMPARISON IN HEIGHTS OBTAINED BY TWO METHODS

Method	*Test Block No.*	*Total Nails*	*Max. diff.*	*Difference in height from actual value*			
				above 0.3 in.	*above 0.2 in.*	*above 0.1 in.*	*above 0.05 in.*
Author's	1	23	0.14 in.	—	—	4	2
	2	12	0.08 in.	—	—	—	3
Hallert's	2	12	0.6	11	—	1	—
	1	23	0.39	4	4	7	2

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