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Time Considerations for Digital Plotters

From the photogrammetric designer's point of view, the hybrid type of solution seems to be more attractive than the fully digital controlled system.

ABSTRACT: *In automated plotters, critical conditions occur in those operations that require rapid motions in the mechanical tracking (tracing) devices. If tracking is controlled in a fully digital manner, then the frequency of sampling, computing, and actuating the motion is high, demanding high dynamic capability. The conditions are considerably better in hybrid systems where only small differential displacements are controlled digitally. From the point of view of the instrument designer, hybrid systems seem to be more attractive than fully digital controlled systems.*

THE DESIGN OF hybrid plotters involves some crucial kinetic problems inherent in digitally controlled tracking. In photogrammetric plotters provided with digital control units, the limiting factors are the speed of digitizing movements, the repetition rate of computing cycles, and in particular the time response of the output servos. Systems of high performance can be achieved by establishing a good balance between the size of the kinetic steps for quantizing the input movements, the time period for computing cycles, and the time response of the actuators for tracking. Such a balance might be established by fulfilling one of the two alternate approaches:

- By increasing the performance and matching the relevant subsystems, i.e., analogue to digital converters, computer, and the actuators.
- By implementing a hybrid solution so that the digitally controlled tracking provides for only differential small displacements.

The scope of this paper is restricted to the second approach, which seems to be more feasible from the kinetic point of view. Attention is focussed on the differential digitally controlled movements as a part in the total tracking. The investigation includes four characteristic instrument classes:

1. Plotters with analogue control for X, Y and Z; provided with partial analogue (e.g., mechanical) devices for the photo tilts (φ, ω); and digital differential tracking (see Figure 1 and Appendix).
2. Plotters provided with analogue control of X, Y and Z, which neglect the photo tilts, and with digital differential tracking (Figure 2). A simple example is a projection type instrument without analogue means for setting tilts and to which the corrections for tilts are applied through linear displacements of the photographs under computer control.
3. Plotters provided with analogue control of X and Y, and digital differential tracking (e.g., AP-1, Analytical Plotters, Zeiss Jena Digomat; Figure 3)
4. Plotters applying solely digital tracking (e.g., AP-C, AP-2 Analytical Plotters; Figure 4).

This treatment has been restricted to the conventional near-vertical photographs,

* See also "Hybrid Stereo Restitution Systems" in October 1970 issue of PHOTOGRAMMETRIC ENGINEERING. ITC: International Institute for Aerial Survey and Earth Sciences.

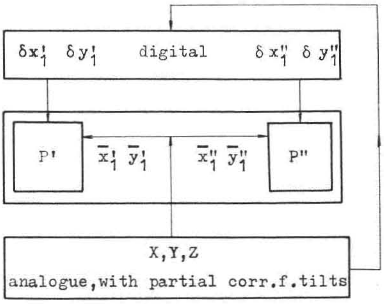


FIGURE 1

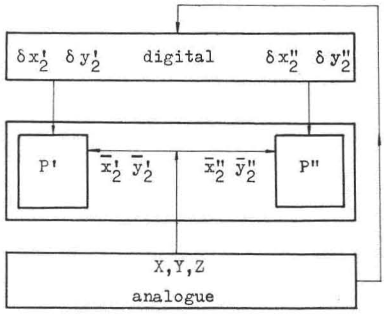


FIGURE 2

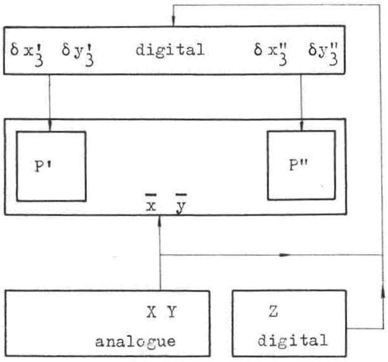


FIGURE 3

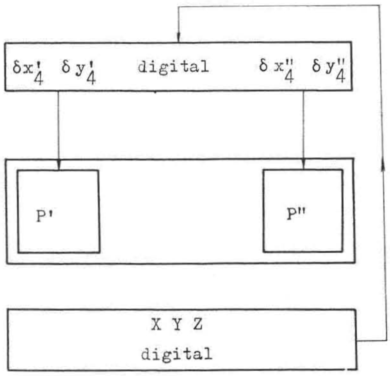


FIGURE 4

representing the objects in a central perspective. Thus, the restrictions correspond more or less with those of the Analytical Plotter AP-C. Extensions to other types of photography, e.g., panoramic, are possible. The following paragraphs deal with the geometric principles and problems concerning the speed of digitally controlled tracking and the repetition rate of corresponding computing cycles for the specified classes of instruments.

GEOMETRIC PRINCIPLES

The metric relations between the points in a photo-plane, representing the central perspective of object points, and the corresponding model points can be represented in a simplified form by the matrix array

$$\begin{bmatrix} x \\ y \\ c \end{bmatrix} = \lambda A \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{1}$$

where x, y, c represent the coordinates of photo-points and X, Y, Z of the model points. The origin of both coordinate systems is the perspective center.

The perspective Equations 1 can also be represented algebracially[1,2] by means of the angles α and β (see appendix):

$$\tan\alpha = X/Z, \tan\beta = Y/Z \tag{2}$$

$$x = c\tan(\alpha - \varphi - \delta)$$

$$y = c\tan(\beta - \omega - \epsilon). \tag{3}$$

Here φ and ω represent the longitudinal and lateral tilts of the photograph, whereas

δ and ϵ are small corrective angles which vary during tracking. The latter two angles are defined as:

$$\tan\delta = \frac{(X\cos\varphi - Z\sin\varphi)[(Z\cos\varphi + X\sin\varphi)\cos\omega + Y\sin\omega - (Z\cos\varphi + X\sin\varphi)]}{(Z\cos\varphi + X\sin\varphi)[(Z\cos\varphi + X\sin\varphi)\cos\omega + Y\sin\omega] + (X\cos\varphi - Z\sin\varphi)^2}$$

$$\tan\epsilon = \frac{Y(Z\cos\varphi + X\sin\varphi) - YZ}{Z(Z\cos\varphi + X\sin\varphi) + Y^2}\,. \tag{4}$$

In Equation 3, the variables are α, β, δ and ϵ for a given photograph. These relations can be realized partly or fully by means of digitally controlled tracking. In general terms the digitally controlled tracking is

$$\delta x = x - \bar{x}, \qquad \delta y = y - \bar{y} \tag{5}$$

where $\bar{x}$ and $\bar{y}$ represent the amount of tracking contributed by analogue means. In the case where tracking is controlled in a fully digital way, it follows that $\bar{x}$ and $\bar{y}$ are zero.

In the following the expressions for digitally controlled tracking will be formulated algebraically for the four characteristic classes of plotters, as defined previously.

CASE 1

Tracking accomplished by analogue means is

$$\bar{x}_1 = c\tan(\alpha - \varphi), \qquad \bar{y}_1 = c\tan(\beta - \omega).$$

The values of δx_1 and δy_1 represent displacements corresponding to the small angles δ and ϵ. The angles δ and ϵ increase towards the perimeter of a photograph. From the theoretical point of view maximum changes of the angles are interesting.

In tracking in the x or y direction, the maximum changes in δ and ϵ are:

$$d\delta_{x\,\max} \text{ for } X \approx Z/\sqrt{3}, \qquad d\delta_{y\,\max} \text{ for } X = X_{\max};$$

$$d\epsilon_{x\,\max} \text{ for } Y = Y_{\max}, \qquad d\epsilon_{y\,\max} \text{ for } Y \approx Z/\sqrt{3}.$$

The changes in digitally controlled tracking can then be estimated accordingly as:

$$d\delta x_1 \approx -\frac{c}{\cos^2\alpha}\delta = -\frac{c}{Z^2}(X^2 + Z^2)\delta$$

$$d\delta y_1 \approx -\frac{c}{\cos^2\beta}\epsilon = -\frac{c}{Z^2}(Y^2 + Z^2)\epsilon. \tag{6a}$$

CASE 2

The tilts have to be accounted for by the differential (digital) control. Thus, the tracking performed by analogue devices is

$$\bar{x}_2 = cX/Z, \qquad \bar{y}_2 = cY/Z.$$

Consequently the digitally controlled movements will be:

$$\delta x_2 = x - cX/Z, \qquad \delta y_2 = y - cY/Z. \tag{7}$$

In these expressions δx_2, δy_2 represent the total corrections for the tilts (φ, ω) of a photograph. The changes of δx_2, δy_2 can be estimated as follows:

$$d\delta x_2 = -\frac{c}{\cos^2\alpha}(\varphi + \delta) = -\frac{c}{Z^2}(X^2 + Z^2)(\varphi + \delta)$$
$$d\delta y_2 = -\frac{c}{\cos^2\beta}(\omega + \epsilon) = -\frac{c}{Z^2}(Y^2 + Z^2)(\omega + \epsilon). \tag{7a}$$

CASE 3

The tilts and the Z-movement have to be accommodated by the differential digital control. The tracking accomplished by analogue means is

$$\bar{x} = cX/Z_0, \qquad \bar{y} = cY/Z_0$$

where Z_o is constant. Supposing $Z_o = c$, then $\bar{x} = X$ and $\bar{y} = Y$.

The differential (digital) tracking will in this case be:

$$\delta x_3 = x - cX/Z_0 = x - X, \qquad \delta y_3 = y - cY/Z_0 = y - Y \tag{8}$$

The values δx_3, δy_3 include both the correction for tilts and for terrain relief. Equation 8 can be written by substituting Equation 3 in an extended form, thus:

$$\delta x_3 = c\tan[\alpha - (\varphi + \delta)] - X \approx c\tan\alpha + \delta x_2 - X = (X/Z)(c - Z) + \delta x_2$$
$$\delta y_3 = c\tan[\beta - (\omega + \epsilon)] - Y \approx c\tan\beta + \delta y_2 - Y = (Y/Z)(c - Z) + \delta y_2. \tag{8a}$$

The terms with $(c-Z)$ include the corrections for both scale and relief, whereas δx_2, δy_2 represent the corrections for the photo-tilts (see Case 2). The changes of the value δx_3, δy_3 during tracking are defined by

$$d\delta x_3 \approx -\frac{X}{c}dZ - \frac{(X^2 + c^2)}{c}(\varphi + \delta)$$
$$d\delta y_3 \approx -\frac{Y}{c}dZ - \frac{(Y^2 + c^2)}{c}(\omega + \epsilon). \tag{8b}$$

CASE 4

For plotters which apply only to digital tracking, the condition is

$$\bar{x} = 0, \qquad \bar{y} = 0$$

and the digitally controlled movements will be

$$\delta x_4 \equiv x = x_0 + \delta x_3, \qquad \delta y_4 \equiv y = y_0 + \delta y_3. \tag{9}$$

In these relations x_o, y_o represent the idealized photo-coordinates on exactly vertical photographs of horizontal terrain; and δx_3, δy_3 represent the corrections for photo tilts and relief as in Case 3. Such a representation is convenient for simplifying the analysis. From Equations 9 and 8b the changes in digitally controlled movements are

$$d\delta x_4 \equiv dx \approx dx_0 - \frac{X}{c}dZ - \frac{(X^2 + c^2)}{c}(\varphi + \delta)$$
$$d\delta y_4 \equiv dy \approx dy_0 - \frac{Y}{c}dZ - \frac{(Y^2 + c^2)}{c}(\omega + \epsilon). \tag{9a}$$

Corrections for arbitrary known errors can be accommodated by the differential tracking; as the corrections change at a relatively slow rate, they do not essentially affect the ratio between the digitally and analogue controlled movements. Therefore these corrections will be disregarded here.

In the following paragraph the extreme conditions of application for the digitally controlled tracking will be appraised for each of the four classes of plotters.

Partition of Digitally Controlled Tracking

The ratios of incremental movements $d\delta x/dX$ and $d\delta y/dY$ are of prime importance as seen from the performance point of view. The most critical are the maximum digitally controlled movements occurring at the photo perimeter, particularly in super-wide angle photographs. The geometric relations, derived in the preceding paragraph, show that the amounts of the digitally controlled tracking increase with greater photo-tilts for all four cases and with height differences in the model for the Cases 3 and 4.

For the design and evaluation of the instruments the extreme circumstances (which might appear in real operations) should be considered. Such limiting conditions in digitally controlled tracking occur in the corners of a photograph. In this paragraph the relations under extreme circumstances will be examined for the four characteristic classes of instruments. For this purpose the following assumptions for the instrument parameters will be made: $X=Y=Z=c$; $\varphi=\omega=5^g$ and 3^g; and the terrain slopes $\delta X/\delta Z=\delta Y/\delta Z=1$.

CASE 1

The maximum values of the small variable angles δ and ϵ, and of the corresponding digitally controlled differential movements δx_1, δy_1, can be determined for the above assumptions by means of the Equations 4 and 6. The results are entered in Table I and are represented graphically in Figure 5. From this figure is it apparent that the relation between the photo-tilts φ, ω and the corresponding differential tracking is virtually linear. The *fractional movements* in x and y-directions are defined (by differentiating of Equation 6) as:

$$\frac{d\delta x_1}{dX} \approx \frac{cY}{Z^2}\,\omega, \qquad \frac{d\delta y_1}{dY} \approx \frac{cX}{Z^2}\,\varphi. \tag{10}$$

By substituting the assumed values for the instrument parameters in the Equation 10 the extreme values will be as shown in Table II. Thus the maximum partition of the digitally controlled movement corresponds to 8 percent of the input movement.

TABLE I

$\varphi=\omega$	δ	ϵ	δx_1	δy_1
5^g	2.3^g	2.4^g	7.6 mm.	8.1 mm.
3^g	1.4^g	1.5^g	4.4 mm.	4.6 mm.

TABLE II

$\varphi = \omega$	5^g	3^g
$\left(\frac{d\delta x_1}{dX}\right)_{max} = \left(\frac{d\delta y_1}{dY}\right)_{max}$	8%	4.7%

CASE 2

Without the use of analogue means for the photo tilts, the maximum amounts of the digitally controlled differential movements δx_2, δy_2 will be larger than in Case 1. They can be estimated according to Equations 7. The results are entered in Table III and illustrated in Figure 6. In this case the magnitude of digitally controlled movements is about 2.7 times of that in Case 1. The relation between the photo tilts and the corresponding tracking correction is nearly linear (Figure 6). The partition of the digitally controlled movements is defined by the differential coefficients

$$\frac{d\delta x_2}{dX} \approx \frac{c}{Z^2}\,(2X\varphi + Y\omega), \qquad \frac{d\delta y_2}{dY} \approx \frac{c}{Z^2}\,(X\varphi + 2Y\omega) \tag{11}$$

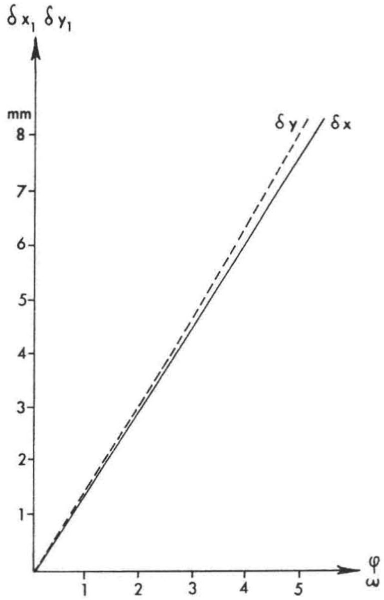


FIGURE 5

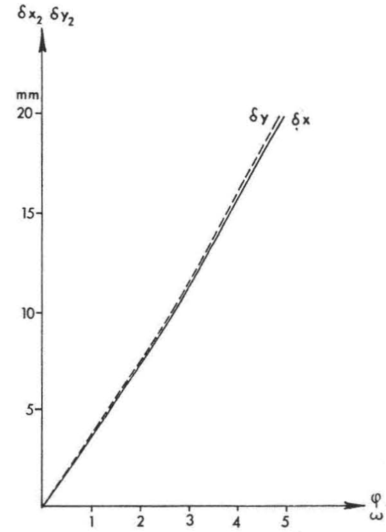


FIGURE 6

TABLE III

$\varphi=\omega$	δ	ϵ	δx_2	δy_2
5^g	2.3^g	2.4^g	20.5 mm.	20.9 mm.
3^g	1.4^g	1.5^g	12.4 mm.	12.8 mm.

TABLE IV

$\varphi=\omega$	5^g	3^g
$\left(\frac{d\delta x_2}{dX}\right)_{max}=\left(\frac{d\delta y_2}{dY}\right)_{max}$	23.7%	14%

and the maximum values are given in Table IV. In most critical circumstances the maximum partition of the digitally controlled tracking is about 24 percent (or 14 percent, respectively) of the input movement.

CASE 3

If in addition to the photo tilts, also the relief displacement is to be taken into account by the digitally controlled tracking, the values in Table III should be increased according to the effect of the terms $X(c-Z)/Z$ and $Y(c-Z)/Z$ of Equations 8a.

The partition of the digitally controlled movements is determined by

$$\begin{aligned}\frac{d\delta x_3}{dX} &\approx \frac{(c-Z)}{Z}+\frac{c}{Z^2}(2X\varphi+Y\omega)-\frac{cX\partial Z}{Z^2\partial X}\\ \frac{d\delta y_3}{dY} &\approx \frac{(c-Z)}{Z}+\frac{c}{Z^2}(X\varphi+2Y\omega)-\frac{cY\partial Z}{Z^2\partial Y}\,.\end{aligned}\tag{12}$$

By substituting the assumed values into Equation 12, the maximum partitioning is as indicated by Table V. Here the digitally controlled movements may *exceed* the basic input movements by about 24 percent (or 14 percent, respectively). This is caused mainly by the effect of terrain slope.

CASE 4

To this case belong those instruments where tracking is controlled fully by digital means. In Equations 9 the movements were resolved into ideal x_o, y_o and differential δx_3, δy_3 displacements. Hence, the partition of the digitally controlled movements can be represented by

$$\begin{aligned} \frac{d\delta x_4}{dX} &= \frac{dx}{dX} = \frac{dx_0}{dX} + \frac{d\delta x_3}{dX} \\ \frac{d\delta y_4}{dY} &= \frac{dy}{dY} = \frac{dy_0}{dY} + \frac{d\delta y_3}{dY} . \end{aligned} \tag{13}$$

By substituting for $x_0 = X$ and $y_0 = Y$, it follows that $dx_0/dX = dyc/dY = 1$. The terms $d\delta x_3/dX$ and $d\delta y_3/dY$ are defined by the Equations 12. Under extreme conditions the values of the differential coefficients are as shown in Table VI. Thus in the extreme case the digitally controlled movements are about 124 percent (or 114 percent, respectively) *greater than* the input movements. The requirements in the performance of the digital control system are therefore about twice as high as those in Case 3 (compare Tables V and VI).

The evaluation of the partitions of digital control for the four classes of instruments is possible by, e.g., comparing the values in the Tables II, IV and V with the corresponding values of the Table VI, which is taken as a common reference. These relative partitions of the digital tracking control are summarised in the Table VII.

TABLE V

$\varphi = \omega$	5^g	3^g
$\left(\frac{d\delta x_3}{dX}\right)_{max} = \left(\frac{d\delta y_3}{dY}\right)_{max}$	123.7%	114%

TABLE VI

$\varphi = \omega$	5^g	3^g
$\left(\frac{dx}{dX}\right)_{max} = \left(\frac{dy}{dY}\right)_{max}$	223.7%	214%

TABLE VII

$\varphi = \omega$	5^g	3^g
Case 1	4%	2%
Case 2	11%	7%
Case 3	55%	53%
Case 4	100%	100%

The tabulated values represent the fractional part of the tracking which is controlled by digital means in comparison with the total tracking that would be required under the extreme conditions assumed.

SPEED REQUIREMENTS IN DIGITAL CONTROL

The rate of digitizing the input movements, repetition rate of computing, and of the subsequent servo control required are determined by the size of steps and the accepted tracking velocity. Hence, the dynamic properties can be estimated after the digital steps and the tracking velocity have been assessed. The conditions are most stringent for the smallest steps and the maximum speed of tracking.

In practice the size of steps and the velocity of movements are often interrelated. In some modes of operation it is desirable to have variable steps which are related

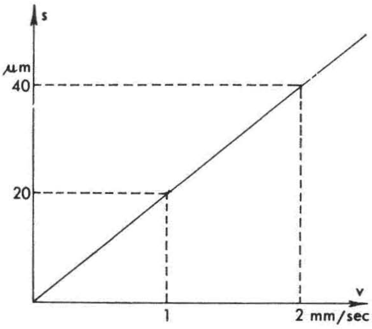


FIGURE 7

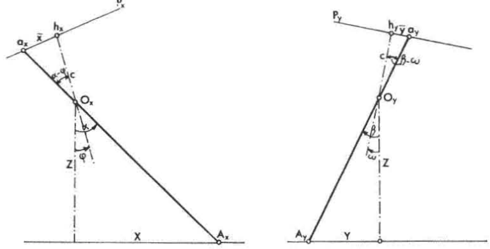


FIGURE 8

to the tracking velocity; in others (e.g., in the profiling mode) constant steps are indicated. However, the minimum frequency of the stepwise motion is limited in manually operated instruments by the need for a continuous (thus nonintermittent) movement of the measuring marks with respect to the corresponding images in the fields of view of the eyepieces. A perceivable stepwise movement would disturb the stereoscopic view. However, this limitation is not serious in practice. Another limitation to the size of the steps are the specifications pertaining to the output (e.g., accuracy if the enlargement from photographs to the map sheet is the maximum). The maximum tracking velocity might be restricted by the inertia forces.

For the purpose of illustration, the following quantitative assumptions have been made:

The size of steps in the photo-plane $s=20\ \mu$m (a half of the diameter of an average measuring mark) for the tracking velocity $v=1$ mm/sec.

It is assumed that the size of steps changes proportionally with the speed (Figure 7). By this, a constant rate (or frequency) of sampling the input movements, digital computing and generating the output movements are accomplished.

The frequency n required can be obtained according to the ratio: $n=v/u$ sec^{-1}, where u is the (total) increment of tracking. In hybrid instruments the tracking increment u is the sum of the step s, controlled by digital means, and the corresponding displacement d, controlled by analogue means. Thus

$$u = s + d = 20\ \mu m + d.$$

The proportions between the digital and analogue steps (s and d) for each of the four characteristic classes of instruments differs greatly. Assuming the extreme conditions, namely, $\varphi=\omega=5^g$ or 3^g, $X=Y=Z=c$, $\delta Y/\delta Z=\delta X/\delta Z=1$, the corresponding values of d, u and n are summarised in Table VIII. It is evident that the required frequency n in digital control varies over a wide range. In the Analytical Plotter AP-C, $n=30$, whereas in the new version of the Analytical Plotter AS-11A, $n=100$.

TABLE VIII

Instrument class	$\varphi=\omega=5^g$				$\varphi=\omega=3^g$			
	$s=20\ \mu$m	d	u	n	$s=20\ \mu$m	d	u	n
Case 1	8% d	250 μm	270 μm	4	4.7% d	426 μm	446 μm	2
Case 2	24% d	83 μm	103 μm	10	14% d	143 μm	163 μm	6
Case 3	124% d	16 μm	36 μm	28	114% d	18 μm	38 μm	27
Case 4	u	0	20 μm	50	u	0	20 μm	50

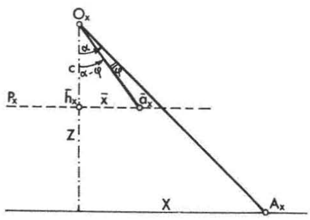
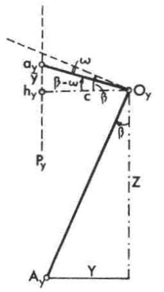


FIGURE 9



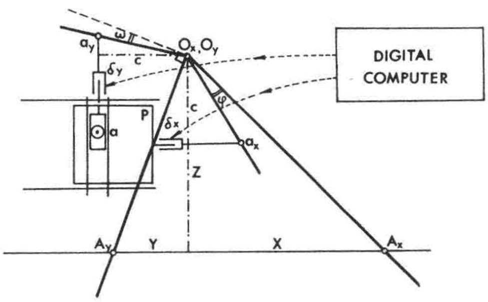


FIGURE 10

CONCLUSION

In digital tracking control, the most critical conditions arise in those operations that require rapid changes in the mechanical location of the tracking devices. An example is the profiling mode of operation, controlled automatically or manually.

If tracking is controlled in a fully digital manner, then the frequency of sampling, computing, and actuating the required movement is high (see Case 4, Table VIII). This, in turn, raises the demands on the dynamic capability of the equipment.

The conditions are considerably better in hybrid systems where only small differential displacements are controlled digitally. In instruments such as the Analytical Plotter AP-1 and the Digomat-Zeiss Jena (Case 3) the needs for digital differential tracking *in extreme conditions* is reduced to 55 percent of the need in fully digitally controlled tracking systems (e.g., Analytical Plotter AP-C, Case 4). If, in addition, an analogue Z-motion is used (Case 2), the requirements are reduced even more (to 11 percent). This solution is very feasible for practical implementation. A further reduction of the digital differential tracking can be achieved by the use of simple mechanical devices which partially account for the photo tilts (Case 1). The need for digitally controlled tracking is in this case only 4 percent of that in fully digital tracking control.

At present the requirements on the speed of operation in fully digital control systems can only be met at the expense of considerable technological effort and cost. It may be concluded that, from the photogrammetric instrument designer's point of view, the hybrid solutions appear to be the more attractive.

REFERENCES

1. Schoeler, H., "Über die perspektive Umbildung," *Jeaner Jahrbuch* 1957, S. 29–34.
2. Van den Hout, C. M. A., "Punttransformatie in de fotogrammetrie" *Tijdschrift voor Kadaster en Landmeetkunde,* 85e j. nr. 1, Febr. 1969, p. 47–58.

APPENDIX

THE PRINCIPLE OF CASE 1 PLOTTERS

The geometric relations in the XZ and YZ planes is shown in Figure 8, and a modified set of relations is illustrated by Figure 9. The principle of the design is shown for one projector by Figure 10.

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