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Propagation of Error in Blocks

The maximum random rms error of horizontal coordinates of passpoints is bounded and independent of the size of block analytic aerotriangulations with dense perimeter ground control.

Abstract: *Previous results obtained by various investigators regarding the precision of adjusted passpoint coordinates have been extended to simulated blocks of 406 photographs using an XDS Sigma 5 computer. The present results continue to confirm the conjecture that the maximum root-mean-square error of the horizontal coordinates of the passpoints is bounded and independent of the size of blocks with dense perimeter ground control. The error in determining passpoint elevations is not bounded but increases linearly with the number of photos in the block. Present indications are that augmenting the ground control, increasing the passpoint density, and/or using 60 percent side overlap will not result in bounds on the vertical error but serve to lower the rate of increase of the error with block size. The high precision of the results implies that the use of external sensors (such as TPR's or precise inertial reference units) for the control of vertical error buildup will be limited to blocks larger than 1,000 photos.*

Published results regarding the unexpected precision and accuracy that may be obtained by simultaneous block analytical aerotriangulation have been limited to date to blocks of moderate size, i.e., 100 to 150 photographs. In order to test whether any deterioration in precision or numerical difficulty occurs when the block size is increased, a series of simulations were performed for blocks up to 406 photographs. The simulations were performed under the following conditions:

- Camera altitude: 6,000 feet.
- 60 percent forward and 20 percent side overlap.
- Standard deviation of the photo coordinates: 5μm in x and y.
- Ground control was assumed errorless and limited to the block perimeter as illustrated in Figure 1.
- A regular 9 point passpoint pattern was used.
- Camera: 6 inch focal length, 9×9 inch format.

The simulations consisted of rigorously computing the standard errors of the adjusted passpoint coordinates. The method employed is described by Gyer and Kenefick[1] and consists in essence of rigorously computing the diagonal entries of the inverse of the normal equations matrix of the block adjustment. The rigorous computation is made feasible by exploiting the patterned structure of the normal equations.[2,3,4,5]

The computations were performed on DBA's Xerox Data Systems Sigma 5 computer which is comprised of 32,000 32-bit words of core, 3 magnetic tape units, a 3 million byte random access disc, and other peripherals. The speed of internal arithmetic operations of the Sigma 5 is approximately the same as an IBM 7094.

The simulations were performed for the five blocks shown in Table 1. The results of the simulations may be summarized as follows:

- ⋆ No numerical instabilities occurred using double precision arithmetic on the Sigma 5.
- ⋆ The conjecture (first broached by Ackermann in Reference 6, and then further explored more rigorously by Kunji in Reference 7 for blocks up to 105 photos) that the horizontal precision is independent of block size, when the block

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is subject only to dense perimeter ground control as illustrated in Figure 1, is completely corroborated for blocks up to 406 photos.

* The increase in vertical error is considerably less than expected. Perimeter control only seems adequate for blocks up to 300 photographs.

Regarding horizontal precision, a standard error of less than 0.2 foot in the X and Y coordinate was obtained for the midpoint of all five blocks (Figure 2). The distribution of the error is shown in Figures 3 and 4 for the 406 photo block. These results make us confident that the steady state condition for horizontal precision will hold for blocks of thousands of photographs.

In order to convert our standard horizontal error to blocks of identical configuration but different camera altitude, camera focal length, and photo coordinate precision, one simply multiplies the values given by $\sigma H/5{,}000c$ where

σ = standard error of photo coordinates in micrometers
c = focal length in inches
H = camera altitude in feet
$5{,}000 = \sigma H/c$ for $\sigma=5$, $c=6$, $H=6{,}000$.

Because a horizontal passpoint accuracy of one foot would be more than adequate for compiling planimetry at a scale of 200 feet per inch from the 6-inch focal length, 6000-feet photography, it seems that the perimeter control illustrated in Figure 1 is more dense than required. We shortly intend to investigate those perimeter control configurations which yield accuracies of about 1/6000 of the flying height for blocks up to 500 photos.

TABLE 1. THE BLOCKS ON WHICH THE SIMULATIONS WERE COMPUTED

Strips	*Photos/Strip*	*Total Number of Photos*
3	7	21
5	11	55
7	15	105
10	21	210
14	29	406

The vertical standard error at the midpoint of the block is plotted against block size in Figure 5 and contours of the vertical error are shown for the 105 and 406 photo blocks in Figures 6 and 7. Although the vertical error of the midpoint of the block increases with block size, its rate of increase seems to become nearly constant after 105 photographs as shown in Figure 5. The results for the midpoint are shown in Table 2.

Conversion of σ_z as given to different flight altitudes, format dimensions, and values of photo coordinate precision, may be accomplished by multiplying σ_z by $\sigma H/3.333d$ where d is the format dimension in inches. Extrapolating the above results for the vertical error at the midpoint based on the assumption of a constant rate of increase of error with block size gives the results shown in Table 3. Use of

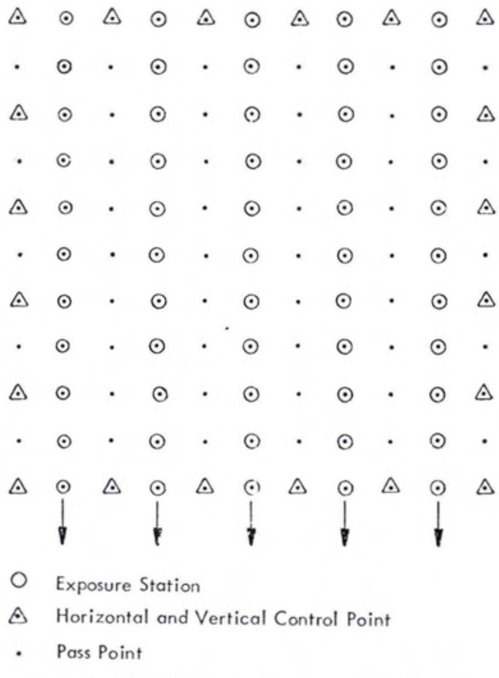

FIG. 1. The dense perimeter control scheme employed in simulations. This example is for the 55-photo block.

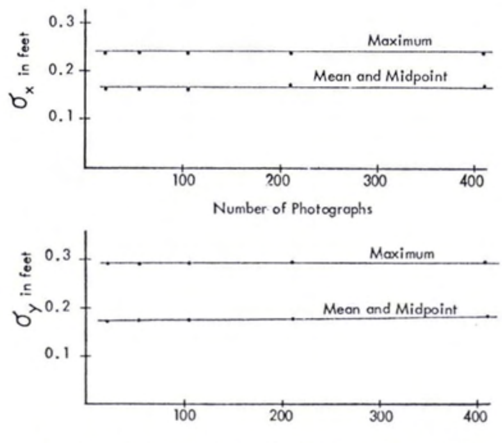

FIG. 2. Horizontal standard error as a function of block size. The maximum errors occur along the perimeter.

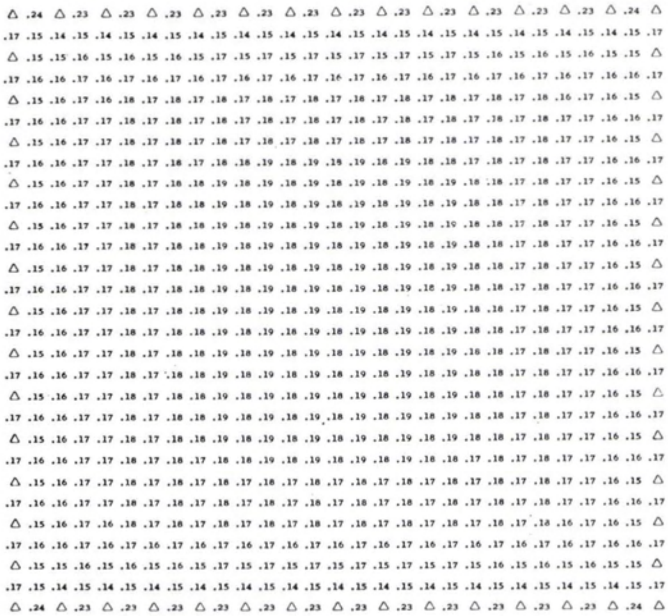

FIG. 3. Distribution of the X-standard errors in feet for the 406-photo block.

photography of the same scale (1/12,000) but obtained with a super-wide angle camera such as the Wild RC-9 (3.5 inch Super Aviogon lens) at 3500 feet altitude would theoretically result in vertical errors of 0.583 σ_z. Thus for the 528-photo block a precision of 1.0 foot might be obtained which would be just about adequate for 5-foot contouring. However the assumption of a 5 μm photo coordinate standard error is questionable over the entire format of a super-wide angle camera in view of the finding by various authorities such as Moren,[8] Hallert[9] and Brown[10] that there is a significant increase in the photo coordinate

FIG. 4. Distribution of the Y-standard errors in feet for the 406-photo block.

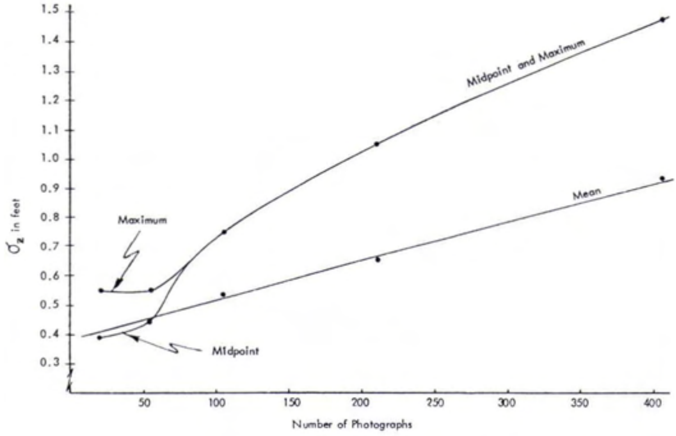

FIG. 5. Vertical standard error as a function of block size. For small blocks the maximum error occurs along the perimeter. As the block size increases the midpoint error increases and becomes dominate for blocks greater than about 75 photographs.

error with radial distance. It is planned to re-run some of the above simulations using a model of photo coordinate error, such as that given by Moren or Brown, to determine if any significant changes occur in the results given above.

For the wide-angle camera the block size limit for perimeter only vertical control seems to be a $12\times25=300$ photo block which results in a midpoint vertical standard error of 1.25 feet. For blocks with more than 300 photos the vertical error is larger than that

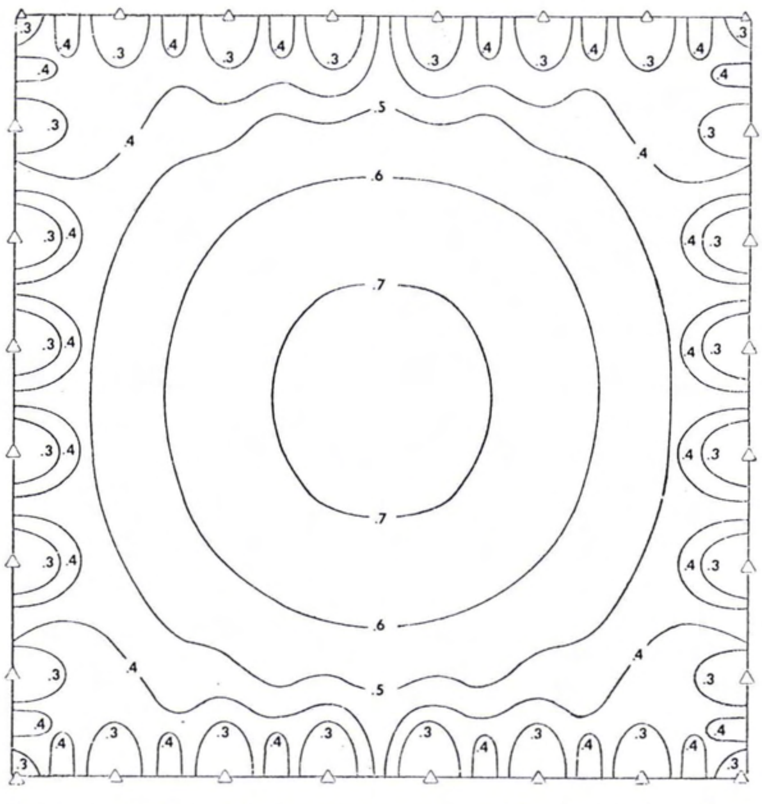

FIG. 6. Contours of vertical standard errors in feet for the 105-photo block.

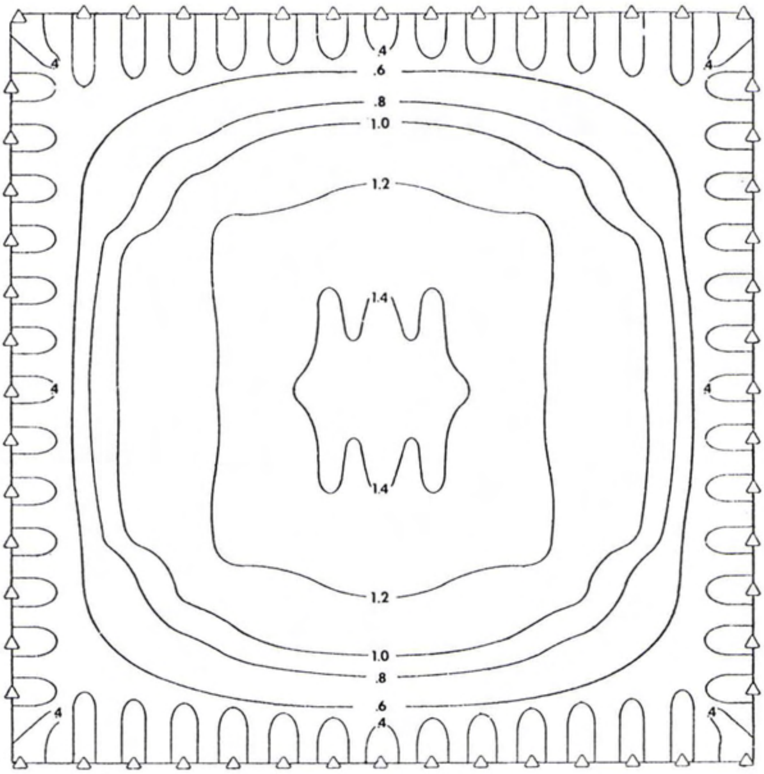

FIG. 7. Contours of vertical standard errors in feet for the 406-photo block.

generally considered adequate for 5-foot contouring.

Various methods may be considered for bounding the vertical error or lowering its rate of increase for blocks with perimeter control. Some of these are as follows:

- Increasing the passpoint point pattern to 15, 25, or 50 points per photograph.
- Increasing the side overlap to 60%.
- Augmenting the perimeter control with ground control in the interior of the block.
- Utilizing external sensors such as terrain profile recorders, statoscopes, or camera attitude determination systems such as star trackers or precise inertial vertical reference units.

TABLE 2. VERTICAL STANDARD ERROR AT THE MIDPOINTS OF BLOCKS

Block Size	σ_z = *Vertical Standard Error*	*Proportion of Camera Altitude* $=\sigma_z/6,000$
3× 7= 21 photos	0.39 feet	1/15,400
5×11= 55	0.54	1/11,100
7×15=105	0.74	1/ 8,100
10×21=210	1.05	1/ 5,700
14×29=406	1.47	1/ 4,800

Increasing the passpoint pattern will lower the rate of increase of vertical error with block size but should not result in a steady state condition. With regard to blocks with 60 percent side and forward overlap, we would be surprised, on the basis of present results, if such blocks achieved a steady-state condition with control limited to the perimeter. Such blocks however should exhibit a marked decrease in the rate of increase of error with block size as compared with the results given above for the 20 percent side overlap case as was demonstrated by Kenefick[11].

The introduction of control into the interior the block (as was done by Kunji, for example, who performed simulations with

TABLE 3. EXTRAPOLATED VALUES OF TABLE 2 ASSUMING A CONSTANT RATE OF INCREASE OF ERROR WITH BLOCK SIZE

Block Size	σ_z	$\sigma_z/6,000$
16×33= 528 photos	1.7 feet	1/3,500
20×41= 820	2.4	1/2,500
23×47=1081	2.9	1/2,100

perimeter control and a line of vertical control down the center of the block) will not result in bounds on the vertical error but again only lower its rate of increase.

The value of a TPR for blocks up to 528 photos seems doubtful because the combined errors in measuring the terrain clearance, deviations from the isobar, and determining isobaric slope between checkpoints is greater than the vertical standard error of (1/3500) H given above. For blocks up to 1000 photos a significant improvement in the maximum vertical standard error of $(1/2100)H$ for the 1081 photo block would still be difficult to attain using presently available TPR equipment and techniques.

In order to attain significant improvement in the vertical error through use of external camera attitude sensors, the sensors must be of very high order of precision. This may be seen from the magnitudes of the standard errors of camera pitch and roll shown in Table 4. An attitude sensor must have an accuracy significantly better than above, i.e., in the 10-second range, in order to significantly decrease the vertical error of the block. As an example of the effect of precise attitude information the 105-photo block adjustment simulation was rerun assuming the camera pitch and roll were known to 10 arc seconds. The result was to reduce the vertical standard error at the midpoint of the block from 0.74 to 0.33 feet. However a verticality sensor capable of 10 arc seconds (1σ) precision costs between one-half to one million dollars.

Running times required to perform the above error propagation computations on the XDS Sigma 5 are shown in Table 5. The above times include formation of the normal equations, inversion of the reduced normal equations by the Method of Recursive Partitioning, and computation of the standard errors of the passpoint ground coordinates. A more detailed breakdown of the time for the 406-photo block is shown in Table 6.

TABLE 4. ERRORS IN EXTERNAL ATTITUDE SENSORS

Block Size	*Pitch Standard Errors in Seconds*		*Roll Standard Errors in Seconds*	
	Max	*Mean*	*Max*	*Mean*
21 photos	23	14	19	13
55	25	15	19	15
105	26	15	20	16
210	27	15	24	17
406	28	16	29	22

TABLE 5. COMPUTING TIMES ON THE XDS SIGMA 5

Block Size	*Total Running Time*
3× 7= 21 photos	10 minutes
5×11= 55	20
7×15=105	45
10×21=210	120
14×29=406	6 hours 14 minutes

Our investigations to date may be summarized as follows:

▲ The steady-state effect for horizontal passpoint errors in blocks with dense perimeter control has been corroborated for blocks up to 406 photos and doubtless continues to hold for blocks of thousands of photos.

▲ The rate of increase of vertical error is constant for larger blocks with dense perimeter control. For a 9-inch format, 6000-feet flight altitude, and plate sigma of 5 μm the rate of increase is 0.21 feet per 100 photos.

▲ For blocks of up to 300 wide-angle photographs with dense perimeter control the vertical accuracy is adequate for mapping purposes. For blocks with more than 300 wide-angle photos recourse must be had to additional ground control, increase of passpoint pattern, 60 percent side overlap, and/or use of a precise attitude sensor. Further investigation is planned in these directions. The value of a TPR in improving vertical accuraies in blocks up to 1000 photos with dense perimeter control seems doubtful because the TPR system accuracy is not much greater than the vertical accuracy obtainable from the block adjustment.

▲ For super-wide angle cameras, the verticle accuracy obtainable in blocks of up to 800 photographs with dense perimeter control is adequate for mapping purposes, assuming a uniform plate sigma of 5 μm over the entire

TABLE 6. BREAKDOWN OF COMPUTER TIME FOR A BLOCK OF 406 PHOTOGRAPHS

Sub-Program	*Running Time Includirg I/O*
Normal equations formation.	5 minutes
Triangularization of the reduced normal equations.	120
Computation of the inverse of the reduced normal equations.	170
Computation of the 3×3 covariance matrices of the ground coordinates of all the passpoints.	79

format. This is a doubtful assumption and it is planned to perform additional simulations to test the effects of increasing plate sigma with radial distance.

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