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External Block Adjustment of Planimetry

The method is easily programmed and is suitable for a very small computer.

Abstract: *This paper discusses current methods for the adjustment to ground control of an internally adjusted block and related methods for the determination of a surface from its heights at discrete points. A new method for planimetric block adjustment is developed from one of the latter. However, a distinguishing feature of the method is the adjustment of planimetric position rather than that of the coordinates separately. The method is easily programmed and is suitable for a very small computer. For experimental purposes, the method has been combined with an internal block adjustment consisting in polynomial transformation of strips. Results are compared with those obtained with the standard polynomial block adjustment used at the author's organization.*

Introduction

The adjustment of a block of aerial photography can be performed in two separate steps: internal adjustment and external adjustment. The internal adjustment serves to adjust the units of which the block is composed relative to each other, without considering ground control. The external adjustment serves to adjust the resulting block to ground control without appreciably altering the relative positions of points in any small area. One likely advantage of this adjustment in steps is that it will require less computer time and storage than a simultaneous internal and external adjustment.

The photogrammetric literature on external block adjustment is extremely scarce. This adjustment is, however, closely related to another photogrammetric problem: that of determining the shape of the earth's surface in a given area from the given heights of a number of its points. If the internally adjusted block is placed in an approximately correct position, the corrections which remain to be applied are relatively small and are continuous functions of position in the block. If, for each coordinte separately, these corrections are plotted as Z-coordinates at the position of the points, the plotted points from a *correction surface* and the problem of determining the shape of this surface is identical with that of determining the shape of the earth's surface. In addition, many problems in other disciplines concern the determination of a quantity as a function of position and can be formulated in the same way.

It appears from the available photogrammetric and nonphotogrammetric literature that at least four different approaches apply to the solution of this problem. These are discussed in the next section of this paper.

The method of external block adjustment of planimetry, which is the main subject of this paper, is based on the most promising one of these methods. However, a separate adjustment of easting and northing is undesirable. Instead, a simultaneous adjustment is performed. In contradistinction to the separate adjustment, this simultaneous adjustment can, if appropriate, apply an exact rotation and an exact conformal transformation.

To evaluate this method, it has been incorporated in an experimental program, together with an internal adjustment which consists in the polynomial transformation of

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strips. The program has been used for the adjustment of the available test blocks and the results obtained are compared with those obtained with a program for the simultaneous block adjustment of strips by polynomial transformation. In addition, the method has been applied to the correction of measured photograph coordinates for film distortion.

Discussion of Methods

If the planimetric easting and northing coordinates are to be adjusted separately, as in the following four methods, the internally adjusted block must first be approximately positioned and scaled. This step may consist in a simple similarity transformation. It may contain also a correction for deformation, for instance by means of a polynomial transformation. The external adjustment has now the task of eliminating residual errors in position and residual deformation. Of the following four methods, the first and the third ones are in use for the interpolation of terrain heights; the second and the fourth ones were designed for use in strip and block adjustments.

THE SEQUENTIAL DETERMINATION OF A GRID

Here, as the first step in the calculation, the required corrections at the points of a uniform square grid are determined in a sequential manner.[1] First, corrections are determined for each corner of each square in which a control point occurs. For this purpose, in each of these squares the plane is constructed which can best represent the correction surface in the area of the square. The heights of the plane at the corners of the square are provisionally accepted as the required corrections. Next, the corrections at grid points which are at progressively greater distances from the control points are computed. This computation is done in the same way, and makes use of previously calculated corrections at grid points. An averaging procedure and a smoothing procedure are included to improve the results and to eliminate irregularities.

A satisfactory result can be obtained only by sophisticated programming. Much attention must be paid to the method of calculating the planes and to the sequence in which the planes are calculated.

The second step in the computation consists of the calculation of the corrections at all required points, or in the calculation of contour lines on the surface with the help of the established corrections at the grid points. Here, an interpolation procedure will be used. If the grid is sufficiently fine, a simple type of linear interpolation is satisfactory.

POLYNOMIAL TRANSFORMATION OF THE BLOCK

Here, the adjustment consists in the transformation of all points in the block by means of the same same set of transformation formulas. The parameters in these formulas are determined first, with the help of all available control points. Usually a polynomial transformation will be selected because this simplifies the computation of the parameters.

The pre-selection of a specific set of transformation formulas has serious disadvantages. If a polynomial of low degree with respect to the planimetric coordinates is used, the number of parameters will be smaller than the number of control coordinates and as a result a satisfactory fit at the control points is not assured. If a polynomial of high degree is used, large errors can be produced locally in uncontrolled areas of the block.

These disadvantages can be alleviated by the use of orthogonal polynomials.[2,3] These make it possible to recognize and select only terms which contribute significantly to the fit at the control points. On the other hand, because the control points in a block will never be located exactly at the corners of a uniform grid, the use of orthogonal polynomials makes the derivation of transformation formulas more complicated. Reference 1 contains also a program for surface fitting with orthogonal polynomials. If here the grid points are used as control points, the calculation becomes very simple.

INDIVIDUAL TRANSFORMATION OF POINTS

This method differs from the preceding one in that a separate set of transformation formulas is derived for each point in the block. The transformation formulas can be polynomials with respect to the planimetric coordinates and they need not be of higher than the second degree.

As is the case in the preceding method, the coefficients in the transformation formulas will be derived by the procedure of the method of least squares: each required coordinate correction at a control point leads to one correction equation, normal equations are formed from the correction equations and are solved for the coefficients. Here, however, the contribution of a correction equation to the normal equations is given a weight that is a function of the distance between the point that will be adjusted and the control point.[4,5,6] This makes the coefficients in the transformation formulas different for each point which is transformed.

This method makes it possible to obtain a good fit at any number of control points, simply by means of attaching a sufficiently large weight to the contributions from nearby control points. A good overall result of the adjustment can be obtained by careful selection of a weight function and a polynomial.

The same method is used in Reference 7 to compute the heights at the points of a uniform grid. Further computations are then based upon the heights of these grid points.

D. W. G. Arthur's Interpolation Method

With this method, each point is given a coordinate correction that is a linear function of *correlates*.[8] There is one correlate for each control point. The coefficient that is attached to a correlate is a function of the distance between the point whose coordinate correction is being determined and the control to which the correlate is assigned.

First, these linear equations are formulated for the control points themselves. They contain the correlates as unknowns. By a proper arrangement of the equations and of the correlates, this gives a set of linear equations with a symmetric matrix of coefficients. The correlates are solved from these equations. Next, the values obtained are used to correct all points in the block.

This method would be attractive both because of its computational simplicity and because it gives an exact fit at the control points. However, it has two major drawbacks.

First, taking the instance where four control points are situated at the corners of a square, the symmetric matrix turns out to be singular. Consequently, the correlates cannot be solved from the equations. Because this should be a well-defined case, this throws doubt on the validity of the method. The same difficulty has occurred also in a test made with 21 control points in random locations.

Second, even if the matrix is not singular, the adjustment does not apply a correction which is a linear function of position even in instances where this is obviously warranted. An example of this is the case of a height adjustment with three control points located at the vertices of an equilateral triangle. A tilt correction is here accompanied by a complicated height deformation.

In Arthur's method as well as in the two preceding ones, the correction at any point can be written as a linear function of the required corrections at the control points. In Arthur's method, the coefficients in this function are functions of distances only. One can devise other methods in which the coefficients are functions of distances only and which do not have the first of the above drawbacks. However, it is difficult to see how in the case of three control points such a method could give points on the line between two of these points a correction which is independent of the correction at the third one. Therefore, any method which makes use of distances only appears to be unsuitable.

Description of the Developed Method

Transformation Formula

It follows from the preceding discussion that the third one of the four methods is the most promising one for use in an external block adjustment of coordinates. However, as indicated in the introduction, the separate adjustment of the two planimetric coordinates is undesirable. Instead, a simultaneous adjustment can be performed by operating not on these coordinates, X and Y, separately but on the complex number $X+iY$. This has the advantage that in any small area the transformation formula represents a conformal transformation.

Therefore, let the external block adjustment of planimetry consist in the polynomial transformation of the planimetric coordinates of a point by means of the formula for conformal transformation of the second degree:

$$(E + iN) = (X + iY) + (e_1 + ie_2) + (e_3 + ie_4)(X + iY) + (e_5 + ie_6)(X + iY)^2 \qquad (1)$$

Here, E and N are the adjusted coordinates (easting and northing),
X and Y are the coordinates before the adjustment,
i is the imaginary unit, $\sqrt{(-1)}$, and
e_1 to e_6 are variable parameters.

From this equation, one can obtain separate transformation formulas for X and for Y by performing the multiplications in the second part, replacing i^2 by -1, and separating the real and the imaginary terms. The two separate transformation formulas contain the same six parameters. In addition, these parameters will be computed from the required corrections to both coordinates of control points. These two features make the adjustment a simultaneous adjustment of eastings and northings.

The actual separation is not needed at any stage of the computations and it is even undesirable. The use of complex numbers eliminates duplication of computations and storage during the formation and solution of normal equations. Even if the available computer software does not include operations on complex numbers, the duplication can be avoided by performing the corresponding operations on real numbers.

For the computation of the parameters, the formalism of the method of least squares is used. Therefore, it involves the formation of a correction equation for each planimetric control point, the association of a weight with each correction equation, and the formation and solution of normal equations.

The weights are functions of distances, as described in the section, *Individual Transformation of Points*. This feature makes the values of the parameters different for each point that is transformed. Therefore, although Equation 1 with constant values of e_1 to e_6 represents a conformal transformation, the block adjustment is in general not a conformal transformation.

CORRECTION EQUATIONS AND WEIGHTS

The first step in the computation of a set of parameters is the formation of one correction equation for each planimetric control point. For this purpose, its given corrdinates E and N and the obtained coordinates X and Y are substituted into Equation 1.

To enhance the influence of nearby control points on the adjusted position of a point, a weight is associated with each correction equation. The weight should be a monotonic decreasing function of the distance from the point to the control point. To obtain the coordinate corrections $E—Y$ and $N—Y$ that are continuous and smooth functions of position in the block, the weight must be a continuous and smooth function of distance. In addition, if one specifies that for the adjustment of a point only control points within a certain maximum distance shall be used, the weight must approach tangentially to zero if the distance approaches its maximum value.

Although the selection of a suitable weight function is the main problem in this adjustment, no information on this has been found in the literature. It has required an extensive investigation. This investigation was conducted initially for a different project: the computation of terrain contours and profiles from discrete terrain heights.

The above requirements are satisfied by the weight function

$$w = 1 + \cos \pi r.$$

Here, r is the ratio between the distance to a control point and the specified maximum distance. This function was the first to be used in the computation of a terrain profile from discrete heights in a given area. However, using a second-degree transformation and various specifications for the maximum distance, it failed to give a reasonable approximation of the profile.

Some improvement was obtained by adding a factor $(1-r^2)$ to the weight function. This decreased the influence of the more distant control points. A satisfactory result was finally obtained when a factor $1/r$ was added. This greatly increased the influence of the nearest points.

The unrestricted use of the factor $1/r$ would produce an exact fit of the adjusted block at all control points, even at erroneous ones. To prevent this and possibly appreciable distortion in the neighborhood of control points, ratios smaller than 0.01 were replaced by this value.

Because of these departures from the trigonometric function, the factor $(1+\cos\ \pi r)$ was next replaced by the computationally simpler factor $(1-r)^2$. The graph of this function also, is horizontal at $r=1$. This replacement did not adversely affect the adjustments.

Further experiments were performed with weight functions which contained various powers of $1-r$ and of $1-r^2$ as factors, in addition to a factor $1/r$. Optimum results were obtained with the following function:

$$w = (1 - r)^3(1 - r^2)^3/r. \qquad (2)$$

In experiments with the external adjustment of planimetry, also, this weight function has given satisfactory results. Optimum results have been obtained by making the specified maximum distance somewhat larger than the maximum distance which occurs in the block.

NORMAL EQUATIONS

Conventionally, in the correction equation the terms which contain the parameters are placed in the first part and the known terms are placed in the second part. The correction equation can now be represented by the matrix equation

$$\boldsymbol{a}_r\boldsymbol{x} = b. \qquad (3)$$

Here, $\boldsymbol{x}$ is the column vector whose components are the three complex parameters e_1+ie_2, e_3+ie_4 and e_5+ie_6,

$\boldsymbol{a}_r$ is the row vector whose components are the coefficients 1, $X+iY$, and $(X+iY)^2$ of the parameters, and

b is the sum $E-X+i(N-Y)$ of the terms in the second part.

Let further $\boldsymbol{a}$ be the column vector which is the complex conjugate of the transpose of $\boldsymbol{a}_r$. Its components are 1, $X-iY$, and $(X-iY)^2$.

Each correction equation is now used to compute a 3×3 matrix $w_r\boldsymbol{aa}$ and a 3-component column vector $wb\boldsymbol{a}$. The matrix of coefficients of the normal equations and its vector of second parts are the sum of these matrices and the sum of these vectors, respectively. Thus, indicating a summation by square brackets, the normal equations are

$$[w\boldsymbol{a}\boldsymbol{a}_r]\boldsymbol{x} = [wb\boldsymbol{a}]. \qquad (4)$$

The matrix of coefficients of the normal equations is hermitian. This means that the elements on the main diagonal are real and that the elements which occupy positions symmetric with respect to the main diagonal are each others' complex conjugate. Therefore, there is no need to compute and store the elements below the main diagonal.

The normal equations are solved by successive elimination of the unknowns, using successive elements on the main diagonal as pivotal elements. Elements below the main diagonal, if needed, are found as the complex conjugate of an element above the diagonal. After each elimination of un unknown, the matrix of coefficients is again hermitian.

Some advantage may occur in temporarily shifting the origin of the planimetric coordinates to the point that is being adjusted. This makes the last two terms in Equation 1 for that point equal to zero. Consequently, the corrections to its planimetric coordinates become simple e_1 and e_2. Taking e_1+ie_2 as the last unknown in the normal equations, back substitution for solving the other unknowns is not needed.

Experiments

The external block adjustment must be preceded by an internal adjustment. A separate program for this purpose has not been developed at National Research Council of Canada (N.R.C.). However, the existing N.R.C. program for block adjustment of strips by polynomial transformations[9] can be used as such. To obtain an internal planimetric adjustment, one uses the planimetric control points either not at all or only in one of the strips.

A comparison of this combination of internal and external adjustment with the polynomial block adjustment of strips can now give some practical information on the usefulness of this method of external adjustment. The combination is of interest especially in instances of light or medium dense ground control. In the case of very dense control, the internal adjustment should not consist in polynomial transformation of strips.

This comparison has been completed for four blocks of aerial photographs covering two separate test areas, and for two blocks of fictitious photography. Where photograph coordinates were measured or were provided, they were first converted to strip coordinates by analytical strip triangulation.

Each block has been subjected to four different adjustments. In the tables of results, these are labelled as follows:

i. Internal adjustment of the second degree followed by external adjustment.
ii. Internal adjustment of the third degree, followed by external adjustment.
iii. Polynomial block adjustment, as in ref. [9]. of the second degree.
iv. Polynomial block adjustment, as in ref. [9], of the third degree.

International Test 1960–1964

For this test, 300 non-targeted control points were determined in an area of about 80×80 km with a maximum height difference of 1,100 m. In the N.R.C. participation the eight east-west strips of Wild RC7 plate photography were used. These strips contain 213 photographs of 140×140 mm at scale 1:80,000.

For the present tests, the Zeiss Jena 1818 Stereocomparator measurements and the Wild A7 measurements were used. For use with the present programs, this data had to be copied into new cards, re-sorted and partly re-named and debugged. Subsequently, it has been used not only in the internal and external block adjustments but also in new polynomial block adjustments.

The planimetric adjustments have been performed with two control points in each corner of the block and in the middle of each side. In the earlier adjustments, the selection of two control points in each location served as a safeguard against identification errors. In the present experiments, some control points which proved to be unreliable have been replaced by others. The average distance between the two control points in the same location is 6 km.

The two internal adjustments have been

TABLE I. INTERNATIONAL TEST 1960–64, WILD RC7 PLATE PHOTOGRAPHY

Strip	Check Points	Internal and External Adjustment		Polynomial Block Adjustment	
		i	*ii*	*iii*	*iv*
Zeiss Jena 1818 stereocomparator measurements					
1	26	3.9 m	4.7 m	4.2 m	4.4 m
2	35	4.0	4.7	4.3	4.5
3	35	4.5	5.2	4.7	5.1
4	39	4.3	4.9	4.6	4.7
5	30	4.4	4.9	4.7	4.8
6	38	4.9	5.0	4.8	4.7
7	38	4.5	4.2	4.7	4.1
8	29	4.5	4.7	4.8	4.0
block	270	4.4 m	4.8 m	4.6 m	4.6 m
Wild A7 triangulations					
1	29	7.6 m	8.5 m	7.8 m	6.0 m
2	37	6.2	6.1	5.8	4.9
3	28	6.8	5.8	5.2	4.4
4	36	6.5	5.6	6.7	5.1
5	34	6.1	6.0	5.8	5.0
6	37	8.5	7.7	8.4	6.4
7	35	5.5	5.7	5.5	5.2
8	32	7.8	5.8	8.8	6.1
block	268	6.9 m	6.5 m	6.9 m	5.4 m

started with the positioning of strip 1 in the ground-control system. Because this strip has ground-control points in essentially only three locations, a planimetric transformation of the second degree has been used in both adjustments. For all following strips, the specified degree has been used.

Table I gives the root-mean-square values of the residuals in the position of all reliable check points after the adjustments. These values are listed for the points in each strip indvidually as well as for the whole block. To simplify the computations, the values for the block have been derived directly from the values for the strips, without taking the mean of the coordinates of points which occur in the overlap of two strips.

Table Ia gives the root-mean-square values of the residuals in position at the 16 control points. The table shows that the weighting in the external adjustment has satisfied the requirement that the residuals at control points whould not be zero if neighboring points disagree with each other.

TABLE IA. INTERNATIONAL TEST 1960–1964. ROOT MEAN SQUARE VALUES OF THE RESIDUALS IN POSITION AT THE CONTROL POINTS

	Internal and External Adjustment		Polynomial Block Adjustment	
	i	*ii*	*iii*	*iv*
Zeiss Jena 1818	0.6 m	0.8 m	2.2 m	2.0 m
Wild A7	1.0	1.1	3.6	2.3

FREDERICTON TEST AREA

The Fredericton (New Brunswick) test area was established in 1963. It measured about 20×30 km and contained about 120 targeted ground-control points, located mostly in the flat cultivated northern and eastern parts. The remainder of the area is hilly and covered with a fir forest which makes relative orientation somewhat difficult.

The area was covered with Wild RC8 photography in 3, 4, and 6 usable east-west strips at scales 1:40,000, 1:24,000, and 1:12,000, respectively. The photographs were triangulated in the Wild A7 and the Stereoplanigraph C8 of the N.R.C. Photogrammetric Section and by different operators.

The coordinates of the 1:40,000 and 1:24,000 strips needed correction for differential shrinkage of the photographs. For this purpose, the strip Y-coordinates were given a scale correction, determined separately for each strip from measurements of distances between fiducial marks on a few photographs.

The planimetric adjustments have been performed with eight well-separated control points: four along the northern edge of each block and four along the southern edge. The results obtained at the check points are listed in Table II. Because of the wide separation of the control points, the residuals at these points after the external adjustments are practically zero.

ITC* BLOCK 1964

The ITC-publication A27/28 of 1964 contains strip coordinates of 10 fictitious strips of 30 models each. The strips cover an area of 80×120 km. The assumed photograph size is

* International Training Center, Kanaalweg 3, Delft, The Netherlands.

TABLE II. FREDERICTON TEST AREA, WILD RC8 FILM PHOTOGRAPHY

Block	Check Points	Internal and External Adjustment		Polynomial Block Adjustment	
		i	*ii*	*iii*	*iv*
Wild A7 triangulations					
1:40,000	119	1.2 m	1.2 m	1.2 m	1.1 m
1:24,000	102	1.1	1.8	1.2	1.7
1:12,000	95	1.6	1.7	1.5	1.7
Zeiss C8 triangulations					
1:40,000	124	2.0 m	2.5 m	2.1 m	2.2 m
1:24,000	102	2.5	2.3	2.5	2.8
1:12,000	96	2.8	2.3	3.0	1.7

TABLE III. ITC BLOCK, FICTITIOUS DATA

Strip	Check Points	Internal and External Adjustment			Polynomial Block Adjustment	
		i	*ii*	*ii**	*iii*	*iv*
With correction for differential shrinkage						
2	90	2.5 m	4.7 m	4.9 m	3.2 m	5.6 m
3	92	6.4	5.6	5.7	6.0	6.6
4	93	3.7	5.3	5.2	3.7	6.2
5	93	3.3	4.4	4.3	4.1	5.7
6	91	4.6	4.0	4.0	6.4	5.5
7	93	4.2	3.1	3.1	5.5	5.3
8	93	4.8	3.3	3.0	4.6	5.1
9	93	3.6	3.4	3.0	3.4	4.5
10	90	3.1	3.6	2.6	2.7	4.4
block		4.2	4.2	4.1	4.6	5.5
Without correction for differential shrinkage						
2	90	3.1 m	4.1 m	5.4 m	3.5 m	3.6 m
3	93	6.6	5.5	5.8	6.6	5.9
4	93	4.1	5.5	5.4	5.4	6.3
5	93	3.3	4.9	4.5	6.0	6.5
6	91	4.3	4.2	4.2	7.7	6.8
7	93	4.3	3.1	3.2	7.0	6.0
8	93	4.7	3.8	3.2	5.8	5.4
9	93	3.3	3.5	3.2	4.3	4.3
10	90	3.2	3.8	3.2	2.9	2.9
block		4.2	4.4	4.4	5.7	5.5

* With external adjustment of the third degree.

230×230 mm, the scale 1:43,500, the focal length 152 mm. The ground control is a level grid of 4×4 km. Each model contains six grid points in the usual locations for relative orientation and six orientation points in virtually the same locations. The latter points serve also as tie points between strips. The photograph coordinates, derived from this data, have received various systematic and random errors. The strip coordinates are the result of an analytical triangulation performed at the ITC with the thus modified photograph coordinates. In the N.R.C. calculations, strip 1 showed a break at or near model 12 and has not been used in the adjustments.

Each of the planimetric adjustments has been performed with eight ground-control points: one in each corner and one in the middle of each side. Adjustments were performed both with and without applying a correction for systematic differential shrinkage of the photographs. This shrinkage amounts to 0.18 parts per thousand. In each adjustment a correction for earth curvature has been included. The root-mean-square values of the residuals at the check points are listed in Table III.

INTERNATIONAL TEST 1968

For this test, fictitious photograph coordinates for a block of 9 by 20 photographs were provided. The photograph size is 230 ×230 mm, the focal length 152 mm, and the scale about 1:65,000. The photographs are approximately vertical and their perspective centers form an approximately rectangular grid of 6×6 km. The ground-control points form a similar grid of 3×3 km, with elevations of up to 1,800 m. Each perspective center is located approximately above a ground-control point. This arrangement gives 25 image points per photograph. The photograph coordinates are affected by various systematic and random errors.

In the N.R.C. participation, the photographs were arranged in 9 strips of 20 photographs each. The strips were triangulated analytically, using the available 15 control points in each model for relative orientation. The odd-numbered strips and the even-numbered strips were used as two separate blocks, each with normal sidelap. These blocks measure about 60×120 km and 48×120 km, respectively.

The planimetric adjustments of the second and of the third degree have been performed

TABLE IV. INTERNATIONAL TEST 1968, FICTITIOUS DATA

Strip	*Check Points*	*Internal and External Adjustment*		*Polynomial Block Adjustment*	
1	117	1.2 m	0.8 m	1.2 m	1.0 m
3	117	1.1	1.1	1.1	1.5
5	117	1.0	1.2	1.3	1.8
7	117	1.5	1.4	1.7	1.6
9	117	1.0	1.3	1.1	1.2
block 1	585	1.2	1.2	1.3	1.5
2	117	1.7	1.0	1.9	0.9
4	117	1.2	1.0	1.2	1.2
6	117	1.0	1.0	1.0	1.1
8	117	1.1	1.0	1.1	1.0
block 2	468	1.2	1.0	1.4	1.1

with six and with eight ground-control points, respectively. These points are equally spaced along the outside edges of the two outside strips. The points on the axis of the strip and the points midway between the axis and the edges were used as check points. The control points and the tie points between the strips are located on the strip edges. The use of the latter as additional check points would have complicated the computations. Each of the block adjustments includes a correction of the strips for earth curvature.

Table IV gives the root-mean-square values of the residuals at the check points.

CORRECTION FOR FILM DISTORTION

The method of external adjustment of planimetry can be applied also to correct measured photograph coordinates for film distortion. In this case, the fiducial marks serve as control points. To evaluate the accuracy of the corrections, reseau photographs can supply the necessary check points.

In a small experiment, using a linear conformal transformation (that is, omitting the second-degree term in Equation 1) and using reseau photographs collected by Dr. H. Ziemann of the N.R.C. Photogrammetric Section, the introduction of weights according to Equation 2 improved the results by 30 percent on the average. Further experiments and more detailed results will be included in a forthcoming paper on this subject by Dr. Ziemann.

TABLE V. LARGEST RESIDUAL IN POSITION AT A CONTROL POINT AFTER INTERNAL ADJUSTMENT

Block	*i*	*ii*	*i*	*ii*
Int. test '60–'64	Zeiss Jena 1818		Wild A7	
1:80,000	18 m	55 m	60 m	200 m
80×80 km				
Fredericton	Wild A7		Zeiss C8	
1:40,000	7m	8m	12m	11m
1:24,000	9	8	20	25
1:12,000	8	20	21	41
20×30 ,m				
ITC block	with correction		without correction	
1:43,500	31 m	77 m	41 m	90 m
72×120 km				
Int. test '68	block 1		block 2	
1:65,000	10 m	9 m	11 m	11 m
60 and 48×120 km				

DISCUSSION OF RESULTS

ACCURACY OF THE APPROXIMATE POSITIONING

Before evaluating the results of the external block adjustment, it is necessary to investigate whether the approximate positioning which is performed during the internal adjustment is sufficiently accurate. Here, planimetric ground control has been used only in the first transformed strip. Because this strip is located on one side of the block, the error in planimetric position in the following strips can be expected to become progressively worse.

Table V shows the largest residual in position at the control points after each of the internal adjustments. In some instances, especially in the third-degree internal adjustments, this error is rather large. Therefore, one may wonder whether the internal adjustment should not be followed by a planimetric transformation of the block as a unit, using all control points. Such a transformation reduces the largest residual to a fraction of its present size. However, a test in which such a transformation was included showed that its omission does not affect the results of the external adjustment.

ACCURACY AND USE OF THE EXTERNAL ADJUSTMENT

The root-mean-square values of the residuals in position for all blocks, listed in Tables I to IV, have been collected in Table VI and are here expressed in microns at photograph scale. In evaluating these results, those of the Fredericton blocks should be given a somewhat smaller weight than the others because of the very inhomogeneous distribution of ground-control points.

The tables show that on the average the two combinations of internal and external adjustment have practically the same accuracy. It seems evident that, with the density of ground-control points that has been used, the larger deformation introduced by the third-degree internal adjustment is just about eliminated by the external adjustment.

TABLE VI. ROOT MEAN SQUARE VALUES OF THE RESIDUALS IN POSITION, EXPRESSED IN MICRONS AT PHOTOGRAPH SCALE

Block	*Internal and External Adjustment*		*Polynomial Block Adjustment*	
	i	*ii*	*iii*	*iv*
Int. test '60–'64				
Zeiss Jena 1818	55 μm	60 μm	58 μm	57 μm
Wild A7	87	81	86	68
Fredericton				
1:40,000 A7	31	31	30	27
1:24,000 A7	45	74	50	70
1:12,000 A7	130	142	124	138
1:40,000 C8	50	64	53	54
1:24,000 C8	104	95	105	116
1:12,000 C8	237	193	248	144
ITC block				
with correction	96	98	105	126
without correction	98	100	131	126
Int. test '68				
block 1	18	18	20	23
block 2	19	15	31	16

The discrepancies in planimetry between strips after internal adjustment of the second degree are quite sufficiently small for topographic mapping. Still, those after the third-degree adjustment are somewhat smaller and therefore, if the block is sufficiently well fenced in by ground control, one may prefer this adjustment.

To evaluate the accuracy obtained in the external block adjustment, one should compare each internal-external adjustment with a polynomial block adjustment of the same degree as the internal adjustment. Thus, one should compare the adjustments *i* and *iii*, and also the adjustments *ii* and *iv*.

This comparison shows that in the case of the second-degree adjustments there is a slight tendency for the results of the internal-external adjustment to be better. In no case are the results appreciably worse and in one case they are considerably better.

In the case of the third-degree adjustments, there is a wider spread in the outcome of the comparisons. However, on the average the result of the internal-external adjustment is, here also, definitely not inferior to that of the polynomial block adjustment.

An increase in the number of planimetric ground-control points over the present minimum would increase the accuracy of both the internal-external adjustment and the polynomial block adjustment. The internal-external adjustment would benefit most because of its greater flexibility.

The computer time required by the internal and external adjustment varied between 1/3 and 1/2 of the time required by the rather slow iterative polynomial block adjustment. It included a strip adjustment of heights during the internal adjustment, but no block adjustment of heights. The inclusion of such a height adjustment would still keep the time requirement smaller than that of the polynomial block adjustment, especially in view of the fact that the polynomial height adjustment converges in about half the number of iterations required by the corresponding planimetric adjustment.

Therefore, the conclusion seems justified that this combination of internal and external adjustment of planimetry can with advantage replace the polynomial block adjustment in topographic mapping.

In large-scale mapping, and in the case of a dense net of planimetric ground-control points, the internal adjustment may leave undesirably large discrepancies between strips. Here, a different method of internal block adjustment should be used. As such, a method of internal block adjustment of sections proposed by Roelofs[10] may be suitable. Both this method and the external block adjustment require only a small program and could be used on a very small computer.

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