ROBERT A. MATOS*
USA TOPOCOM ETL
Fort Belvoir, Va. 22060

Multiple-Station Analytical Triangulation

The MUSAT programs are used as research tools in simulation studies leading to further technique refinement and extended applications and as major production tools in topographic map production.

ABSTRACT: *The subject of this paper presents the most recent development by the U.S. Army Engineer Topographic Laboratories of a technique and computer program for simultaneous analytical block triangulation. Two variations of the program have been recently completed. The first is called MUSAT and the second, Expanded MUSAT, represent an accumulative effort by this Laboratory, starting in 1960, to develop a practical, unlimited version of a simultaneous block triangulation technique suitable for production use by the U.S. Army Topographic Command. The Expanded MUSAT program features automatic error correction of photographic observations, blunder elimination, control verification, data edit, simultaneous triangulation and adjustment of up to 2,000 photographs, statistical analysis of results and error propagation. The programs are coded in the Fortran IV language to run on the IBM 7094 and UNIVAC 1108 computers and are currently undergoing engineer design and service testing. Results are given for the simultaneous block triangulation of 500 photographs and for a block triangulation of 64 Lunar Orbiter photographs.*

INTRODUCTION

THE MULTIPLE STATION Analytical Triangulation program provides a highly sophisticated technique for the automatic triangulation and adjustment of a model, strip or block of aerial photographs in a single computer run. The technique was given the name MUSAT, an acronym for Multiple Station Analytical Triangulation.

Two variations of the MUSAT program have been recently completed by Autometric Operation, Raytheon Corporation, under contract with USAETL. The first completed in June 1968, employs the coplanarity condition for the intersection of two rays; and the second, completed in February 1969, and known as Expanded MUSAT, uses the collinearity condition which assumes that the object point, image point and exposure station lie on the same line. The latter version also includes blunder elimination, data edit and a new technique called AutoRay† for the rapid solution and inversion of large systems of normal equations.

CHARACTERISTICS

The MUSAT programs are coded in the Fortran language for the IBM 7094 computer and contain over 17,000 Fortran statements in up to 140 subroutines requiring 11 computational links to fit the 32K memory of the computer. They are also coded to run on the UNIVAC 1108 computer which provides twice the internal storage and a significant speed

* Presented at the ASP/ACSM Semi-Annual Convention in Portland, Oregon, October 1969.

† The AutoRay technique was developed by Dr. Atef A. Elassal, Autometric Operation, Raytheon Corporation, while working on the development of the MUSAT program.

advantage over the 7094 computer system. As the programs are coded in Fortran, they can be adapted to most other large scale computer systems with a Fortran capability. In their present form the programs will perform a simultaneous triangulation and adjustment of up to 2000 aerial photographs.

In the MUSAT programs the geometric and physical conditions are enforced simultaneously with *known* data influencing the solution in accordance with their preassigned weight in a massive least squares solution. A single program pass may include: the preprocessing of the initial photographic observation data to eliminate systematic errors; the elimination of blunders due to gross mismeasurements, errors in data transcribing and misidentification of control; simultaneous block triangulation, adjustment and a comprehensive data edit of the image data to identify and remove the remaining less obvious errors. The programs also provide result analysis, error propagation and output of absolute or relative position and photograph orientation data in a form compatible for input to the Universal Automatic Map Compilation Equipment (UNAMACE).

PHILOSOPHY

From the several feasible approaches to analytical triangulation, that used in the MUSAT June 1968 system has the following philosophy. The basic least-squares solution computes simultaneously the positions and attitudes of all photographs. By incorporating ground control equations in the solution, the problem is solved in the coordinate system in which the control is given. By appropriate selection of the weights, it is possible to treat the parameters as externally observed values, influencing the solution statistically according to the accuracy with which they are observed. An extension of this technique makes it possible to utilize partially given control data by assigning high weights to the known coordinates and very low weights to the unknown parameters. In most procedures, corrections to photograph orientations are found in terms of the angles by which the matrix is originally defined. This has the disadvantage that at certain positions, the fundamental angles become ambiguous. In the Herget approach, on which this technique is based, the orientation correction is accomplished by means of a differential rotation vector, which is always defined, and which operates directly upon the orientation matrix rather than upon the fundamental angles. The formulation of the photo orientation matrix in terms of x tilt, y tilt, and heading, although not the simplest procedure, was retained from earlier work. Subsequent to the final solution for air station positions and camera attitudes, the positions of all triangulated points are computed by intersection using the coplanarity condition.

With the introduction of the automatic data edit feature into the MUSAT program it was found to be less cumbersome for system analyzation and reprogramming to redefine and expand the basic mathematical model used in the previous MUSAT version.

In the Expanded MUSAT program the photographic block is described by two different mathematical models; one model for the blunder-edit phase and a second model for the least-squares adjustment operation. The blunder-edit model is designed to allow for a two-level checking of the data pertaining to a specific ground point. The first level blunder edit is performed on the internal consistency of the photographic data of the point. In other words, the point is treated as a pass point. The edit operations are performed employing the conditions of coplanarity and scale restraint. If a survival of the first-level test is a control point; a second level blunder edit, which checks the consistency of the photographic data with the given ground control coordinates, is conducted. The second level test is based on the collinearity condition equation. Besides the primary blunder edit operations which are performed on the measured image coordinates and ground control data, a secondary blunder edit of the supplied lengths of camera station base lines is performed. A condition equation based on a three-dimensional version of the pythagorean theorem is used. To summarize, the mathematical model used by Expanded MUSAT for blunder operations is based on four types of condition equations (coplanarity, scale restraint, collinearity and base-line) and the least-squares adjustment employs a mathematical model which is based on two types of condition equations (collinearity and base-line).

In both mathematical models, the notion of differential rotation vectors, retained from earlier work, is used for deriving linear forms of the various condition equations.

INPUT/OUTPUT

As a minimum, input data may consist of estimated exposure station position and orientation, survey ground control, and measured coordinates of well defined image points for which the precise horizontal and vertical positions are desired, however, much additional information can be input if available. A complete list of observations and parameters that may be used as input is given in Table 1.

The output of the programs are basically the camera station and ground point position data, the orientation matrix for each camera station and plate residuals for each measured image point. Position data may be output in the geographic, UTM, geocentric and local coordinate systems. If the programs have sufficient information, all four may be given regardless of the input data form.

The output position data may be used: to form a control base of known positions for topographic mapping; for input into automatic compilation instruments; for position refinement, densification and extension of ground control or for the determination of absolute or relative position of preselected ground image points. Output in addition to photogrammetric control data, includes: statistical quantities derived from the least-squares adjustment to provide a measure of plate coordinate accuracy, and automatic analysis of selected points based on known survey control.

Results

The results of a block triangulation of Lunar Orbiter photography which was run by the Autometric Operation, Raytheon Corporation, under contract with the National Aeronautics and Space Administration is included to indicate the diversity of current applications of the program and to highlight the problem of program overhead versus system automation. In the Lunar Orbiter mission the photographic system carried two cameras, one with a 3.2-inch focal length lens and the other with a 24-inch focal length. Each camera was exposed simultaneously, providing one set of high-resolution frames and another set of medium-resolution frames from each exposure station. For this particular test 36 high-resolution frames, in which over 150 points were measured on each plate, and a covering set of 28 medium-resolution frames (in which over 60 points per plate were measured) were combined for block solution. The total block consisted of 64 photographs and over 7,000 points for triangulation and adjustment. The interest of this example is twofold; first, to showing the diverse application of the MUSAT program, and second to provide an insight to understanding the magnitude of the heretofore manual operations now shifted to the computer to speed analysis and reduce production costs.

The complete block solution took a little less than two hours on the IBM 7094 computer. A breakdown of the time required in the various operations is shown in Table 2. From Table 2 it is shown that the actual simultaneous triangulation and adjustment of the 64 photograph block (matrix of order 384) and the data edit of over 7,000 pass and control points took less than 12 minutes per iteration or about 30% of the total computer time used. It is interesting to note from Table 2 that the greatest single amount of time consumed in these runs, occurs in the sort and merge routines that are included to automatically assemble, order and reorder the great mass of data handled by the system. This situation results from an effort to ease the burden of manual data preparation and manipulation and to provide orderly output to the user. Thus, automating much of the

Table 1. Acceptable Input Data to MUSAT

Photograph Data	*Exposure Station Data*
Reseau Coordinates	Airbase
Standard Deviations	Position
Fiducial Coordinates	Orientation
Image Point Coordinates	Standard Deviations
Comparator Calibration Data	*Camera Calibration Data*
Weave of the Ways	Focal Length
X & Y Scale Factors	Reseau Master
Non-orthogonality of Axes	Lens Characteristics
	Frame Characteristics
Ground Control Data	*Atmospheric Refraction Data*
Geodetic Stations	
Standard Deviations	
Equal Elevation Points	

Table 2. Block Triangulation of 64 Lunar Orbiter Photos With Over 7000 Image Points

Iterations 1	*2*	*3*	*Operation*	*Time Minutes*
1			Preliminary Reading Input Tape	1.59
1			Read, Preprocessing, Input List	4.76*
1			Automatic Coding of Input List	5.22
1			Rearrange Sort & Merge	24.37
1			Blunder Elimination	4.85
1	1	1	Least Squares Adjustment and Data Edit Per Iteration	11.82
		1	Result Analysis	0.57
		1	List Triangulation Results	18.56
		1	Sort & List Plate Residuals	22.04
			TOTAL	117.42

* Preprocessing was not performed.

data assembly, both input and output, will save the user many hours of manual effort but must be paid for through longer computer operating times and larger computer commitments.

In another test example the Expanded MUSAT program was used to triangulate and adjust 500 photographs as a single block on the UNIVAC 1108 computer. The run was made to determine the computer time required and program accuracy in the solution of an extremely large block. The photographic block was generated using a program prepared by ETL based on the fictitious data generator program used to generate the 1968 International test block. The new program was coded to generate data directly on magnetic tape for input into the Expanded MUSAT program.

Twenty strips of 25 photographs were simulated to provide the test block. The strips simulated flights flown in an East to West flight pattern, with a nominal flight height of 30,000 feet above mean terrain, a 9×9-inch camera format, a 6-inch focal length lens and tilts of up to 3°. Plate coordinates were perturbed to provide a standard error of 10 micrometers. Nine points, in a 3×3 array, were distributed over the picture format. Fifty-two control points of known latitude, longitude and elevation, uniformly distributed over the entire block area, were used for orientation and adjustment. The total computation time was a little over 1 hour for a total computing cost of less than $300. Three iterations were performed requiring slightly less than 10 minutes each for solution of the 3000×3000 matrix of 3000 unknowns. The maximum standard error for each of the 500 exposure station positions and for each of the 1500 ground point positions was less than 0.3 of a meter, which was consistent with the directed precision of closure. The solution of blocks of this size within a reasonable computer time and cost provides for the first time a fast, accurate and reliable technique for the economical adjustment of very large photograph blocks. Blocks of 1000, 1500 and 2000 photographs could as easily be reduced with this program, although the practicality of data preparation and solution of the extremely large block even by a simultaneous technique may be of questionable value in production practice. The Expanded MUSAT program does provide extreme accuracy in this situation and would provide a technique for elimination of bias that may occur between smaller block triangulations where control is sparse.

APPLICATION

Fundamentally, the MUSAT programs are designed for use with near-vertical photography of conventional focal length, format, and flight configurations. However, they are also adaptable to other problems. The programs will accept data from photographs of a variety of focal lengths and formats, there are no requirements for the photographs or image points to be arranged in a specific or uniform order, nor is there a restriction to a standard photograph overlap or sidelap. Consequently, photographs and data from a number of flights from different missions in almost any pattern may be combined in a single simultaneous adjustment. Also, because the primary photograph input to the program is not restricted in position and attitude, the programs may be adapted for nonphotographic applications such as ballistic and industrial photogrammetric problems.

The programs are also capable of accommodating low- and high-oblique and convergent photography. For convergent photography the two photographs exposed at each air station may be mathematically linked by means of the enforced airbase equation or, if the airbase is unknown, the orientation of each photograph may be solved for independently.

The Expanded MUSAT program is currently being modified to include functional restraints of air and ground distance measurements and of equal elevation points to supplement standard control requirements. The input/output routines are also being modified to accept and display a wider range of descriptive data for use in advanced mapping applications.

The MUSAT programs are used by the U. S. Army Engineer Topographic Laboratories, as a research tool in simulation studies leading to further technique refinement and extended photogrammetric applications and by the U. S. Army Engineer Topographic Center as a major production tool in topographic map production.

ADDENDUM

In a recent test of the Expanded MUSAT program, a block of 1,000 fictitious photographs was computed. The block combined two strips of 500 photographs containing 68 complete control points, 1,000 exposure stations and 9,000 image points. Three iterations were performed, each requiring the simultaneous solution of 6,000 equations with 6,000 unknowns. The solution was completed in 57 minutes on the UNIVAC 1108 computer.