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Pointing Accuracies to Blurred Signals

A minimum target size should be chosen such that annulus widths are not less than one miliradian subtended at the eye.

INTRODUCTION

PRACTICAL PHOTOGRAMMETRY requires the determination of photograph or model co-ordinates using photogrammetric instruments for a variety of purposes, particularly in aerial triangulation, and numerical relative and absolute orientation. Even with the introduction of automation into some aspects of photogrammetry, coordinates of control points must, under normal circumstances, still be observed by human operators. The capabilities of the human operator may therefore be an important factor in determining the ultimate accuracy of the photogrammetric process.

ABSTRACT: *An instrument has been constructed to test pointing accuracies for artificial blurred targets with gaussian blur characteristics. Observations on this instrument indicate that for target blur above a specific level, which depends on width of annulus of the target, pointing accuracies depend primarily on the grade of the density profile of the target and secondly on annulus width. Below this specific level of blur, pointing accuracies depend on annulus width alone. Background density has a negligible effect in pointing accuracies. Based on results in this study, assuming spread functions of photogrammetric systems to be approximately gaussian, the optimum size of ground targets to give maximum pointing accuracies has been found to be approximately 2.5 times the scale number of the photograph, times the σ-width of the gaussian spread function.*

Extensive investigations have been conducted on pointing to sharp targets of low, medium and high contrasts (O'Connor, 1962), (O'Connor, 1967) and (Roger et al., 1969). O'Connor found that pointing accuracies linearly depend on the width of the target annulus or edge ribbon around the measuring mark, over a wide range of annulus widths. Further, he found that accuracies were independent of target background density, except where the annulus of the target was very small. Roger et al. (1969) confirmed the relationship between annulus width and pointing accuracies for sharp targets, and also investigated the effects of nonhomogeneous target backgrounds on pointing accuracies. Very little is known, however, on the effect of *blur* on pointing accuracies. This is despite the fact that considerable research has been conducted on many aspects of vision. Pointing, however, is a task specifically related to photogrammetry, surveying and similar fields of metrology. Its accuracy is a function of the acuity of the visual system, and may vary under different visual conditions.

The purpose of this research was to investigate monocular pointing accuracies (using a black circular measuring mark) on artificial circular targets with blur characteristics and background densities similar to those that occur in photogrammetric practice. Pointing experiments have been conducted on simple equipment specially constructed to simulate pointing observation procedures in photogrammetry. Target annulus widths and pointing accuracies, measured in angular subtense at the observer, have been expressed in radians, whereas target blur has been expressed in terms of change of density per milliradian (ΔD/mrad) on the target density profile. For comparison with sizes of targets occurring in photogrammetry, dimensions of targets have been expressed as equivalent linear measurements at an optical magnification of 10× in a photogrammetric instrument.

Experimental results derived from this

study have been related to existing knowledge on the visual system, so that factors affecting pointing accuracies may be better understood. It is hoped that such investigations may ultimately lead to an improvement in the design of the shape and size of ground signals and hence an increase in accuracies of location of the photographed signals.

EXPERIMENTAL PROCEDURES

INTRODUCTION

The pointing experiments to be performed in this work entailed the design of an instrument which could make precise measurements on blurred and sharp targets. It was proposed to construct an instrument, as simply as possible, keeping in mind the limited finance available, which would give measuring accuracies of 1 sec. of arc (5 μrad) or better. To simplify the design it was decided to investigate movements of the measuring mark (*MM*) parallel to the eye-base only (approximately parallel to the x-direction in a photogrammetric instrument). O'Connor's (1967) results indicate that for annulus widths greater than 300 μrad and sharp targets, accuracies in the x- and y-directions for several observers, prove to be statistically similar. In some cases they prove to be different for annulus widths smaller than 300 μrad, but there is no conclusive evidence for this finding. Roger et al. (1969) found that pointing standard deviations in the y-direction were significantly larger than those in the x-direction, with average increases for different observers ranging from 12–143 percent. The percentage increases appeared to be a constant, and independent of annulus width. In any investigations on visual tasks, it is reasonable to assume that the pattern of results obtained by one or two experienced observers is indicative of the pattern which would occur for all experienced observers. Results of standard deviations of pointing to blurred targets obtained in these investigations may therefore be assumed to indicate the form of the relationship between pointing accuracies and blurred targets for different observers, although small variations will be expected from these results. It is proposed that the results obtained in these experiments will be applicable to observations in both the x- and y-direction. As y-direction pointing standard deviations tend to be a constant factor larger than those in the x-direction, the results obtained in this research will indicate the pattern of y-direction accuracies, though actual values may tend to be 30 percent larger. O'Connor (1968) and Roger et al. (1969) have also shown that pointing errors in the x- and y-directions are uncorrelated. The separation of observations in the x- and y-directions in this study is also based on this finding.

DESIGN OF EXPERIMENTS—STATISTICAL APPROACH

The variables to be investigated in these experiments, were the size of the annulus, the degree of blur of the target and its background density. An efficient approach to the investigation of these variables is to conduct an analysis of variance based on a factorial design of experiment (Moroney, 1951, Ch. 19). Applying a factorial design however, to psychophysical investigations is difficult for the following reasons. Firstly, the exact width of the target as seen by the observer is impossible to predict before observations commence. This is deduced from the fact that different observers determine the target edge by psychophysical observations at different positions on the target (Trinder, 1965). The use of a set of targets of specific widths for a number of observers is therefore impossible. The accuracy of width measurements of blurred targets is however very low, and hence some range in the widths may be allowed. Secondly, small targets become invisible after a relatively small amount of blurring while large targets can be blurred considerably before they become invisible. It is therefore impossible to use all combinations of degree of target blur and target width for the observations.

An analysis of variance has been performed in this research in two groups, involving two variables each, as follows. Three levels of background density were chosen—0.25, 0.40 and 0.70—which cover the normal range of density levels on aerial photographs. Targets were printed for each of these densities to give an annulus width of approximately 2 mrad at different grades of blur. By selecting suitable targets from the total observed, a variance analysis was conducted between the slopes of the target density profile (in $\Delta D/$mrad) and the background densities for annulus sizes of 2 mrad. Five levels of target blur were chosen at each of the three background density levels: Test No. 1. The actual levels of blur and background density of each target may vary slightly from the nominal values. Effects of these differences are minor and merely introduce a small amount of *noise* into the analysis.

Three separate analyses, testing the effect of width of annulus against grade of blur for

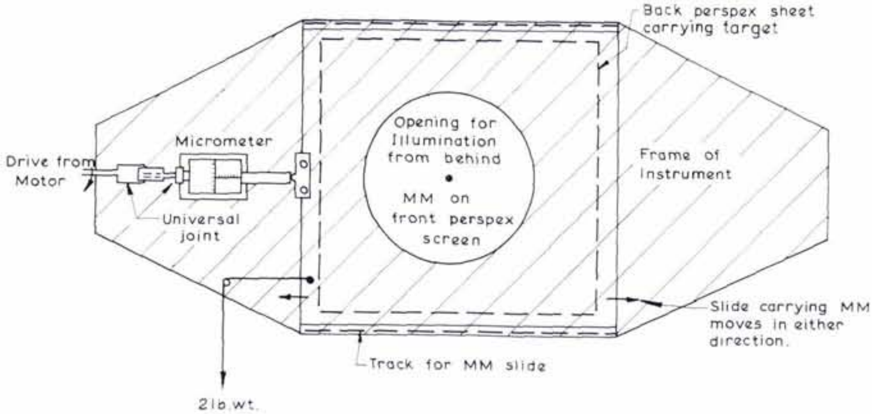

FIG. 1. Instrument used in pointing observations.

targets of 0.25 background density, were necessary. The first involved 0.8 mrad and 2.0 mrad annuli at four levels of blur: Test No. 2. The second tested targets with 2.0 and 5.0 mrad annulus widths against three levels of blur of the target profile: Test No. 3. Test No. 4 analyzed annuli of 2.0 mrad and 5.0 mrad against two levels of blur at a higher adaptation level. As Test No. 1 indicated that background densities have only a minor effect on pointing accuracies, they were not included in the remaining tests. Following the completion of the analysis of variance, a linear regression of logarithm (slope) against logarithm (pointing standard deviation) has been evaluated.

INSTRUMENTATION

In the design of the instrument for the research, it was decided to use direct viewing with no optical elements between the observer and the target. Although this may not be of the utmost importance for sharp targets, optical elements may well influence results for blurred targets. The target in the instrument as shown in Figure 1 is set in a vertical plane such that the horizontal viewing line is at right angles to the plane of the target. The measuring mark (*MM*) is attached to a clear perspex* sheet which slides along a rail located 1 mm. in front of the target and parallel to it. A small spherical bearing attached to the perspex screen is the point of contact with a micrometer screw 25 mm. long. The micrometer is driven by a 1/25 horse-power electric motor with rotation speeds ranging from 60 rpm to zero (30 mm. per minute to zero). Speed is controlled by means of an electronic switching device manipulated by the observer at the viewing stand 10 meters away. Least count of the micrometer drum is 0.001 mm. or 0.1 μrad (0.02 sec. of arc). However, it is believed that the accuracy of the instrument is approximately 0.5 μrad (0.005 mm. on the micrometer drum).

Repeatability tests on well-defined targets using theodolite telescopes have given standard deviations of better than 1 μrad. The accuracy of the instrument is therefore more than 10 times higher than the pointing accuracies being investigated in this research. Observations have been made under controlled temperature and humidity conditions, and therefore no corrections were necessary for atmospheric conditions to either the micrometer or the target. Movements of the *MM* can be made in either direction because the *MM* slide bears on to the micrometer spindle with the force of 2 pounds which is attached to the slide by a cord via a small pulley. As there were no linkages between the micrometer spindle and the *MM* slide, no backlash existed in the instrument. Backlash in the micrometer drum itself was also negligible.

Illumination of the target was by a tungsten filament lamp set 2 feet behind the target, and diffused by a diffusing screen 6 inches behind the target. Luminance of the target area, approximately 10 inches in diameter, was found to be constant at 12 mL based on measurements on an SEI Photometer. Pointing observations were made in a laboratory, the lighting of which was used to illuminate an adaptation screen, 5 feet by 7 feet, 2 feet in front of the target. The luminance of this screen was 15 mL. The whole instrument was obscured from the observer by the screen except for the target area and the micrometer, which was read using a theodolite telescope. Observations were made from a stand with

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* Transparent.

a head rest 10 meters from the target. The left eye was used by the observers because it was the eye normally used by them for monocular viewing; the right eye was restricted from seeing the target by a small screen on the viewing stand, but otherwise could see the area around the target, thus providing for equal adaptation of both eyes.

Before commencing an observation, the *MM* was moved well off to the edge of the target. In the process of centering, the target was approached with care and at a slow speed of the motor. If an overshoot was suspected, the whole operation was repeated. Initially 4 sets of 25 observations were taken. After some experience, however, with very blurred targets, 10 sets of 10 observations were found to avoid fatigue more effectively, and therefore the majority of targets were observed in 10 sets of 10. O'Connor (1967) indicated that statistically no difference existed between results of 5 sets of 50 observations, and 10 sets of 10 in his experiments. Similar results were noted in these investigations where both 4×25 and 10×10 observations were taken.

In addition to the blurred targets observed in this research, a number of sharp targets were also observed. This was done, firstly, to compare the two observers in these investigations with O'Connor's (1967) observers and, secondly, to give a basis for judging the effects of blur on pointing accuracies.

DETERMINATION OF WIDTHS OF TARGETS

As widths are an important factor in determining pointing accuracy, it was necessary to locate the edge of the target as part of the observational procedure. Edge location is a subjective determination (Hempenius, 1968, 18), depending on the illumination, observation distance and particularly the observer. Although a detailed study of the characteristics of edge location was not intended in this work, an estimate of widths was necessary to relate pointing accuracies to characteristics of the target.

The measuring mark used for these observations was as shown in Figure 2. The gap between the two sections gave the observer an unobstructed view of the whole target edge. Though it was necessary to interpolate visually the line across the edge of the target, it was considered that this arrangement was preferable to an unbroken line which tends to bias the observations. The width of the line was 1 mm. or 100 μrad. Target widths were determined by approaching the edge of the target from both directions in separate sittings. An extended rest period was taken between each set. The location of the edge was then taken as the mean of the two sets.

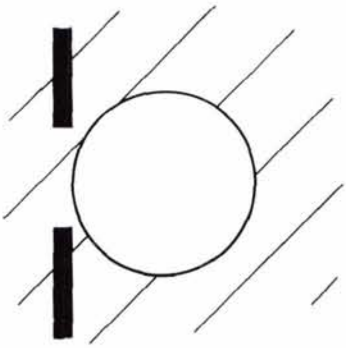

FIG. 2. Measuring mark used for target edge location observations.

PRODUCTION OF TARGETS AND MEASURING MARK (MM)

The sharp targets were cut from photographic film of a specific density using a sharp compass. Background density of the targets was chosen as 0.3, although pointing accuracies are not influenced by background density (O'Connor, 1967) for the range of annulus widths being investigated. The *MM* 1.0 mrad in diameter was made of aluminium sheeting and adhered to the *MM* slide with black paint.

Trinder (1965) developed an optical anologue computer (termed *Umbrascope*) for the purpose of determining convolutions of annuli and point-spread functions. It was intended to use targets with gaussian blur characteristics in this research because the general blurring characteristics of photographic systems closely approach a gaussian function, i.e., the spread functions of photographic systems are approximately gaussian (Hempenius, 1965), (Wolfe and Tuccio, 1960). The Umbrascope equipment was used to conduct similar convolutions of sharp circular targets and a range of gaussian spread functions (abbreviated to SF) to produce blurred targets of different image qualities for this research.

Although small discrepancies exist in the comparisons between gaussian curves and the profiles of targets from the Umbrascope (Trinder, 1970), these did not affect the assessment of pointing results, because all targets were measured on a Joyce Lobel Automatic Recording Microdensitometer and therefore the exact profile of the target was obtained. The approximation to a gaussian function however is sufficiently close for comparison with photogrammetric systems.

PRESENTATION OF RESULTS

Pointing accuracies for two observers expressed as standard deviations of repeated observations are plotted against degree of target blur in Figures 3, 4 and 5. Both standard deviations and target blur, expressed as ΔD/mrad, have been plotted on logarithmic scales. Linear regressions in Figures 3 and 4 have been computed with weights inversely proportioned to (standard deviation)^2, whereas in Figure 5, the line for 0.8 and 2.0 mrad has been computed using equal weights. The regression line for 5 mrad in Figure 5 was only approximately located.

Figure 3 represents pointing accuracies plotted against target blur for targets with 2 mrad annuli and 3 background densities. Figures 4 and 5 represent accuracies plotted against target blur for 3 target annulus widths. Results of all targets in Figure 3 for 2 mrad annulus widths have been included in Figure 4. This step was justified because background density had a negligible effect on results as seen from the regression lines. This point was also borne out in the variance analysis.

With regard to the use of statistics in psychophysical research, the large number of possible variables entering into the investigations must be kept in mind. Graham (1950) indicated that the psychophysical response to stimuli may depend on many factors such as motivation, attitude, time, etc., as well as the stimulus conditions being investigated. Statistics require the treatment of homogeneous data of the same statistical registration. If additional variables enter into the investigations, then theoretically the application of statistics is invalid, unless these variables are included in the statistical tests.

In this study statistical testing has been used as a guide in the analysis of variance. This analysis indicates that target width and blur are significant whereas background density is only a very marginally significant property of the target affecting pointing accuracies. A very much larger sample of observations than was used in this research, taken over a very extended period, would be necessary to investigate all possible factors affecting accuracies. Such an amount of work would be impractical considering the very limited advantages gained. Statistical theory therefore is considered as a very useful tool

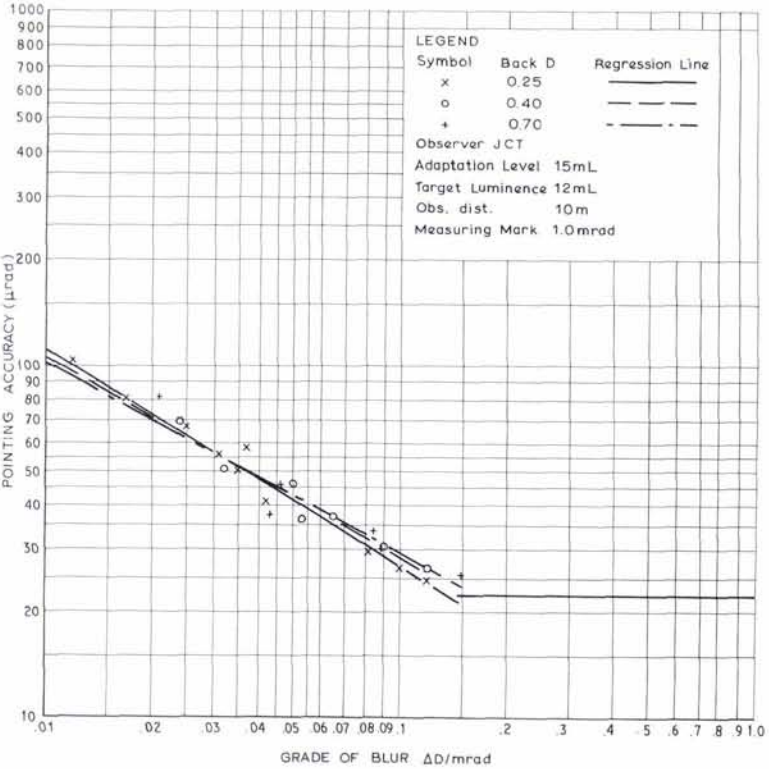

FIG. 3. Pointing standard deviations against target blur for three different background densities, and annulus sizes of 2mrad. Regression lines have been computed using weights proportional to the (standard deviations)^2.

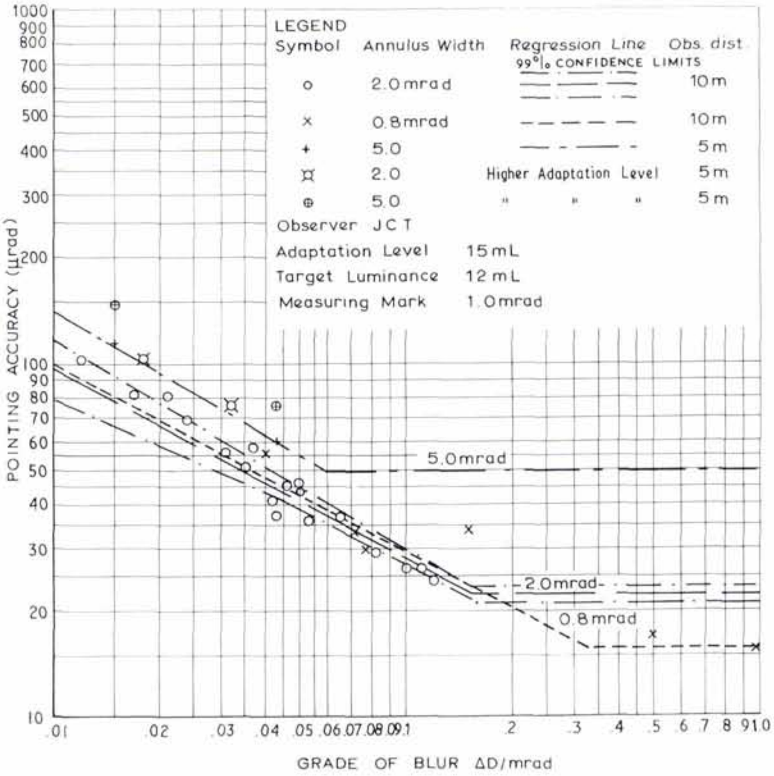

FIG. 4. Pointing standard deviations against target blur, for three target annulus widths, regression lines have been computed using weights inversely proportional to the (standard deviations)².

in psychophysical testing, but one which must be used carefully.

Conclusions based on the variance analysis in this study are as follows. The most significant effect on pointing accuracies over the ranges tested is the blur of the target. Widths of the annulus are a secondary effect, where the interaction between these factors is also marginally significant. Some interaction may occur between background density and grade of the blur, although this effect is very small. It is possible that further minor interactions may exist between all three elements, but these effects are not worthy of investigation. Upon considering the practical applications of this research, only the most significant features need be investigated because one may expect variations from one observer to the next for the same target. This is clearly indicated by the results of O'Connor (1967) as well as the present research. In addition, other observers may indicate different characteristics with respect to the interactions mentioned above. O'Connor, for instance, found that pointing accuracies in the x- and y-directions were significantly different for himself for annuli smaller than 0.3 mrad, but not for other observers. The visual system, and the psychophysical aspects involved in visual tasks are very complex and over-simplification of the processes involved are unwarranted. The two aspects of blur and width only will therefore be discussed in the following section.

DISCUSSION

Figure 6 contains data derived by O'Connor (1962) and O'Connor (1967) plus the results in Figures 4 and 5. The curves for sharp targets by JCT and AHC clearly follow the same pattern as the results of O'Connor (1962) and O'Connor (1967). Significant variations were detected by O'Connor in results for different observers, though the general patterns were still the same. Such differences are clearly due to the many factors, both physiological and psychological, involved in psychophysical tasks. The very good agreement by JCT and AHC with the results presented by O'Connor is an indication that pointing accuracies obtained in this research are indicative of the trend of results by average observers, although variations of 10 to 20 percent may be expected. Further variations in standard deviations may occur due to changes in adapta-

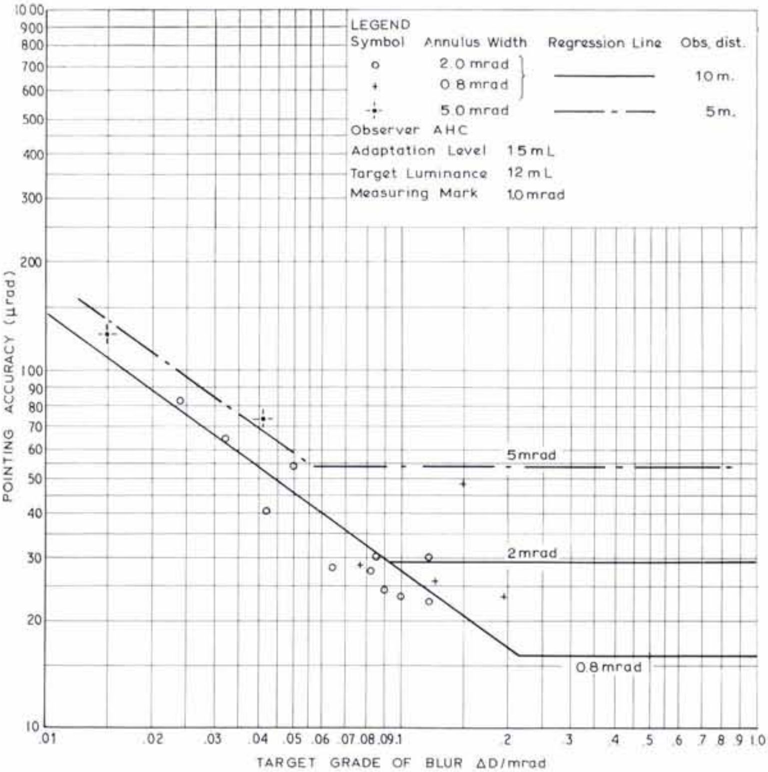

FIG. 5. Pointing standard deviations, against target blur, for three target annulus widths. Regression lines have been computed using equal weights.

tion level. Increases by a factor of 1.3 were noted in the higher adaptation results derived in this study. This higher adaptation level was caused by side illumination entering the observer's eye, a situation which could occur during normal photogrammetric observations. The observer's ability to discriminate luminance differences in blurred targets is evidently reduced if the eye is adapted to a higher luminance level.

Curve II in Figure 6 derived by O'Connor (1962) follows almost exactly the same line as the .05 ΔD/mrad curve derived in these experiments. The curves of O'Connor (1962) were derived using a Zeiss Pulfrich Stereocomparator. O'Connor (1967) stated that inaccuracies in this instrument obscured the true pattern of results which could be obtained by the observer's visual system. The similarity between the two curves is however quite significant. The experiments conducted by O'Connor (1962) involved sets of ten observations only on a large number of targets. This led to a significantly lower confidence interval than for results obtained by O'Connor (1967) or those derived in this work. Curves III and IV are difficult to define accurately, but some tendency is for them to follow shapes similar to the curves derived in this research. This is particularly the situation for Curve IV, which also may follow a line closer to the 2-percent line than is actually shown. It is impossible to compare directly the two sets of experiments, especially considering the completely different experimental conditions of the two sets of data. The relative agreement between the shapes of the curves is nevertheless very good.

The three straight lines plotted for the sharp, low and medium contrast curves derived by O'Connor (1967) for annulus widths less than 250 μrad, which apparently represent near threshold results, agree with the pattern of reduced accuracies in Figure 6 obtained for near visibility threshold blurred targets. In both situations luminance discrimination seems an important criterion for pointing (Trinder, 1968), and therefore a similar pattern of results for near threshold targets would be expected.

Composite curves in Figure 6 clearly indicate the very complex non-linear behavior of the visual system in respect to pointing. Various criteria are apparently adopted by the visual system for pointing depending on the size and luminance profile of the target.

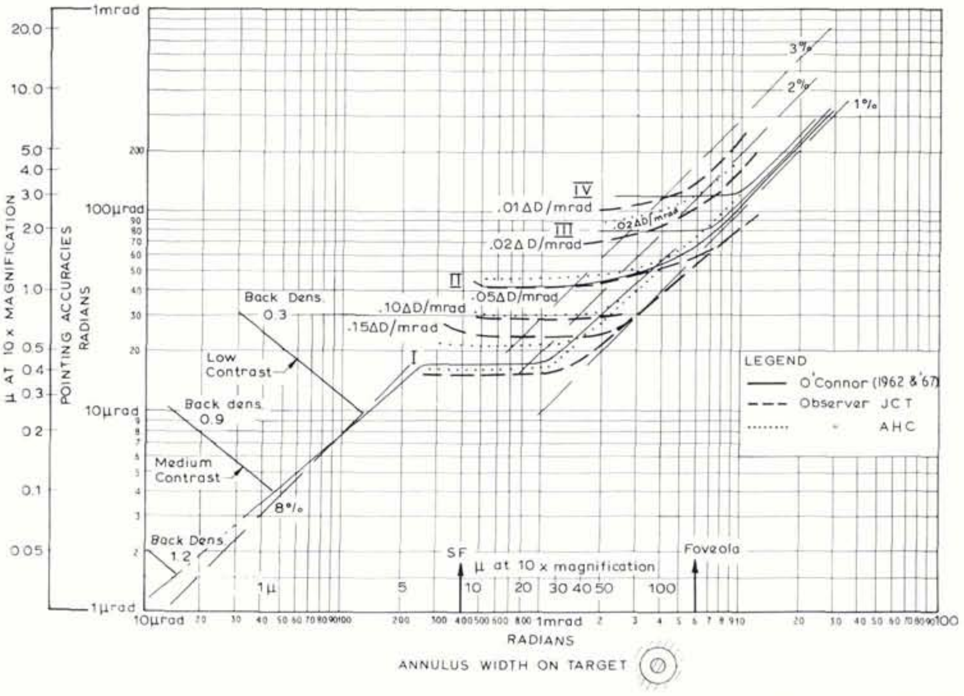

FIG. 6. Results from figures 4 and 5 superimposed onto curves derived by O'Connor (1962, 1967). Curves derived by J.C.T. and A.H.C. have been marked according to the legend. The grade of blur (in ΔD/mrad) corresponding to each line is shown.

The general criterion used by the visual system for pointing seems to be based primarily on width discrimination, the accuracy of which is reduced by increased blurring of the target with perhaps some assistance from the luminance levels of the target.

With regard to contour-sharpening mechanisms referred to by O'Connor (1967), Mach bands were not consciously observed on any of the targets, though the edges of the targets always seemed to be moderately well defined. The accuracy of location of target edges varied from 50 µrad to 200 µrad. Subjective positions of the edges on the density profiles, however, were not at an abrupt change as might have been expected by the subjective impression of the targets. The main influence of the contour-sharpening mechanisms therefore seems to be in aiding the definition of the target edges, and hence the width of the annulus. This, however, seems to be only a secondary effect as is indicated by the fact that the ability to discriminate width is substantially reduced by large degrees of blur.

The basic explanation for pointing accuracies seems to lie in the proposals to explain vernier acuities put forward by Andersen and Weymouth (1923) together with the minor influences of neural processing effects such as inhibition and contour sharpening mechanisms, in locating the target edge. Luminance patterns projected onto the retinal mosaic lead to the discrimination of a mean local sign according to Andersen and Weymouth, such that the *MM* is located centrally, despite the fact that edges of the targets can only be located with an accuracy of 50 to 200 µrad. No explanation is available as to how the mean local sign is resolved from the stimulated cones, particularly for blurred patterns. The neural processing in the retina and higher centers of the visual system are expected to influence the manner in which this mean local sign is resolved, as these functions are basic to the visual system. Although contour-sharpening mechanisms are significant, it seems they should not be given undue importance. These were also the general findings of the investigations in Trinder (1970). Further descriptions of the complex manner in which the visual system performs pointing tasks are impossible at this stage because of the many unknown factors in the visual system, but it is important to indicate the factors that may influence accuracies.

The results in Figures 3 to 5 may be used to derive pointing accuracies obtainable with specific targets on aerial photographs or alternately, to design ground target sizes which when photographed will produce optimum pointing accuracies. These factors will be discussed in the following section.

PRACTICAL APPLICATION OF RESEARCH

INTRODUCTION

The results presented earlier outline monocular pointing accuracies which can be obtained by the visual system using blurred targets of low and medium contrast. For binocular viewing, improvements in pointing accuracies may be expected, particularly for very blurred targets. O'Connor (1962) and Roger et al. (1969) indicate that there is not a significant difference between monocular and binocular pointing results for sharp targets. From experiences in this study, the use of binocular vision lowers the threshold and therefore should improve pointing accuracies at least for very blurred targets. The exact relationship, however, is unknown. The following discussion will therefore relate to monocular pointing results. Such a discussion may be applied directly to observations on monocomparators. As the relationship between monocular and binocular observations is likely to be simple, except perhaps for near threshold targets, the conclusion reached in the following discussion should also be applicable to binocular pointing results.

If sufficient information is known on the image quality of the target being viewed by the observer in the photogrammetric instrument, pointing accuracies may be predicted. The many steps which may be involved in predicting pointing accuracies based on Figure 6 are given by Hempenius (1964, 310). A more useful approach to the practical problem is to be able to choose a target size which, when viewed in the photogrammetric instrument, will give a predictable maximum pointing accuracy. This approach requires a knowledge of the luminance characteristics of the target and its background, details of the imaging characteristics of the photographic and photogrammetric systems, and also the results in Figure 6. As there are many unknown variables involved in such a problem, it is impossible to treat it in detail in this work. However, examples of targets artificially produced by the Umbrascope will be discussed. This discussion will indicate the manner in which the problem can be treated, and also estimate a formula for optimum target sizes. Another application of pointing results may be in the choice of an economic scale of photography to give maximum pointing accuracy for certain specific ground objects. The effects of granularity on pointing accuracies are outside the scope of this work and therefore cannot be included in this investigation.

Before proceeding with investigations on target size a brief résumé of existing literature on signalized targets will be given.

SIGNALIZED TARGETS

Eekhout (1963) conducted a comprehensive investigation on signalized targets with the purpose of finding the most suitable and economic types of targets for local conditions. He recommends a white square target with a black surround approximately the same width as the target. To aid in identification, four bars should form a cross through the target center with their length about 5 times their width. The whole target should be surrounded further by a dark background, produced artificially if necessary. Ackerl and Neumaier (1959) recommend a yellow centered target with a black border with no cross bars for identification. The border of their target is narrower than that recommended by Eekhout above, but the two targets are basically similar. A high contrast between the target and its background should give the steepest target density profile, and therefore the best pointing accuracy, provided target width is also controlled.

Very few recommendations are available for the size of targets. Pastorelli (1959) recommends that the minimum size of target in meters should be—

Scale No. of Photograph/40,000.

Hempenius (1964, 314) suggests that the target size should equal the scale number of the photograph times the 2σ-width of the combined spread function of the photographic and photogrammetric systems. As the quality of the image of the target depends on the spread function of the total system, and pointing accuracies depend on the grade of the blur, this criterion is clearly more acceptable than that of Pastorelli. Pastorelli's formula would normally prescribe smaller targets than Hempenius' rule.

Target profiles given by Hempenius (1964, 311) are not directly applicable to Figure 6 because the spread functions of the optical systems of photogrammetric instruments are unknown. No actual pointing results however were reported. Pointing observations to such targets made on a photogrammetric instrument would include the inaccuracies of the instrument, e.g., Visser (1964). O'Connor (1967) commenting on results of his earlier work, indicated that the stereocomparators obscured the true capabilities of the visual

TABLE 1

2σ-value of SF	*3.0 mrad*		*2.4 mrad*		*1.8 mrad*		*0.7 mrad*		*0.2 mrad*	
Ground Target	*Blur ΔD/mrad*	*St. Dev. μrad*	*Blur ΔD/mrad*	*St. Dev. μrad*	*Blur ΔD/mrad*	*St. Dev. μrad*	*Blur ΔD/mrad*	*St. Dev. μrad*	*Blur ΔD/mrad*	*St. Dev. μrad*
Density of Background = 0.3										
7 mrad	0.037	84	0.060	65	0.12	45	0.18	38	0.8	35
4 mrad	0.029	71	0.050	51	0.095	35	0.40	22	0.8	22
7 mrad	0.024	70	0.042	53	0.085	28	0.40	22	0.8	20
2 mrad	0.01	> 100	0.021	68	0.05	34	0.40	16	0.8	16
1 mrad	<.01	—	0.012	—	0.025	51	0.40	16	0.8	16
Density of Background = 0.6										
7 mrad	0.070	58	0.12	45	0.18	38	0.57	35	>1.0	35
4 mrad	0.051	50	0.085	39	0.15	35	0.57	22	>1.0	22
3 mrad	0.027	61	0.049	42	0.095	33	0.50	22	>1.0	20
2 mrad	<.01	—	0.021	65	0.05	41	0.35	16	>1.0	16
1 mrad	—	—	0.01	—	0.024	61	0.17	16	>0.9	16

system, particularly for small targets. The grades of the density profiles of the signalized targets presented by Hempenius (1964, 311) range from 0.15 to 0.32 ΔD/mrad if viewed in an instrument at 10× magnification with perfect optics. As such grades of the density profile are larger than at the points of discontinuity in Figure 3, neglecting instrumental errors, pointing accuracies would be determined by size rather than blur characteristics.

In the following section an attempt will be made to relate the size of the target on the ground to the target blur, and therefore predict the accuracy for different ground target sizes, using targets artificially produced in the Umbrascope.

PREDICTION OF POINTING ACCURACIES

The situation reproduced artificially in the Umbrascope was that of a white target on a dark background photographed at different exposures with photographic systems of different qualities. As this is similar to the recommended situation described in the previous section, targets produced in the Umbrascope may be related to practical cases. The spread functions used in the reproduction of these targets were gaussian, as mentioned earlier.

It has been pointed out that some inaccuracies do exist in the Umbrascope (Trinder, 1965). These inaccuracies were not important in assessing pointing results because each target was accurately measured on a microdensitometer. They may however influence slightly the grades of blur estimated for large targets in Table 1. These results therefore represent the approximate relationship between ground target sizes and target blur, for gaussian spread functions. Inasmuch as the discussion will be mainly centered around smaller targets, this limitation tends to be reduced.

Table 1 lists the grades of blur obtained in the Umbrascope for 5 different spread functions, using 5 ground target sizes and exposures producing target background densities of 0.3 and 0.6. These results have been obtained from microdensitometer traces of approximately 120 targets. Based on these grades of blur and the corresponding annulus sizes derived from each target, pointing standard deviations have been determined from Figure 6 as shown in Table 1.

Three factors enter into the determination of pointing accuracy in Table 1:

i. The size of the signalized ground target,
ii. The grade of blur for the different target sizes in *i*,
iii. The width of the annulus.

For large 2σ-widths of the *SF*, factor *i* is clearly important in determining the grade of blur and therefore the pointing accuracy. In contrast, for small *SF*'s, ground target size has no effect on the grade blur, and therefore pointing accuracies are determined by annulus size only. The optimum size of ground target for the three *SF*'s of 3.0, 2.4 and 1.8 mrad is between 2 and 3 times the σ-value of the gaussian SF. This agrees approximately with Hempenius' rule (1964, 314) mentioned in the previous section. Adopting an average value of 60μm (2.4 mrad at 10× magnification) as the 2σ-width of the SF of a photogrammetric system, the recommended ground target size is approximately 3.0 mrad or 75μm at the scale of the negative, assuming a MM of 25 μrad.

If the system SF has a 2σ-width less than 1 mrad, adoption of a target smaller than 1 mrad will not improve pointing results because accuracies reach a constant value of approximately 16 μrad for annulus widths between 0.25 and 1.0 mrad. (Figure 6). If annulus widths can be decreased to less than .25 mrad an improvement in accuracies may result provided the targets are of very high contrast and very sharp. The intention to use annuli smaller than 250 μrad would have to be studied very carefully since pointing accuracies may decrease rapidly if the ideal conditions are not satisfied.

For a given ground target size and spread function, pointing accuracies depend on the blur of the target. This varies according to the background density of the photographed target, and therefore the exposure and characteristics of photographic emulsion. From the results in Table 1, maximum accuracies are obtained when the targets are photographed at high contrast. Further work on specific practical problems, however, must be carried out to confirm this statement.

CONCLUSIONS

Though results from the Umbrascope are only approximate, good agreement has been obtained between different spread functions and the derived empirical formula for optimum ground target size, neglecting the effects of granularity of photography. This formula, for practical photogrammetry is as follows:

> Optimum target size is equal to the photograph scale number times 2.5 times the σ-width of the SF of the photogrammetric system. A minimum target size should be chosen such that annulus widths are not less than 1 mrad sutended at the eye. Targets should be photographed at high contrast, for maximum accuracy.

The above rule has been derived for the particular case of a theoretically bright target on a black background and a gaussian SF. Different target configurations and SF's may produce a slightly different relationship. Since SF's are normally approximately gaussian, and targets high contrast, the above work gives a reliable estimate of a rule for optimum ground target size.

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