Sten Nyberg*
Torleiv Orhaug*
Harald Svensson†
Institute of Physical Geography
Lund, Sweden

Optical Processing for Pattern Properties

An arrangement for measuring the angular content of the wiener spectrum, applying coherent optics, may be of considerable help in analyzing imageries for earth sciences.

Introduction

Various methods of multidimensional imaging are utilized in different branches of science to obtain information about phenomena, systems and objects under investigation. In the earth sciences two-dimensional imageries are of significance for the study of patterns and structures of different size. The problem of extracting useful or significant information with respect to the problem being investigated is thus a problem of processing the pictorial data.

Although two-dimensional pictorial information has been in use for a long time, no satisfactory method has been available for extracting and for measuring various properties of pictorial information. The only methods available have been manual methods, which are particularly time-consuming due to the large amount of data contained in imageries. As our ability to obtain and record pictorial information increases, so also does the need for faster methods to process this information.

Abstract: *The method of using coherent optics for generating fourier transforms of pictorial information is applied to imageries obtained in earth sciences. The invariance of different properties of linear structures in a two-dimensional pattern is discussed both mathematically and experimentally. Five samples of images of interest to earth sciences are investigated by this method. The properties of the structural information thus obtainable from the patterns are discussed in some detail. It is concluded that this method of image processing may be useful for investigating two-dimensional structures and for measuring their properties.*

During the last years two methods for processing two-dimensional information have been under development. The first is the use of large computers having a memory large enough for handling an appreciable part of an image. The advantage of this method is its great flexibility; its disadvantage is the sequential mode of operation which often make the computing time, and therefore the cost, unacceptable. The second method is the use of coherent optical systems (see, for instance, Pincus and Dobrin, 1966 and Dobrin, 1968). The advantage of this method is the two-dimensional mode of operation; the computing time is consequently extremely low. The disadvantage, on the other hand, is the limitation to particular sets of operation.

The present paper describes the application of coherent optics for studying properties of two-dimensional patterns. Five sample images, each containing a structural pattern of some kind, have been investigated by recording the spatial fourier spectrum and by measuring some properties of this spectrum.

Method of Investigation

We consider the pictorial information in the form of a photographic transparency, the

* Research Institute of National Defence, Stockholm, Sweden
† Department of Physical Geography, University of Lund, Sweden.

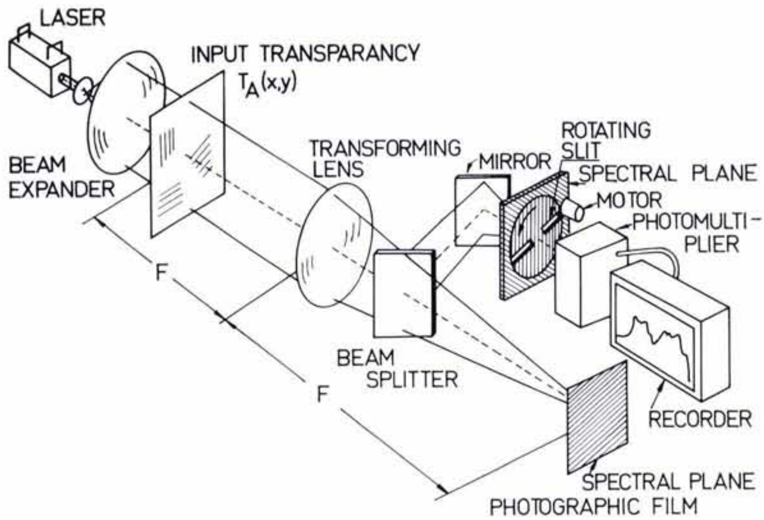

FIG. 1. The principle of generating spatial fourier and wiener spectra of a photographic transparency using coherent optics. The beam is divided into two parts in order to show the two-dimensional recording of the spectral plane and the angular scanning using a rotating slit.

amplitude transmission coefficient of which is $T_A(x, y)$. When this transparency is placed in a collimated beam of coherent light, the spatial amplitude distribution behind the transparency is given by

$$E_1(x, y) = E_0 T_A(x, y) \tag{1}$$

where E_o is the (constant) amplitude of the incident light and x, y are measured in a plane perpendicular to the direction of the beam. If now a spherical lens is placed anywhere in the beam after it passes through the transparency the complex amplitude distribution in the *second* focal plane is approximately equal to the fourier transform of the amplitude distribution at the first focal plane (see Figure 1). Mathematically

$$E_2(u, v) = C_1 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dy\, T_A(x, y) \cdot \exp\left[-j \frac{2\pi}{\lambda F} (xu + yv) \right] \tag{2}$$

where u, v are the coordinates of a point in the second focal plane and C_1 is a complex constant (which we shall put equal to unity). Due to the homogeneity of the beam, the optical field as expressed by Equation 2 gives the fourier transform of the pattern of the transparency $T_A(x, y)$. As the fourier transform (or the spatial spectrum, as it also may be called) is given by a physical quantity (amplitude distribution), it may be effected by putting a mask in the focal plane. An additional fourier transformtion then produces a weighted or filtered version of the original transparency. This method of *spatial filtering* is of considerable interest because it constitutes a convenient way of dealing with and processing two-dimensional information.

The spatial distribution of the complex amplitude may also be recorded. The most direct way of recording is to use a photographic plate. Due to the properties of the film as a detector, the *intensity* distribution

$$I(u, v) = C_2 E_2(u, v) \cdot E_2^*(u, v) = C_2 \left| E_2(u, v) \right|^2 \tag{3}$$

is recorded on the film. The spectral distribution $I(u\ v)$ is often called the *wiener* spectrum of the spatial distribution $T_A(x, y)$ It is also possible, using holographic recording methods, to record the *complex* distribution $E_2(u, v)$. We shall, however, only be concerned with the intensity distribution given by Equation 3. The difference between the wiener spectrum (Equation 3) and the corresponding complex amplitude spectrum (Equation 2) is that all *phase* information has been cancelled. Due to the reciprocity of the fourier transform, the amplitude distribution $T_A(x, y)$ may be recovered through a knowledge of $E_2(u, v)$ (this is in fact done in any coherent imaging system). As there is an infinite number of complex amplitude distributions having the same wiener spectrum, the latter *cannot* be used to recover the original amplitude pattern $T_A(x, y)$. The wiener spectrum nevertheless contains important information about the pattern $T_A(x, y)$, as we shall now discuss.

Consider a train of equispaced slits in the xy-plane. The number of slits is m and the

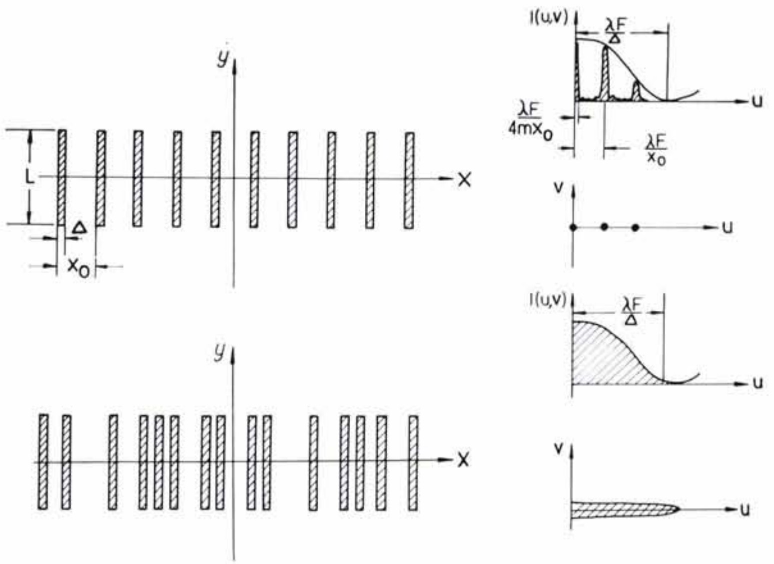

FIG. 2. Two distributions of elongated patterns and corresponding wiener spectra.

length, thickness and amplitude are L, Δ and A_o respectively (see Figure 2). The corresponding fourier transform is then

$$E_2(u,v) = C_1 A_0 L\Delta \operatorname{sinc}\left(\frac{2\pi}{\lambda F}\frac{L}{2}v\right) \cdot \operatorname{sinc}\left(\frac{2\pi}{\lambda F}\frac{\Delta}{2}u\right) \cdot \sum_{k=-(m-1)/2}^{(m-1)/2} \exp\left(j\frac{2\pi}{\lambda F}kx_0u\right) \quad (4)$$

where x_o is the distance between the slits and where

$$\operatorname{sinc}(\alpha) = \frac{\sin\alpha}{\alpha}. \quad (5)$$

Because

$$\sum_{k=-n}^{n} \exp[j\alpha k x_0] = \frac{\sin[(n+1/2)\alpha x_0]}{\sin\left[\frac{\alpha}{2}x_0\right]}, \quad (6)$$

the corresponding wiener spectrum is

$$I(u,v) = C_2^2(A_0L\Delta m)^2 \operatorname{sinc}^2\left(\frac{2\pi}{\lambda F}\frac{L}{2}v\right) \cdot \operatorname{sinc}^2\left(\frac{2\pi}{\lambda F}\frac{\Delta}{2}u\right) \cdot \frac{\operatorname{sinc}^2\left(\frac{2\pi}{\lambda F}2mx_0u\right)}{\operatorname{sinc}^2\left(\frac{2\pi}{\lambda F}\frac{x_0}{2}u\right)}. \quad (7)$$

If we consider only the spectral values of the main lobe (i.e., disregarding sinc $[(2\pi/\lambda F)(B/2)u]$ for $u>\lambda F/\beta$, the wiener spectrum consists of a series of dots as depicted in Figure 2. These dots are situated on a straight-line *perpendicular* to the elongation in the xy-plane (provivded that the c-axis and the u-axis are oriented parallel to each other).

A non-regular distribution of linear slits may be analyzed by assuming that the position of the slits along the x-axis is randomly distributed. The signal along the x-axis can now be written

$$T_A(x,0) = A_0 \sum_i h(x - x_i)$$

where $h(x)$ is the shape of the amplitude transmission. For x_i randomly distributed with uniform density the probability of the number of slits in a given interval is Poisson-distributed. The properties of this process have been examined in considerable detail. The wiener spectrum can be written (see, for instance, Papoulis, 1965), as,

$$I(u,0) = \text{const}\,(A_0L\Delta m)^2 x_0 \Lambda \operatorname{sinc}^2\left(\frac{2\pi}{\lambda F}\frac{\Delta}{2}u\right) \quad (8)$$

where Λ is the density of the slit number (for the previous case $\Lambda = 1/x_o$). This spectrum, which also is shown in Figure 2, is a *continuous random* distribution of lines in the original image.

If the spectral distribution is observed through a narrow slit and the transmitted light intensity is integrated along the elongation of the slit, the resulting angular distribution function $F(\theta)$ gives information about the

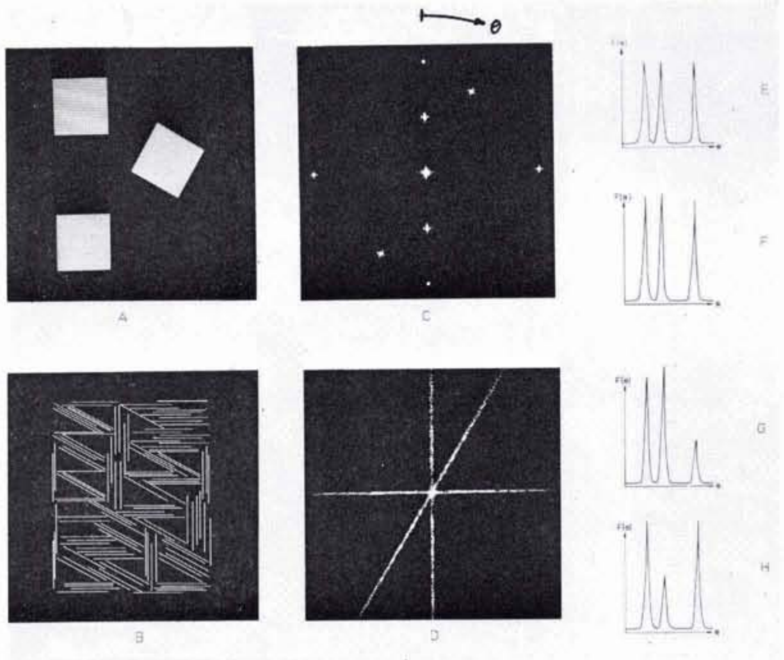

FIG. 3. Two samples of regular A and irregular B distribution of straight lines with corresponding spectra C and D. In E and F are shown the corresponding angular intensity distributions. The curves G and H shows the effect of decreasing the length of the vertical lines to half their original value G and decreasing the number of the inclined lines to half H.

occurrence of line structure in the spectrum and hence about corresponding line structure in the original image.

An arrangement for making such observations is also shown in Figure 1. The beam emerging from the transforming lens is split into two parts. One part is used for photographic recording of the wiener spectrum, the second is used for generating the angular distribution by rotating a disc which contains two slits. The light coming through the slits is integrated by a photomultiplier and recorded on a xy-recorder, the other axis of which records the angular displacement. The use of *two* slits blocks the low frequency components of the spectrum.

By integrating Equation 8 we obtain

$$F(\theta) = \int_0^\infty I(u, 0)du = \text{const } (A_0 Lm\Delta)^2 x_0 \Lambda \cdot \int_0^\infty \text{sinc}^2\left(\frac{2\pi}{\lambda F}\frac{\Delta}{2}u\right) = \text{const } (A_0 Lm)^2 \Delta. \tag{9}$$

The angular distribution, therefore, depends on the total area of the slits $(L\cdot\Delta\cdot m)$ and the contrast of the slit A_o.

In Figure 3 are shown two distributions of straight lines. One of the distributions is regular in position A, the other is irregular B. The angular distributions and the properties of the lines (length, thickness) are the same. In C and D are shown the wiener spectra corresponding to the distributions of A and B. The spectrum corresponding to $A(C)$ is seen to be discontinuous, a series of *sinc*^{2}-functions, while the spectrum corresponding to $B(D)$ has a *continuous* distribution of intensity. In E and F are shown the *angular* distribution of intensity of the spectra corresponding to A and B obtained by the observation method described earlier. The angular distributions are seen to be the same as they should be according to theory. In G and H are seen, respectively, the effect of decreasing the length of the vertical lines of A and decreasing the number of the inclined lines.

Application to Imagery

To understand the dynamics of processes which formed the ground surface and to judge the characteristics of the material, the establishment of linear patterns and preferred orientation of elements—both on a large scale and in microscopic state—is an important task in many earth science disciplines. Structural geology comes first to mind, because linear features and their orientation are an

FIG. 4. Sample of tetrogonal pattern (*top*), corresponding wiener spectrum (*center*) and angular distribution (*bottom*).

essential parameter for the analysis of tectonic movements and deformations. Directtional analysis enters as a rule also into studies of loose deposits, their morphology and morphogenesis, both concerning the topographical configuration and the forming particles. Differences in the dynamic properties of the eroding or accumulating processes which have formed the deposits are often reflected in rectional variations of the components.

Linear patterns are *sometimes* distinct and of only one direction, but nature is often more complicated. Deviating directions may appear in a dominant pattern. This may be due to the qualities of the material, but also to changes in the initial direction of active processes.

Directional changes often reflect age differences of processes. The deviation may be so great or the frequency of newly added directions so intense that the primary directions are difficult to conceive intuitively. In order to find the dynamic system, or at least to ob-

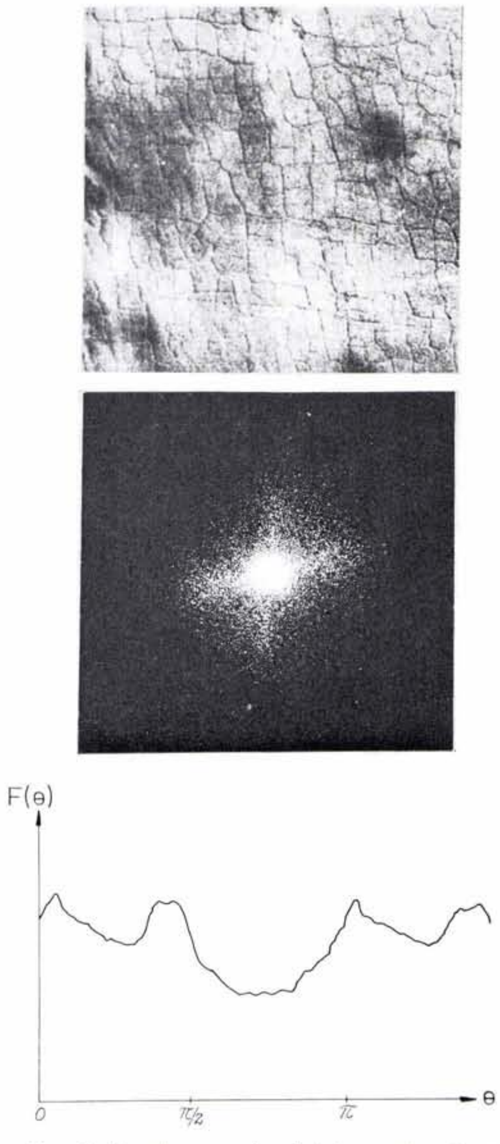

FIG. 5. Another sample of tetrogonal pattern (*top*), corresponding wiener spectrum (*center*) and angular distribution (*bottom*).

tain a diversity of conceptions of the active processes, it is necessary to register the directions which occur.

As a rule the determination of direction is performed in nature or in images, whereupon the data are statistically processed or the results are graphically illustrated, for example in rose diagrams. The frequency distribution is used as a basis for the analysis of the morphogenetic importance of the directional element.

As all the lines or directions *cannot* always be measured, and because the sample may be open to subjective criticism, at the same

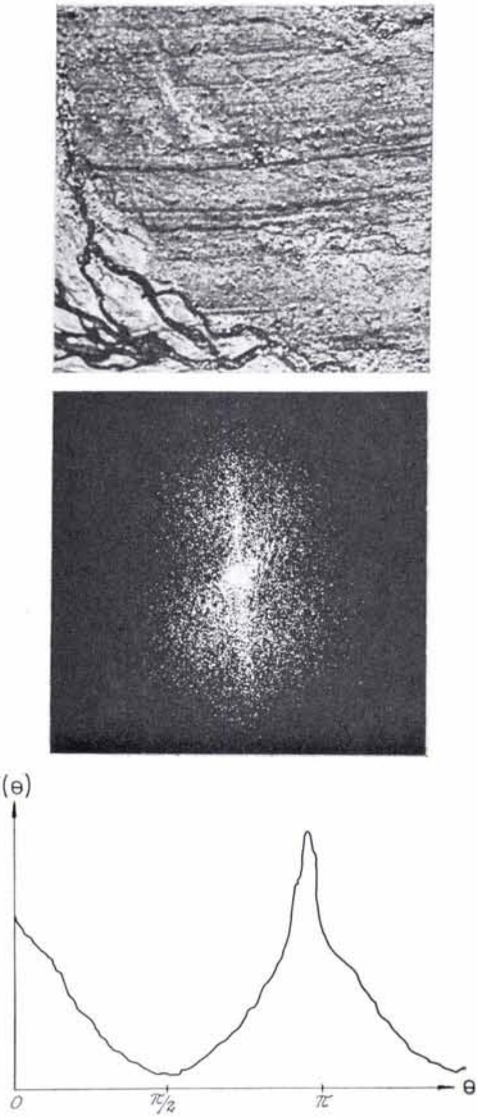

FIG. 6. Sample image of fluted moraine (*top*) with spectral distributions (*center and bottom*).

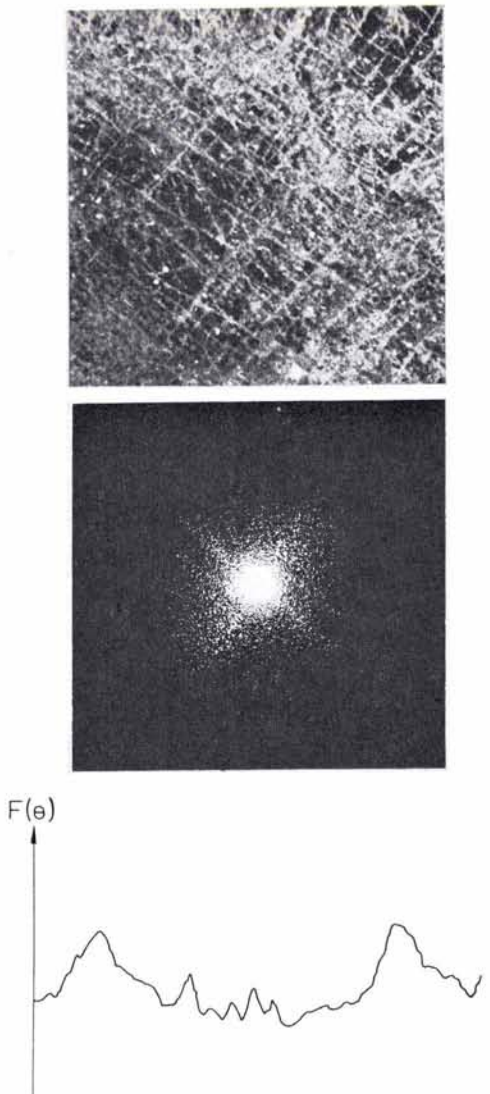

FIG. 7. Sample of linear structure in limestone (*top*) with corresponding spectral distributions (*center and bottom*).

time as errors of measurement may arise, every method which automatically classifies and graphically illustrates directional systems is a step towards an objective analysis of directional elements.

The following examples are shown to show how the method of optical processing operates on images of natural features and to demonstrate the mode of interpreting the spectral and angular distribution records.

TETRAGON-PATTERNED BLOCK FIELD

The spectral feature of this image consists

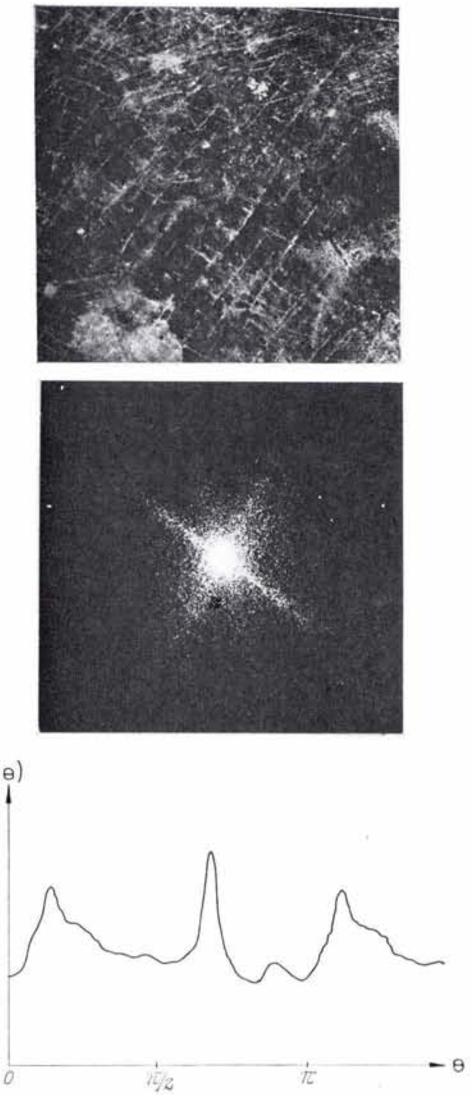

FIG. 8. Another sample of linear structure of limestone.

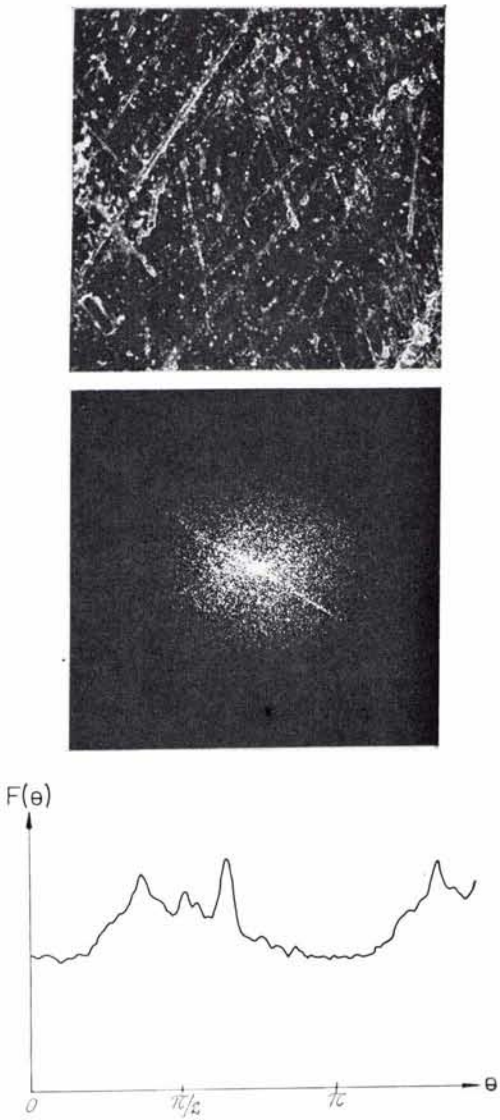

FIG. 9. Sample of striated rock surface (microphotography) (*top*) with corresponding spectral distributions (*center and bottom*).

of a noisy low-frequency spectrum in addition to two linear structures which are almost perpendicular to each other (Figure 4, top and center). The angular distribution (Figure 4, bot.) shows that the horizontal line structure of the image is more intense than is the vertical structure. The *direction* of the linear structure is not very well defined. The other sample of this same structural pattern (Figure 5, top) has a more distinct linear pattern in the spectral distribution (Figure 5, center) which is also evident from the angular distribution (Figure 5, bottom).

FLUTED MORAINE

The structural pattern of this sample (Figure 6, top) consists of an irregular fluvial pattern in addition to a regular horizontal line structure. The former generates a broad-band low-frequency spectral pattern (Figure 6, center), vertically elongated, and the latter a line structure, also vertical. This is very clearly seen in the angular distribution (Figure 6, bottom) where the broad-band structure causes a sinusoidal variation. Superimposed upon this we see the line structure.

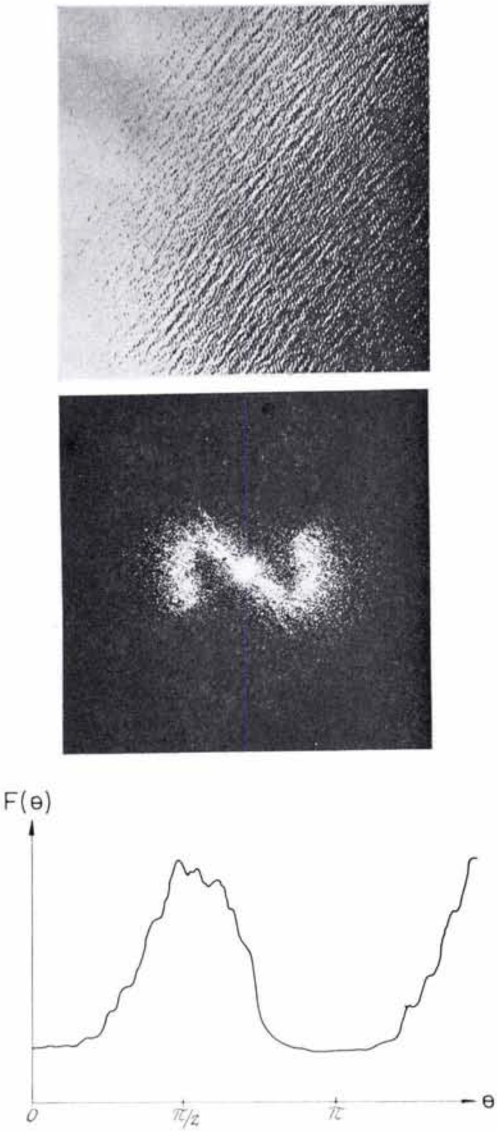

FIG. 10. Image showing ocean waves (*top*) with corresponding spectral distributions (*center and bottom*).

Fractures

The first image sample of this structure (Figure 7, top) incorporates a multitude of linear features; the spectral record (Figure 7, center) shows a predominance of two perpendicular patterns. The angular distribution (Figure 7, bot.) is quite interesting as it shows one predominant feature and a number of smaller features (four or five). The second sample (Figure 8, top) shows lines with greater constrast and smaller thickness. The corresponding wiener spectrum (Figure 8, center) is therefore more intense, having two dominant features and a third, less intense. This is also clearly seen in the angular distribution (Figure 8, bottom).

Glacial striation on a limestone surface

The microphotographic image (Figure 9, top) shows linear features in addition to a speckled pattern. The latter generates a symmetrical low-frequency pattern (Figure 9, center). A strong line component can be seen. The angular distribution also shows other components of smaller intensity.

Ocean waves

Figure 10 shows a sample of ocean waves. The wiener spectrum (center) shows an interesting structure. The low-frequency wave has a well-defined direction. The *periodicity* of the wave is not so well defined; the spectrum is therefore continuous.

The high-frequency wave, on the other hand, has a well defined periodicity whereas the *direction* varies within a rane of approximately 60°. Due to the great variation with respect to spatial frequency, the *angular* distribution (Figure 10) is *not* so easily interpreted. The pattern shows the existence of *fine structure*.

Concluding Remarks

In a surface such as the ground surface with an abundance of information, directional lines may escape detection from a point of observation on the ground. An aerial photograph gives a better view, and there are many examples of the discovery of patterns or objects with the aid of aerial photographs. Even in aerial photographs, however, due to the quantity of information, patterns may be overlooked in ocular inspection. Besides, the conception of an image is different for different people. Optical processing can help in detecting such patterns and in stating their properties. The examples of pictorial information of various kinds treated above show that the relatively simple arrangement of measuring the angular content of the wiener spectrum may be of considerable help for analyzing imageries obtained in earth sciences.

References

Dobrin, M. M. Optical processing in the earth sciences. *IEEE Spectrum 5*, No. 9, Sept 1968, pp. 59–66.

Papoulis, A: *Probability, random variables and stochastic processes*. McGraw-Hill, New York, 1965.

Pincus, H. J., Dobrin, M. M.: Geological applications of optical data processing. *J Ges Res 71*, No 2, 1966, pp 4861–4869.