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Modulation Transfer Functions

Practical considerations for their determination for photogrammetric camera systems.

(*Abstract on next page*)

A PHOTOGRAMMETRIC camera system consists of a mount, camera, filter, film, and magazine operated at a particular aperture and shutter speed. Associated with the system are the vehicle, its speed and flying height, and the method of processing the film. These parameters, plus weather, must be specified in speaking of camera system performance.

Traditionally, camera system performance has been evaluated in terms of photographic resolving power, that is, the number of line pairs per millimeter which can be distinguished in the image. This is an effective way of evaluating the imaging characteristics of photographic systems, and because of its simplicity the determination of high-contrast resolving power has for several years been an integral part of the laboratory testing and camera calibration procedures employed by the U.S. Geological Survey (Figure 1). However, even though resolving power is still an accepted method of evaluating aerial camera systems under both laboratory and operational conditions, its advantages and in particular its limitations have been the subject of many discussions (e.g., Brock *et al.*, 1966, p. 65). Consequently, several alternative methods of evaluating image quality have been developed during the past two decades. Of these, the method which has received the greatest attention concerns the modulation transfer function (MTF), which—unlike resolution—indicates the system response to targets which contain elements whose images range from zero to a finite number of cycles per millimeter. In effect, the MTF is a curve indicating the degree to which image contrast is reduced as spatial frequency is increased, and individual MTF curves can be produced for lenses, films, image motion, and most other variables in the camera system.

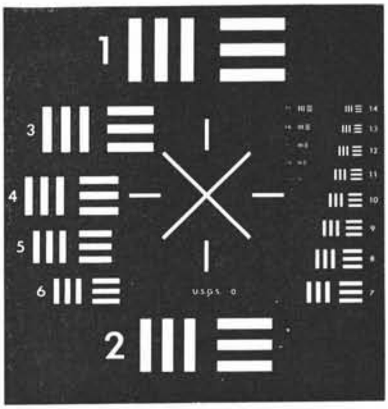

FIG. 1. Collimator target employed in the calibration of cameras by the U.S. Geological Survey.

A total system MTF is derived by multiplying the responses of the appropriate lens, film, and image-motion curves frequency by frequency in a process known as *cascading* (Mees and James, 1966). (See Figure 2a.)

In dealing with MTF's, it must be noted that MTF or *sine-wave response* is associated with a sinusoidal target and not the three-bar target commonly used to test resolving power. However, as most imaging systems will produce an apparent sine-wave image from a bar target (Selwyn, 1948, 1959), particularly at the high frequencies near the limiting resolution of the system, methods of relating MTF to resolution prediction have been devised (Brock, 1966; Scott, 1966). The method most frequently discussed involves a threshold modulation (TM) curve (Lauroesch et al., 1970) in conjunction with a system MTF which excludes the film MTF (Figure 2b). The TM curve is a graphical plot of visually determined *film* resolution for targets of several contrasts, and the indicated system resolution is given by the intersection of the MTF and TM curves.

* Presented at the ASP-ACSM Fall Technical Conference, Denver, Colorado, October 1970.

At this point, the photogrammetrist who is interested in the minimum size of object that can be detected or measured may question the value of MTF's, particularly as they are often used to predict a resolution value which could be determined in practice if the necessary facilities were available. Unfortunately, however, the required test facilities are seldom available, and to compare the imaging qualities of photographic systems, manufacturers' data must be used. At present, MTF, resolution, and granularity data are available for many films (Kodak, 1969), and a few MTF's for photogrammetric camera lenses have been published (Schwidefsky, 1960; Welander, 1962, 1968; Wurtz, 1969). Consequently one can conduct his own analyses to optimize the photogrammetric camera system for a specific task (Hempenius, 1964). Good image quality is, of course, a particularly important consideration in obtaining high-altitude, small-scale photographs for mapping.

In the preceding discussion emphasis has been given to predicting a final MTF curve or resolution value based on known system parameters. It is also possible, however, to determine MTF's that indicate system performance by analysis of aerial photographs containing images of periodic targets or sharp edges. Thus, the photogrammetrist can not only predict the imaging characteristics of a photo-

ABSTRACT: *The subject of modulation transfer functions (MTF's) of photogrammetric camera systems has received very little attention to date because of the complexity of undertaking the required measurements and the largely undemonstrated value of MTF's to the practical photogrammetrist. However, MTF analysis is used extensively in camera system design and can be expected to play an important role in the development of future photogrammetric camera systems. Consequently, in order to judge the improvements in image quality likely to result from new designs, it is first important to establish the MTF's of photogrammetric camera systems in current use and the relation of these MTF's to image quality. The practical considerations in determining MTF's include the design of targets, specification of photographic parameters and evaluation equipment, and the methods of evaluation. Each of these considerations is discussed, with particular emphasis given to an inexpensive but accurate graphical-digital method of MTF analysis of the images of edge targets.*

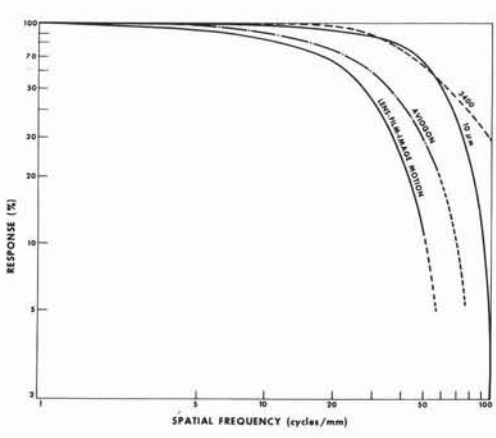

FIG. 2A. MTF curves for a Wild Aviogon lens (Welander, 1968); Kodak 3400 film; and 10 μm of image motion. The total MTF is the product of the lens, film, and image-motion curves.

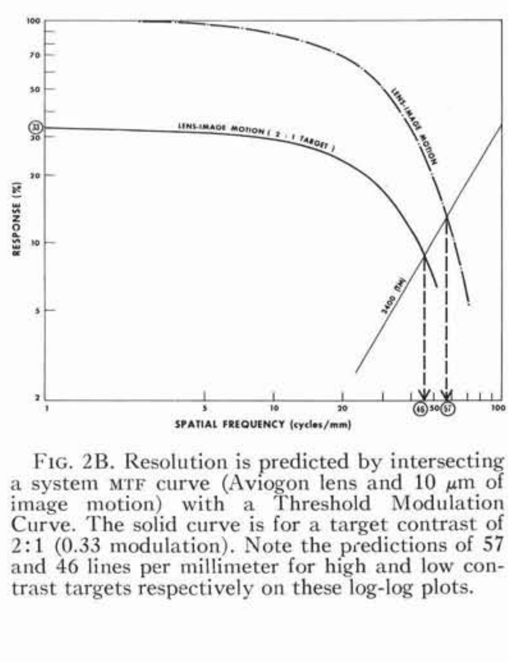

FIG. 2B. Resolution is predicted by intersecting a system MTF curve (Aviogon lens and 10 μm of image motion) with a Threshold Modulation Curve. The solid curve is for a target contrast of 2:1 (0.33 modulation). Note the predictions of 57 and 46 lines per millimeter for high and low contrast targets respectively on these log-log plots.

graphic system but can also compare predicted performance with that obtained in practice. Correspondingly, it seems that by extending the work to include visual evaluations and measurements of objects of known ground dimensions, correlations can be obtained between MTF's and subjective impressions of image quality (Welch, 1969). If adequate correlations can be established, MTF's will likely be of value in judging the level of operational performance of photogrammetric camera systems.

Although MTF's certainly can be useful to the photogrammetrist, fairly complex equipment and data reduction schemes are needed to determine the MTF's from aerial photographs. Perhaps as a consequence, considerable attention has been given to theoretical rather than practical considerations. The purpose of this paper is to examine the possibilities of determining system MTF's from aerial photographs on a practical basis.

METHODS OF DETERMINING SYSTEM MTF'S

Two methods of determining system MTF's will be discussed. The first uses periodic targets and is best suited to laboratory analyses. The second is based on the analysis of sharp edges and is the more useful for determining MTF's from operational photographs.

PERIODIC TARGET METHOD

The theory of MTF analysis is based on the use of sinusoidal targets which, when imaged by a photographic system, retain their sinusoidal shape but are reduced in amplitude at high frequencies. Where the image is traced by a microdensitometer, the resultant pattern is similar to that shown in Figure 3. At each frequency the values of maximum density (D_{max}) and minimum density (D_{min}) are determined directly from the trace, and these values are converted to linear relative exposures by means of the emulsion *D-log-E* curve. The maximum and minimum relative exposure values (I_{max} and I_{min}) are used in

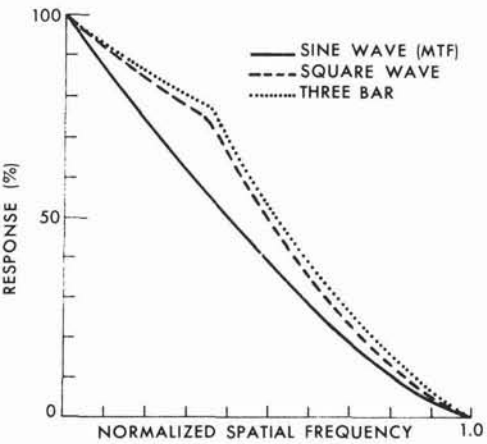

FIG. 4. Response functions for a diffraction-limited lens. Note that these functions are plotted on a linear scale.

the following formula to determine the modulation transfer (MT) factor at each frequency.

$$\text{MT factor} = \frac{I\ \max - I\ \min}{I\ \max + I\ \min} \times \left(\begin{array}{l}\text{Normalization factor to give } 100\% \\ \text{modulation at zero frequency.}\end{array}\right) \quad (1)$$

The normalized MT factors are plotted against spatial frequency to produce the MTF.

This method is easily accomplished if a sinusoidal target is imaged. Unfortunately, precise sinusoidal targets are difficult to produce, and bar targets are often substituted to obtain the square-wave response function (Rosberry, 1959). If desired, the square-wave response can be converted to sine-wave response by equation 2, beginning with the highest frequency (also refer to Figures 4 and 5):

$$M_{(k)} = \frac{\pi}{4} \cdot \overline{M}_{(k)} + \frac{M_{(3k)}}{3} - \frac{M_{(5k)}}{5} + \cdots \quad (2)$$

where k is the spatial frequency, M is the sine-wave modulation (MTF) and $\overline{M}$ is the

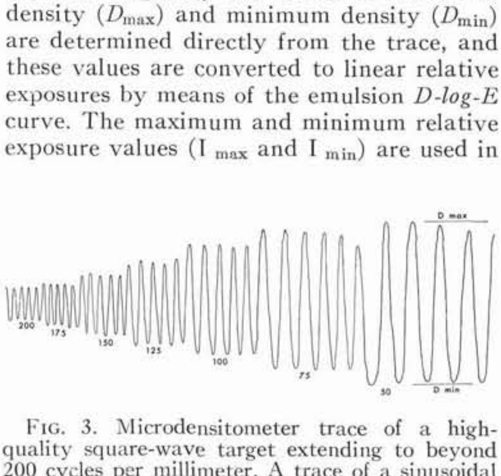

FIG. 3. Microdensitometer trace of a high-quality square-wave target extending to beyond 200 cycles per millimeter. A trace of a sinusoidal target would have a similar appearance.

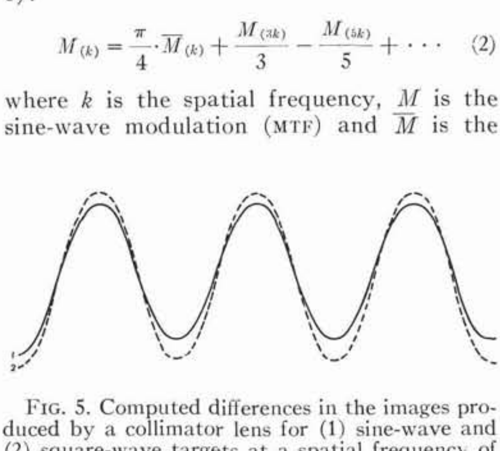

FIG. 5. Computed differences in the images produced by a collimator lens for (1) sine-wave and (2) square-wave targets at a spatial frequency of $33\frac{1}{3}$ lines per millimeter.

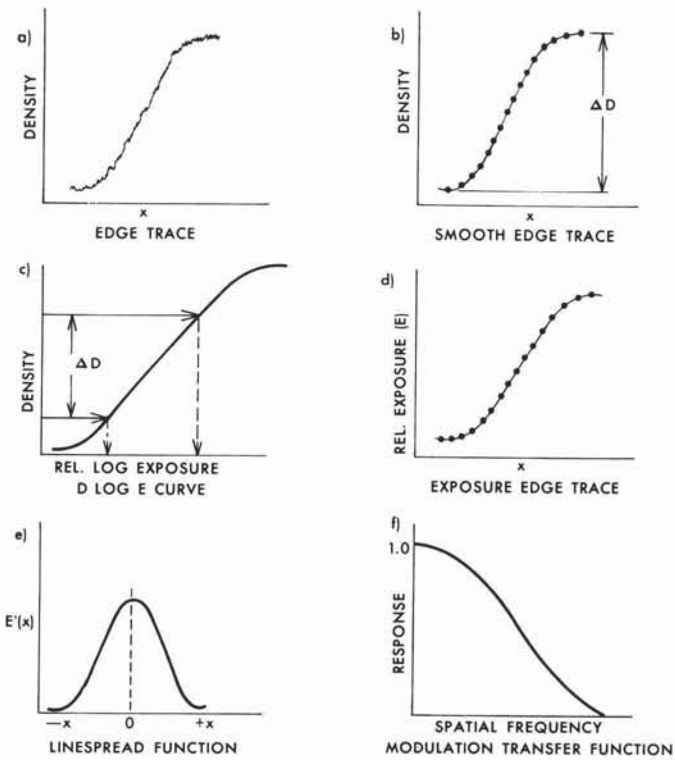

FIG. 6. Steps in the determination of the MTF from the microdensitometer trace of the image of a sharp edge.

square-wave modulation. (Scott, Scott, and Schack, 1963).

EDGE ANALYSIS METHOD

Square-wave functions and MTF's of a photogrammetric camera system can be obtained from a microdensitometer trace of the photographic image of a very sharp edge. The general procedure is diagramed in Figure 6.

After the edge trace (Figure 6a) has been obtained, it may need smoothing (Figure 6b) to eliminate noise introduced by film granularity. The D-log-E curve (Figure 6c) is then determined from step tablets impressed on the film, and the smoothed density versus distance trace is converted to a relative exposure vs. distance trace (Figure 6d) by using the D-log-E curve. The first derivative of this exposure edge trace is then taken to give the line-spread function (Figure 6e) and a Fourier transform of the line-spread function provides the MTF (Figure 6f). The greatest difficulty in determining the MTF of a photographic system by this method lies in obtaining the Fourier transform of the spread function. Ways of doing this will be discussed later.

GROUND TARGETS

The choice and design of targets, particularly for operational use, depends on the contemplated method of analysis as well as a number of practical considerations, such as the imaging capabilities of the system, scale of the photographs, microdensitometer capabilities, and target contrast. The available space for laying out the targets, access to the space, and cost and availability of materials must also be considered.

SINUSOIDAL TARGETS

Sinusoidal targets of varying spatial frequency are ideal for the analysis of MTF's but have two major disadvantages operationally. First, they are very difficult and expensive to produce at the large sizes required; and second, they cannot be analyzed in terms of visual resolving power. Consequently, sinusoidal targets are seldom used in practice, and bar targets or edges are preferred.

BAR TARGETS

Standard three-bar targets of varying spatial frequency are relatively easy to construct and permit the visual determination of resolving power, but they also have some disadvantages. For example, on Air Force resolution patterns displayed at test facilities throughout the United States, the bar length as well as width decreases with increasing spatial frequency, precluding microdensitometer analysis. Consequently, special long-bar targets must be designed if both microdensitometer analyses of MTF's and visual evaluations of resolving power are required. For reliable MTF analysis, the images of the bars should be at least twice the length of the effective slit in the microdensitometer. Effective slit lengths of 80 to 200 μm are common, with 100 μm typical.

Another disadvantage of bar targets is that both shape and amplitude are changed in the imaging process. The differences between images of bar and sinusoidal targets, as imaged by a near-perfect lens, are evident in Figures 4 and 5. Williams (1969) has pointed out the theoretical discrepancies between the various functions and the predictions of resolution derived from them. Barakat and Lerman (1967) have shown, again on the basis of theoretical studies, that some seven bars at a given frequency are needed to obtain a truly square-wave response. However, practical experience, involving an imperfect lens, film, image motion, and the atmosphere, indicates that the differences between square-wave and three-bar functions are negligible in relation to other errors. This is, of course, most useful to our discussion as the construction of more than three bars per frequency is usually impractical.

The number of bar groups and the range of frequencies depend on the scale of the photographs, the number of sampling points required, and the amount of work and material cost that can be expended in constructing targets. As the final MTF plot is derived by drawing a curve through individual MT factors, the spacing of these factors need only be close enough to ensure a reasonably accurate curve. In recent work, 7-bar groups with individual elements having widths of 6, 4, 2.5, 1.5, 0.75, and 0.5 feet have been satisfactorily used at photograph scales of 1:12,000 and 1:24,000. However, a set of targets consisting of legs, one oriented parallel and the other perpendicular to the flight line, required a ground space of 26 by 120 feet for each leg. Targets should be placed on the centerline of the planned flight strip, so as to image near center of at least one photograph.

EDGE TARGETS

Edge targets are easily produced, particularly for large-scale photographs, and may take several forms, including those indicated in Figure 7. In planning the placement of targets, particularly minimum-size edge targets, care must be taken to ensure that one side of the target is laid out parallel and relatively close to a long straight line (such as a road) which will appear in the photographs. This permits the microdensitometer slit to be carefully alined with respect to the road and then transferred to the target for the edge trace measurement.

TARGET CONTRAST

Most resolution test facilities in the United States contain high-contrast targets in which white bars of about 80 percent reflectance are painted on a black background of about 5 percent reflectance to give contrast ratios as high as 16:1. One commercially manufactured edge target has 37 and 4 percent reflectances to give an approximate 9:1 contrast ratio. In the author's opinion, these contrast ratios are too high for resolution and MTF evaluations of photogrammetric camera systems under operational conditions.

Much discussion has taken place over the years concerning the optimum target contrast for operational evaluations of aerial camera systems, and even though it is generally

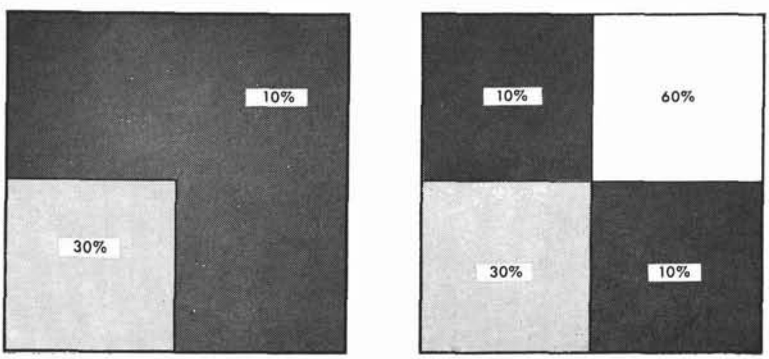

FIG. 7. Two possible edge-target designs.

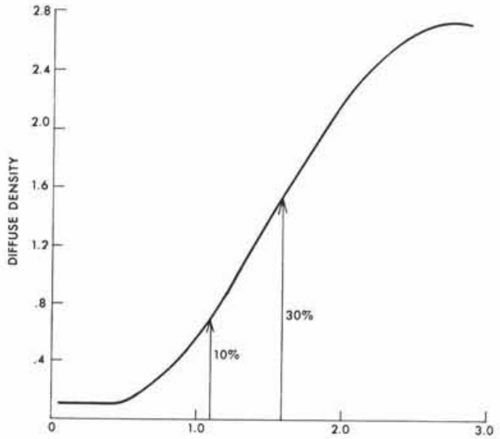

FIG. 8. Desired relationship between a D-log-E curve and the 10- and 30-percent levels of reflectance.

agreed today that a resolution target with a contrast of 1.6:1 or 2:1 should be used for performance tests, few such targets exist in the United States. Furthermore, although contrast ratios are often specified, little thought seems to have been given to optimum reflectance levels for cbtaining the specified contrast ratio. For example, a 50 percent background and 75 percent bar are hardly realistic reflectance values. With respect to edge targets, low-contrast edges are more representative of aerial photographic images,

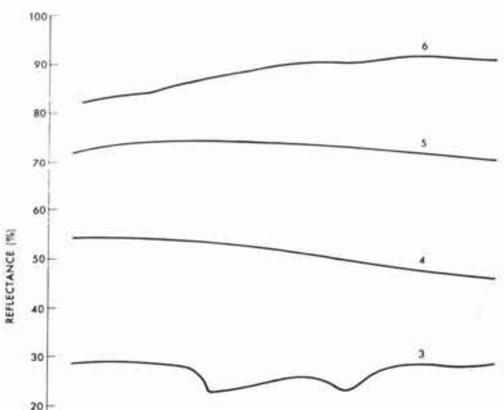

FIG. 9. Spectral curves of some exterior latex house paints and vinyl nylon: (1) black paint; (2) dark-grey paint; (3) gray vinyl; (4) light-gray paint; (5) off-white paint; (6) white vinyl.

but target contrast ratios of less than 2:1 are unlikely to be imaged with sufficient density differences to ensure accurate MTF determinations. It has been proved that the accuracy with which MT factors can be determined falls off rapidly at very low contrast ratios (Attaya *et al.*, 1966). On the other hand, target contrast ratios greater than about 6:1 are unlikely to be suitable for MTF determinations because for clear atmospheres and medium-gamma films the lowlight and highlight reflectances will image outside the linear portion of the D-log-E curve.

On the basis of experience to date, a long bar or edge target having a contrast ratio of approximately 3:1 (perhaps 4:1 or 5:1 for hazy conditions or low-gamma films) with lowlight and highlight reflectances of 10 and 30 percent will be imaged on the linear portion of the D-log-E curve, corresponding to the zone of maximum resolving power (Figure 8). Similar reflectance levels and contrast ratios have proved satisfactory for the evaluation of both MTF's and visual resolving power.

MATERIALS AND CONSTRUCTION OF GROUND TARGETS

Materials for portable targets should be of reasonable weight, flexible, resistent to wind and weather, and receptive to paint. A vinyl nylon (10 oz. per sq. yd.) sewed into tarpaulins of the desired size with grommets at 3-foot intervals around the edges has proved satisfactory. Bars can be efficiently laid out with a measuring tape and carpenter's chalk line, and the edges masked with drafting tape before painting. A latex exterior house paint, for which typical spectral curves are shown in Figure 9, can be easily applied by roller. Experience indicates that about 1½ gallons of paint are needed in applying 2 coats to a 500-square-foot tarpaulin (Figure 10). Steel pegs

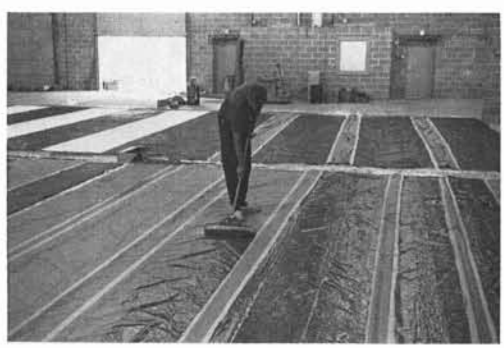

FIG. 10. Painting a target.

12 to 18 inches long, which can be driven through the grommets, are suitable for anchoring the tarpaulins to the ground. A 50- by 50-foot vinyl nylon edge target of the 2-contrast type (Figure 7) was painted and laid out in 1 day at a total cost of about $500.

Aerial Photography

MTF's indicating the performance of an aerial photographic system should be accompanied by complete information concerning the imaging system and the conditions of photographic exposure. Data should include:

- Date, time of day, general weather conditions, and visibility.
- Camera, lens, filter, magazine number, film, and mount.
- Aperture and shutter speed.
- Approximate ground speed of the aircraft and nominal flying height.
- Type of aircraft.

Once the photographs have been obtained, the success of MTF analyses depends on proper laboratory handling of the exposed film. Before the film is processed, a calibrated step tablet must be impressed on unexposed areas at both ends of the roll. A sensitometer with a tungsten light source filtered to daylight color temperature (5,500°K to 6,000°K) plus an additional filter equivalent to that used on the aerial camera is required for this purpose (Carman, 1969). Unfortunately, very few laboratories which process photogrammetric film are at present capable of performing this task.

Evenness of development throughout the roll of film is required if MTF's of reasonable precision and accuracy are to be obtained. Automatic continuous processing machines are excellent in this respect; however, they are quite expensive and may not be available. Information concerning the sensitometric exposures (light source, filters, and exposure time) and development process should be furnished by the laboratory.

Evaluation Equipment

The microdensitometer is the basic instrument required for MTF analysis. The cost ranges from $10,000 to over $200,000, depending on the complexity of the instrument and whether analog or analog-plus-digital output is provided. For the analysis of most photogrammetric camera systems, however, the lower priced instruments, providing analog output, are satisfactory. A typical instrument is shown in Figure 11. To obtain optimum performance from a microdensitometer, precautions must be taken in selecting en-

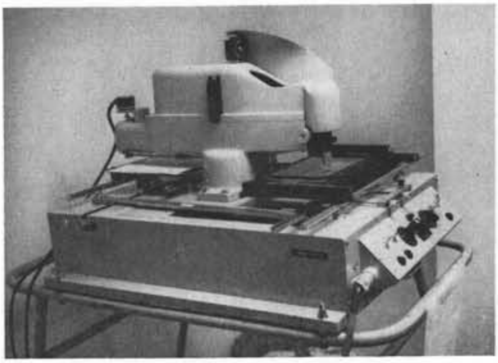

Fig. 11. Joyce, Loebl MK III CS microdensitometer.

largement ratios, density ranges, condensing and enlarging objectives, and effective slit size.

Enlargement Ratios

To some extent the resolution of the instrument, as determined by its mechanical capabilities and the effective slit width, can influence the selection of an enlargement ratio. However, for the normal effective slit widths of 1 to 5 μm, an image-to-plot enlargement ratio of 1:500 or 1:1000 is convenient. The enlargement ratio can be precisely determined by tracing a calibrated glass scale with 0.1-mm divisions and comparing the distance between divisions indicated on the graphical output with the equivalent distance on the glass scale.

Density Range

The useful density range for most properly exposed photographs lies between 0.2 and 2.0 diffuse density units. Consequently, the microdensitometer should give a linear response for densities in this range when tracing a calibrated step tablet on the appropriate type of film.

Objectives

The condensing and enlarging objectives for the microdensitometer must be selected with care. Theoretical considerations indicate that to avoid the effects of partial coherence, the numerical aperture (N.A.) of the condensing objective must be equal to or, if possible, somewhat larger than that of the enlarging objective (Born and Wolf, 1959). To meet this requirement and still maintain a reasonable working distance and depth of focus, relatively low-powered microscope objectives are normally used. In the system currently employed at the U.S. Geological Survey, matched 10× microscope objectives of 0.3 N.A. are used as both the condensing and enlarging objectives. This combination provides a working distance of approximately 5 mm and a depth of focus of several micrometers.

EFFECTIVE SLIT

A long, narrow effective slit is needed to provide maximum resolution in the x-direction yet allow the effects of film granularity to be averaged out in y; that is, the signal-to-noise ratio is increased. The effective slit size is determined by the relationship between the total magnification of the image specimen being scanned and the physical size of the slit as set in the instrument. For example, if the image is magnified by a factor of 10 in relation to a physical slit of 0.1 by 1 mm, the effective size of the slit is 0.01 by 0.1 mm. For most aerial photographs, an effective slit size between 1 by 80 μm and 5 by 200 μm may be selected. Obviously, as the slit becomes shorter, its alinement during analysis becomes less critical (Jones, 1965; also refer to Figure 12a). Also, targets can be reduced in size.

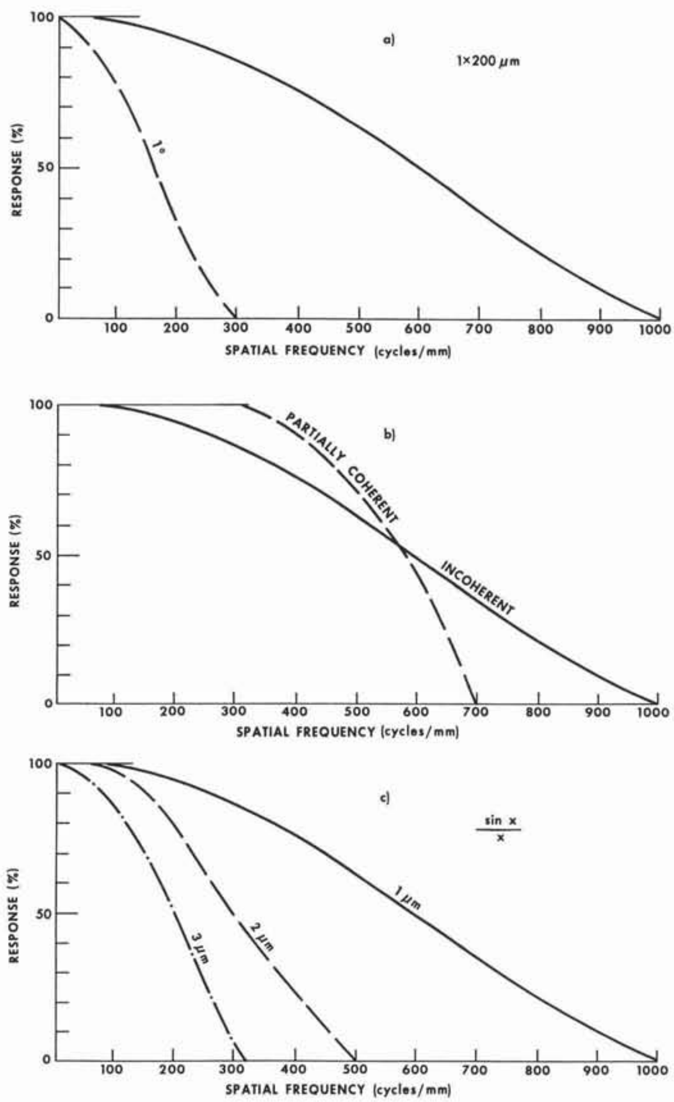

FIG. 12. Factors influencing the response function of a microdensitometer: (a) slit alinement for a 1 by 200 μm slit; note the response falloff for a misalinement of 1°; (b) partial coherence; and (c) slit width as defined by a $(\sin x)/x$ function.

The matter of optimum effective slit width is a controversial subject. For example, a narrow effective slit (1 μm) offers the desirable properties of high resolution and response in tracing narrow bars and spaces. Such narrow slits are commonly used for edge gradient analysis. The disadvantages of narrow slits, depending on the particular type of microdensitometer, light source, optics, and the characteristics of the scanned specimen, are excessive noise and the likelihood of partial coherence which creates a false response as shown in Figure 12b (Becherer and Parrent, 1967). To avoid partial coherence and reduce noise, the effective slit width can be enlarged—although at a corresponding loss of resolution and response (Figure 12c). The lower response of the microdensitometer with a 5 μm slit will obviously affect the edge trace and consequently the final MTF. Although a microdensitometer correction function can be applied to the trace (Jones and Coughlin, 1966) before MTF analysis, it is impractical unless the microdensitometer is connected to a computer. Simpler (but approximate) methods are to determine the response function (MTF) of the microdensitometer from: (1) the known effective slit width using a $(\sin x)/x$ function as in Figure 12c; (2) a trace of a sharp edge having a density difference of 1.0 or greater; or (3) a trace of a high-quality bar target with frequencies extending beyond 200 lines/mm. Once the MTF of the microdensitometer is known, its response can be divided out of the total system MTF.

Although any of these methods of correction may be employed, it would be advantageous to avoid both the effects of noise and partial coherence due to narrow slits and the loss of resolution caused by wider slits. Fortunately, such a compromise seems possible for photogrammetric camera systems with maximum resolving powers on the order of 70 lines/mm.

For these systems, a microdensitometer having a white light source and objectives which were chosen according to the previous discussion will provide an adequate uncorrected, linear, noise-free response to beyond 50 lines/mm with an effective slit width of 2 or 3 μm. In fact, an effective slit of 1.5 by 115 μm has been used extensively without any indication of partial coherence or excessive noise. A practical test for partial coherence is shown in Figure 13.

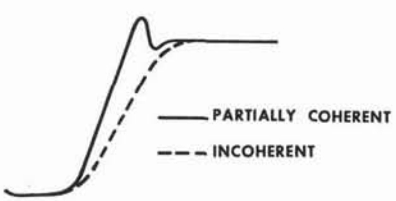

FIG. 13. Simulated edge traces for incoherent and partially coherent light. Practical tests with sharp edges should produce edge traces similar to the one for incoherent light.

Graphical-Digital Method of Edge Analysis

Edge targets may be more easily constructed than periodic targets, but they do present additional problems in data reduction. The primary task in edge analysis is to reduce the edge to its frequency components through Fourier analysis. As direct Fourier methods are extremely tedious, the obvious solution is to use either a computer program or a graphical approach such as that as described by Scott, Scott, and Shack (1963). The latter method is useful for symmetrical edges, but it is still quite tedious and not well suited to the analysis of the asymmetrical edges common in photographs obtained with photogrammetric cameras. By contrast, the computerized approach to edge analysis discussed by Jones (1967, 1970) is highly refined but hardly practical for most organizations. Instead, an approach combining both graphical and digital reduction procedures has been developed and is recommended for use by small organizations having both a microdensitometer and access to a computing facility. The microdensitometer may have either analog or digital-plus-analog output.

This graphical-digital method is currently being used at the U.S. Geological Survey and comprises several steps (see Figure 6). First, three or more traces of the same edge are superimposed on a light-table, and the mean trace is drawn on a separate sheet of graph paper. The density values are then recorded at 2 to 4-mm intervals (2 to 4 μm at image scale), resulting in approximately 20 to 30 sets of (x, density) coordinates defining an edge. The density values are then converted to relative exposure (E) values through the D-log-E curve (as related to microdensitometry) for input to the MTF computer program.

The first step in the program is the determination of the line-spread function, which by definition is the first derivative of the edge exposure trace defined by the x, E coordinates. As the first derivative is simply the tangent or slope, the line-spread function can be obtained by taking the slopes of short line segments made up of successive x, E coordinates according to

$$\tan_i = \frac{E_{i+1} - E_i}{x_{i+1} - x_i} \tag{3}$$

A plot of the tangent values against the corresponding x-coordinates defines the line-spread function as it appears in Figure 6e. The closest x coordinate corresponding to $(E_{max}+E_{min})/2$ is considered the midpoint of the spread function, and all x coordinates are translated so the x midpoint is 0. If the spread function is truly symmetrical, $-x = +x$.

Now, to develop the Fourier transform $A^{\#}$ of the line-spread function, it is convenient to follow the example of Perrin (1960), wherein the basic transform equation is

$$A^{\#} = \int_{-\infty}^{\infty} A(x)e^{-i2\pi vx}dx \tag{4}$$

and can be broken into cosine transform $A^{\#c}(v)$ and sine transform $A^{\#s}(v)$ components for each frequency (v); for example,

$$A^{\#c}(v) = \frac{\int_{-\infty}^{\infty} A(x) \cos 2\pi vx dx}{\int_{-\infty}^{\infty} A(x)dx} \tag{5}$$

by using x-, E-coordinates and tangent values and replacing the integral sign, this formula can be rewritten as

$$A^{\#c}(v) = \frac{\sum_{i=-x}^{+x} \tan_i \cos 2\pi v x_i \Delta x_i}{\sum_{i=-x}^{+x} \tan_i \Delta x_i} \tag{6}$$

where the denominator is simply the area of the spread function and acts as a normalizing factor.

A similar expression can be written for the sine transform, and the two equations can be readily programed. It is obvious that the repetitive calculation and summing operations are ideal for an electronic computer. In this manner $A^{\#c}(v)$ and $A^{\#s}(v)$ are obtained as separate values. If the spread function was truly symmetrical, $A^{\#s}(v)$ would be equal to zero at each frequency, and the MT factor $|A^{\#}(v)|$ or real part of the Fourier transform would equal $A^{\#c}(v)$ alone. However, as perfect symmetry is unlikely, $A^{\#s}(v)$ must be considered. Consequently, an MT factor is given by

$$|A^{\#}(v)| = [\{A^{\#c}(v)\}^2 + \{A^{\#s}(v)\}^2]^{1/2} \tag{7}$$

A plot of $|A^{\#}(v)|$ against the corresponding spatial frequencies produces the MTF. With a program based on the described procedures, numerical values and a graphical representation of the MTF's are plotted between 0 and 100 lines/mm. at 2 lines/mm. intervals on the standard line printer of the computer. This method can be used with any computer having a storage capacity of about 100,000 bytes. Computing time for the MTF, plus computations for phase angle and square-wave response, is less than a minute on the IBM 360/65.

Precision and Accuracy

The precision of MTF determinations based on edge analysis is readily evaluated by tracing the same edge several times and submitting each of these edge traces to the appropriate reduction procedure. Computations based on MT factors at corresponding frequencies from all of the MTF curves will yield the standard deviation from the mean.

Accuracy of MTF curves is more difficult to determine. Methods which have been suggested include (1) comparison of MTF's derived from periodic patterns and edges; (2) comparison of theoretically determined MTF's with actual MTF's; and (3) comparison of static camera-system MTF's with those for which known image motion has been introduced. The most reliable results seem to have been obtained from the last-named method (Hendeberg and Welander, 1963; Jones, 1968).

Studies of the precision and accuracy of MTF analyses based on edge gradient methods are limited. Welander (1970) has summarized investigations to date and indicates that an approximate standard deviation of ± 4 percent in the precision and ± 6 percent in the accuracy can be expected in dealing with aerial photographs. Scott (1968) and Brock (1968, 1970) have indicated that differences of at least 10 percent in the spread functions or MTF's are required before a subjective change in image quality is noticed. Therefore, accuracies of 10 percent or better in MTF determinations by edge analysis should prove satisfactory. Correspondingly, it appears that where system performances are being evaluated on the basis of theoretical or actual MTF's, the range of MTF's produced by system A must not overlap the range of system B before one can be considered superior to the other (Figure 14). Only accumulation and refinement of data will indicate the validity of this premise; however, it should be evident that small differences in system MTF's are not significant.

Summary and Conclusion

MTF's, although increasingly employed by photographic system engineers and by lens and film manufacturers, have received only limited attention from photogrammetrists. The reasons for this include the largely undemonstrated value of MTF's to the photogrammetrist, the complexity of analyzing them, and the lack of standards against which results can be compared. However, MTF's represent a powerful new tool for image-quality evaluation which can be employed by the photogrammetrist to optimize or compare photographic systems. The system MTF is obtained by combining individual lens and film MTF curves with those for image motion and other components of the photographic system to produce a total system MTF. System MTF's when related to other data, such as resolution values and film granularity, provide a more complete description of the capabilities of the photographic system than resolution values alone.

In addition to using MTF's in the prediction of system performance, it is also possible to derive a system MTF from aerial photographs obtained under operational conditions. Before taking the aerial photographs and attempting the analyses of MTF's, however, consideration must be given to the size and type of target; target contrast ratio; conditions under which the aerial photographs must be obtained and processed; type of microdensitometer equipment, including the selection of enlargement ratio, density range, objectives, and effective slit size; and finally the method of evaluation. For organizations having a microdensitometer and access to a computer facility, an evaluation method combining both graphical and digital reduction steps is recommended for economy and accuracy.

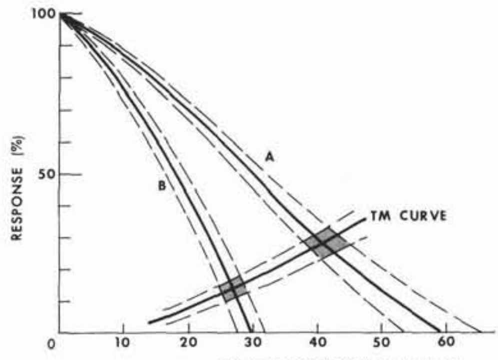

FIG. 14. Two simulated MTF's and a TM curve. The dashed lines indicate the limits of confidence, and the shaded areas are the regions within which predicted values of resolving power are likely to occur.

It must be emphasized that, although the methods described for evaluating MTF's of camera systems may appear exact, many conditions reduce the final precision and accuracy as evaluated. Of these, the nonlinear nature of the D-log-E curve causes the most serious difficulties. However, other factors which may influence the results include limitations of the measuring equipment, the effects of partial coherence, slit width, and slit alinement on the microdensitometer trace, and the computational procedure by which the MTF is determined. Because of these factors and the inability of the human observer to distinguish small variations in spread functions, differences of up to 10 percent in MTF's should probably not be considered significant. On the other hand, if

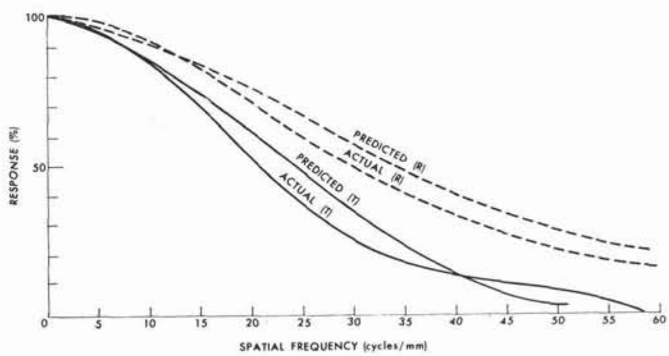

FIG. 15. On-axis predicted MTF's based on the analysis of laboratory photographs compared with on-axis MTF's obtained from actual aerial photographs. A Wild RC8 camera with Universal Aviogon lens and the same magazine, film (2402), filter (500 nm), and aperture (f/8) were used throughout the tests. Eastman Kodak processed the film in a Versamat (MX641 developer) to an average gamma of 1.25.

data were made available to permit comparisons of predicted performances and laboratory tests with the results obtained in practice for typical photogrammetric camera systems, many of the problems of establishing tolerances for precision and accuracy would be eased. In addition, these data would provide a basis for judging the improvements in image quality likely to result from employing new camera system designs (refer to Figure 15).

Acknowledgments

A National Research Council-National Academy of Sciences Post-doctoral Research Associateship in cooperation with the U.S. Geological Survey has permitted the research on which this paper is based. The author is grateful for the assistance given by A. Sorem, F. Jackson, V. A. Baensch, and M. R. Specht of Eastman Kodak Company, and for the invaluable cooperation and assistance given by R. Mullen and J. Halliday, U.S. Geological Survey. The preparation of diagrams by Paul Buckler is gratefully acknowledged.

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