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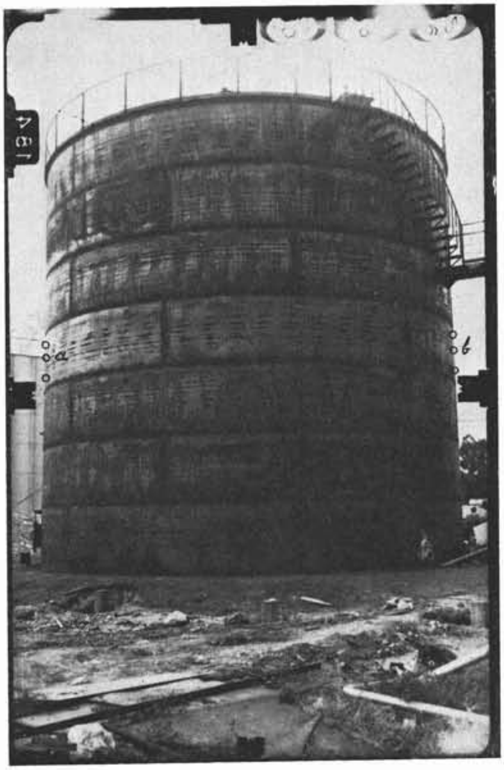
# Calibration of Storage Tanks

**Adequate accuracy is obtained in the application of terrestrial photographs.**

*(Abstract on next page)*



FRONTISPIECE. An oil storage tank was photographed with a Wild phototheodolite, $f = 165$ mm. Points at *a* and *b* indicate the order in which measurements were made.

## INTRODUCTION

CALIBRATION OF cylindrical oil storage tanks is a basic means of measuring oil quantities. The usual calibration method consists of a direct measurement of circumferences of the tank at different levels. This method is applicable only if scaffolding around the tank is available. Recently, tank construction methods were employed which do not require the attachment of scaffolding to the tank. Hence the direct measuring method becomes impossible and the necessity arises to look for other measuring methods. One of the possibilities to determine the tank dimensions necessary for preparing calibration tables lies in the use of photogrammetric tools.

This paper discusses a calibration method based on the evaluation of single photographs. The approach to the evaluation of the photographs is rather unusual, it is not concerned with orientation problems and uses distances measured on the object as control data. The paper presents the derivation of all needed formulas, a brief accuracy analysis and illustrates the method by results taken from practice.

## PHOTOGRAPHY AND MEASUREMENTS

The tank is photographed from a number of stations scattered around its circumference. Photography distance varies according to the size of the tank. Generally two conditions are imposed on its choice, namely that it has to provide a large-scale photograph and ensure that the whole tank will appear on the picutre. The camera axis is usually horizontal, but in certain occasions where high tanks are photographed, a tilted photograph may be needed. The arrangement of the photographic procedure is presented schematically in Figure 1. The number of photographs required for the evaluation of the tank dimensions depends on the size of the tank. For large tanks (diameter of about 50 meters), 10 to 12 photographs are recommended.

The sequence of measurements taken on the negatives is illustrated by the Figure 2.

The pair of points *a*, *b* on the figure defines a chord of a circle which represents the horizontal cross section of the tank at the level *h* above the bottom of the tank. The points *a*, *b* are related to the image reference system and their coordinates are measured on a comparator. Because it is required to determine dimensions of horizontal tank sections, the points *a*, *b* are chosen at equal distances from the horizontal weld. On each photograph chords at different levels along the entire tank are measured, usually three chords per tank course are required.

The photograph stations are chosen arbitrarily, no special measurements to es-

ABSTRACT: *The paper discusses a method of determining dimensions of vertical, cylindrical storage tanks based on the evaluation of a series of single photographs. The accuracy of the tank dimensions provided by the proposed procedure matches the standards required by tank calibration. This is shown theoretically and is also verified by practical tests.*

tablish these stations are made. As a result the scales of the negatives are not uniform. This fact gives rise to the question as to how to choose the chord defining points so that they would represent the same horizontal section on all negatives. In order to answer this question one has to consider the height of the respective course measured on the negatives.

For deriving the dimensions of the tank from the image data it is necessary to measure at least two circumferences directly on the tank, one on the bottom course and the other on the top course. Actually it is advisable to measure on the bottom course several circumferences in order to obtain more reliable information on the negative scales. The measured circumferences serve as control data so characteristic of the photogrammetric measuring process.

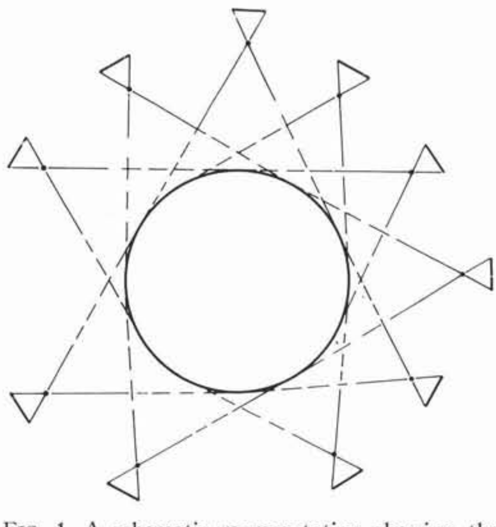


FIG. 1. A schematic representation showing the arrangement of the photographs about the tank.

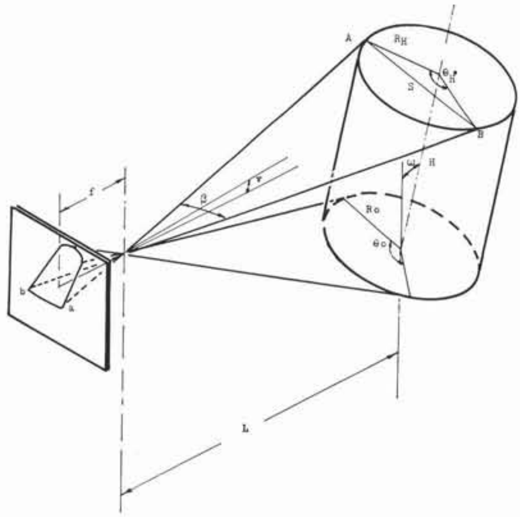



FIG. 2. The projection of a cylindrical tank where the camera is tilted; it is valid to assume that the tank is tilted instead of the camera.

## Computations

The aim of the computation is to obtain the radius of each measured chord at the scale of the tank.

Figure 2 shows a cylindrical tank and its projection on the image plane. The photographic axis deviates from the perpendicular to the tank axis by the angle $\omega$. As only the relative position between the photograph and the object is significant, it can be assumed that the camera axis is horizontal and the tank is tilted at the respective angle.

The angle $\beta$ included between the rays which form the image of the chord $S$ is derived from the scalar product of the vectors $\boldsymbol{r}_a, \boldsymbol{r}_b$:

$$\cos \beta = \boldsymbol{r}_a \cdot \boldsymbol{r}_b / |\boldsymbol{r}_a||\boldsymbol{r}_b|.$$

The letter $v$ in Figure 2 denotes the vertical angle at which the chord is seen from the projection center; it is determined from the expression:

$$\cos v = \frac{k \times \left( \frac{\boldsymbol{r}_a}{|\boldsymbol{r}_a|} - \frac{\boldsymbol{r}_b}{|\boldsymbol{r}_b|} \right)}{\left| k \times \left( \frac{\boldsymbol{r}_a}{|\boldsymbol{r}_a|} - \frac{\boldsymbol{r}_b}{|\boldsymbol{r}_b|} \right) \right|} \cdot \frac{\boldsymbol{r}_a \times \boldsymbol{r}_b}{|\boldsymbol{r}_a \times \boldsymbol{r}_b|} \cdot \tag{2}$$

The cosines of $\beta$ and $v$ have to be expressed by the measured image coordinates. In the image reference system the vectors $\boldsymbol{r}_a, \boldsymbol{r}_b$ are defined as:

$$\begin{aligned} \boldsymbol{r}_a &= x_a \boldsymbol{i} + y_a \boldsymbol{j} - f \boldsymbol{k} \\ \boldsymbol{r}_b &= x_b \boldsymbol{i} + y_b \boldsymbol{j} - f \boldsymbol{k}. \end{aligned} \tag{3}$$

With the angles $\beta$ and $v$ it is possible to determine the angle $\alpha$, which is the projection of $\beta$ on a plane defined by the camera axis and the $x$-axis of the image system. For this purpose consider the spherical triangle of Figure 3.
From Figure 3,

$$\tan \alpha/2 = (\tan \beta/2)/\cos v. \tag{4}$$

Formula 3.4 yields an expression for $\sin \alpha/2$ to be used latter on:

$$\sin \alpha/2 = \sqrt{(1 - \cos \beta)/(1 - \cos \beta)/[1 - \cos \beta + (1 + \cos \beta) \cos^2 v]}. \tag{5}$$

The chord $S$ at the scale of the tank at level $h$ is given by:

$$S = (2R \cos \omega \cot \alpha/2)/\sqrt{(1 + \cos^2 \omega \cot^2 \alpha/2)}. \tag{6}$$

On the other hand $S$ can be expressed by:

$$S = 2\left( L + \ \sin \omega - R \cos \frac{\theta}{2} \cos \omega \right) \tan \frac{\alpha}{2} \cdot \tag{7}$$

In the above formulas the following variables are introduced:

- $L$—horizontal distance from projection center to lower end of tank axis,
- $R$—radius of the section at the level $h$ which corresponds to the chord $S$, and
- $\theta$—central angle corresponding to $S$.

Combining the two Expressions 6 and 7, and performing simple transformations, yields a formula for the radius $R$:

$$R = (L + h \cos \omega)(\sin \alpha/2)\sqrt{(\sin^2 \alpha/2 + \cos^2 \alpha/2 \cos^2 \omega)}/\cos \omega. \tag{8}$$

Formula 8 includes two unknows, $L$ and $\omega$. Both are found by applying the formula to the measured control circumferences. From the circumference measured on the bottom course we compute the corresponding radius $R_0$. Substituting $R_0$ into the left hand side of Equation 8, $L$ may be solved in terms of the unknown $\omega$. Inserting $L$ again in Formula 8 yields the final expression for determining the radius $R$ corresponding to the chord at the level $h$:

$$R = R_0[(\sin \alpha/2)/(\alpha_0/2)]\sqrt{[(1 + x \sin^2 \alpha/2)/(1 + \sin^2 \alpha/_02)]}$$
$$+h(\sqrt{X})(\sin \alpha/2)\sqrt{[(1 + X \sin^2 \alpha/2)/(1 + X)]}.$$

The index 0 refers to the quantities derived from the lower control circumference and $X$ is defined by $X = \tan^2\omega$.

To complete the Transformation of 9 into a working formula, it is necessary to determine the unknown $X$. This is accomplished by applying the above formula to the upper control circumference, whose radius $R_H$ is known:

$$R_H = R_0[(\sin \alpha_H/2)/(\sin \alpha_0/2)]\sqrt{[(1 + X \sin^2 \alpha_H/2)/(1 + \sin^2 \alpha_0/2)]}$$
$$+H(\sqrt{X}) \sin (\alpha_H/2) \sqrt{[(1 + X \sin^2 \alpha_H/2)/(1 + H)]}.$$

$H$ in Formula 10 is the vertical distance between the two cross sections whose circumferences were measured on the tank. Because of the intricate form of Equation 10, it is necessary to solve the unknown $X$ by iterations.

The determination of $X$ completes the solution. Sin $\alpha/2$, which correlates with a certain chord measured on the negative, is found from Formula 5 which in turn uses the Expressions 1, 2, 3. $X$, together with sin$\alpha/2$, enter the Formula 9 for deriving the radius $R$. The quantities $h$ in the Formula 9 is obtained from the known course heights of the tank.

Along the lines described above all radii corresponding to the various cross sections are calculated. Each section is measured on $n$ negatives, hence $n$ different values of $R$ are found. The mean of all determinations represents the circle of the cross section in question. As stated above, on the bottom course several control circumferences may be measured; consequently several solutions are obtained. They differ from each other due to errors in the measurements and due to small irregularities of the tank. The final radii which define the tank dimensions are the averages of the various solutions.

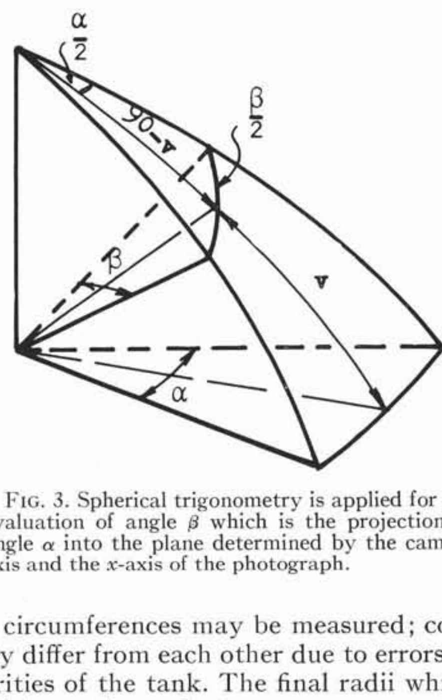



FIG. 3. Spherical trigonometry is applied for the evaluation of angle $\beta$ which is the projection of angle $\alpha$ into the plane determined by the camera axis and the $x$-axis of the photograph.

## ACCURACY CONSIDERATIONS

In the majority of applications the camera axis is nearly horizontal and the angle $\omega$ is small. Therefore for accuracy investigation purposes it is allowable to use an approximate formula for the determination of the radius $R$:

$$R = R_0 \sin \theta/\sin \theta_0 + h \tan \alpha \sin \theta. \tag{11}$$

Applying Equation 11 to the upper control circumference, tan $\omega$ can be eliminated; this yields:

$$R = R_0(\sin \theta/\sin \theta_0)(1 - h/H) + R_H(h/H) \sin \theta/\sin \theta_H. \tag{12}$$

With a fairly good approximation it can be assumed that:

$$\tan\theta = (x_b - x_a)/2f = \Delta x/2f$$
$$\sin\theta = \Delta x/\sqrt{(4f^2 + \Delta x^2)}. \tag{13}$$

Inserting Equation 13 into 12 yields:

$$R = R_0(\Delta x/\Delta x_0)\sqrt{[(4f^2 + \Delta x_0^2)/(4f^2 + \Delta x^2)]}(1 - h/H)$$
$$+ R_H(h/H)(\Delta x/\Delta x_H)\sqrt{[(4f^2 + \Delta x_H^2)/(4f^2 + \Delta x^2)]} \tag{14}$$

We proceed now to examine how the various error sources affect the solved value $R$.

*a.* ERROR IN $R$ DUE TO IMPROPER LOCATION OF THE CROSS SECTION ON THE PHOTOGRAPH

From Equation 11 follows:

$$\partial R/\delta h = \tan\omega \sin\theta$$
$$m_a = \tan\omega \sin\theta \, m_h. \tag{15}$$

Generally, $\sin\theta < 0.5$ and $\tan\omega < 0.1$, hence

$$m_a < 0.05\, m_h. \tag{16}$$

*b.* ERROR IN $R$ DUE TO THE ERROR IN THE FOCAL LENGTH

For error propagation analysis the following assumptions can be made:

$$R_0 = R_H = R$$
$$x_{a0} = x_{aH} = x_a = -d \tag{17}$$
$$x_{b0} = x_{bH} = c_b = +d.$$

These assumptions are valid for cylindrical tanks only, because the variations in the measured $x$ coordinates are due to the irregularities in construction work and due to varying thickness of the plates, and are therefore negligible. It can be easily shown that in this case the errors in focal length do not affect the accuracy of the computed radius.

*c.* ERROR IN RADIUS DUE TO ERRORS IN MEASURING THE COORDINATES OF THE CHORD DEFINING POINTS

Obviously, the measuring accuracy of the coordinates of the chord defining points can be assumed as uniform regardless of the location of the chord on the photograph. With this assumption, Formulas 14 and 17 yield the following expression for $m_c$:

$$m_c^2 = R^2[(h/H) + 1](h/H) - 1]m_{\Delta x}^2/2[1 + (d/f)^2]d^2. \tag{18}$$

The term $M_{\Delta x}$ in the Formula 18 denotes the error in the measured coordinates difference. As $0 \leq h/H \leq 1$, the numerator term that depends on this ratio can be neglected, hence:

$$m_c^2 = R^2(m_{\Delta x}^2/d^2)/2[1 + (d/f)^2]. \tag{19}$$

Summarizing the results of the accuracy analysis, it can be stated that the error in the computed radius depends mostly on the measuring procedure of the image coordinates.

The order of magnitude of the obtainable accuracy can be illustrated by the following example taken from practice: $f = 165$ mm., $d = 55$ mm., $R = 25$ m and $m_{\Delta x} = 0.01$ mm. Inserting these data into Equation 19 yields $m_R = R/7{,}000$. This result is valid for each determined radius. For the determination of the radius of the cross section, $n$ photographs are evaluated and the final radius is the mean of the $n$ derived values, hence the error in $R$ decreases according to the ratio $1/\sqrt{n}$: $m_R = R/(7{,}000\sqrt{n})$.

## PRACTICAL EXAMPLES

A cylindrical tank with a radius of approximately 25 meters was photographed from 9 stations located around its circumference from a distance of about 40 meters. The photographs were taken with a Wild phototheodolite, $f=165$ mm., the camera axis was nearly horizontal (levelling accuracy). For comparison purposes the tank was photographed from one station with a tilted camera, the tilt being 7 grads upwards. On each negative a sequence of measurements was made according to the scheme on Figure 2.

One day before the photographic procedure the tank was calibrated by the common strapping method and all circumferences were measured directly with a special calibration tape. Three measured circumferences on the lower tank course and one on the upper were used as control data, the remaining served for accuracy estimation purposes. After processing the measured data, 16 differences were obtained between the photogrammetric and the direct-measured circumferences. Assuming that the direct measurements are superior from the point of view of accuracy the above differences can be regarded as errors originating from the photogrammetric measurements and as such can be used for evaluating an accuracy estimate of the photogrammetric measuring process.

The mean-square error in determining a circumference of a cross section of the tank from the data measured on the photographs as calculated from the 16 differences was 10 mm. The expected mean-square error which follows from the above accuracy analysis can be computed from Formula 19. Inserting the relevant data into this formula and considering the fact that the final radius of a cross section is an average of nine determinations, $2\pi m_R=8$ mm. This figure is in good agreement with that obtained from the practical test.

The evaluation of the tilted photographs has shown that the accuracy of the computed radii is essentially the same as for the horizontal photographs.