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Slope Measurement from Contour Maps

Approximate slope computation from a digital map is feasible at reasonable computer times.

ABSTRACT: *This paper describes two methods of estimating slope gradients from a digitized contour map. The first method uses amount of contour line per unit area as a slope measure, whereas the second computes slope by measuring distances to nearest contour lines. Both methods have been implemented in PAX II on the Univac 1108 computer.*

INTRODUCTION

CONTOUR MAPS of many kinds are produced to describe spatially distributed data. Such maps contain *isolines*, which are the loci of points at which some measured quantity takes on a given discrete set of values. For example, in a topographic contour map, the isolines are lines of equal terrain elevation; in a weather map, they may be lines of equal pressure (isobars), temperature (isotherms), rainfall (isohyets), etc.; and in other examples they may represent any of a wide variety of data (stress, magnetic variation, radiation, etc.).

Users of contour maps often desire to know the rate of change (i.e., first derivative, or slope) of the data. Calculation of slope along a particular path is a straightforward matter. It is less trivial, however, to determine the slope *gradient* (i.e., the maximum slope in any direction) at a point, let alone at all points of a region. This paper describes computer programs which measure, "in parallel," approximations to the slope gradient at all points of a digitized map, so as to yield a *slope map* of the given region.

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RELATED PRIOR WORK

Fischer[1] has tested an optical-mechanical method of producing a slope map from a given contour map. He used a positive and a negative transparency of the contour sheet separated by a diffusing transparency (semi-matte translucent acetate). An eccentric turntable was constructed and used to nutate the positive transparency relative to the negative at a given nutation radius.‡ A controlled, diffuse light source illuminated the transparencies from below, and a camera, mounted above, recorded the results in a time exposure taken over a period of one or more complete nutations. It is evident that in this arrangement, more light will pass through a region having many contour lines per unit area than through a region having few or none. Because slope is also high where there are many contours per unit area, the photographic recording can thus be regarded as a slope map. It is a fairly straightforward matter to calibrate this system so as to be able to convert any density on the photograph into an equivalent number of lines per unit area. An example of the results obtained by this method is shown in Figure 1.

‡ Fischer suggested that the nutation radius should be C/S, where C is the contour interval divided by the scale of the map, and S is the smallest slope to be detected.

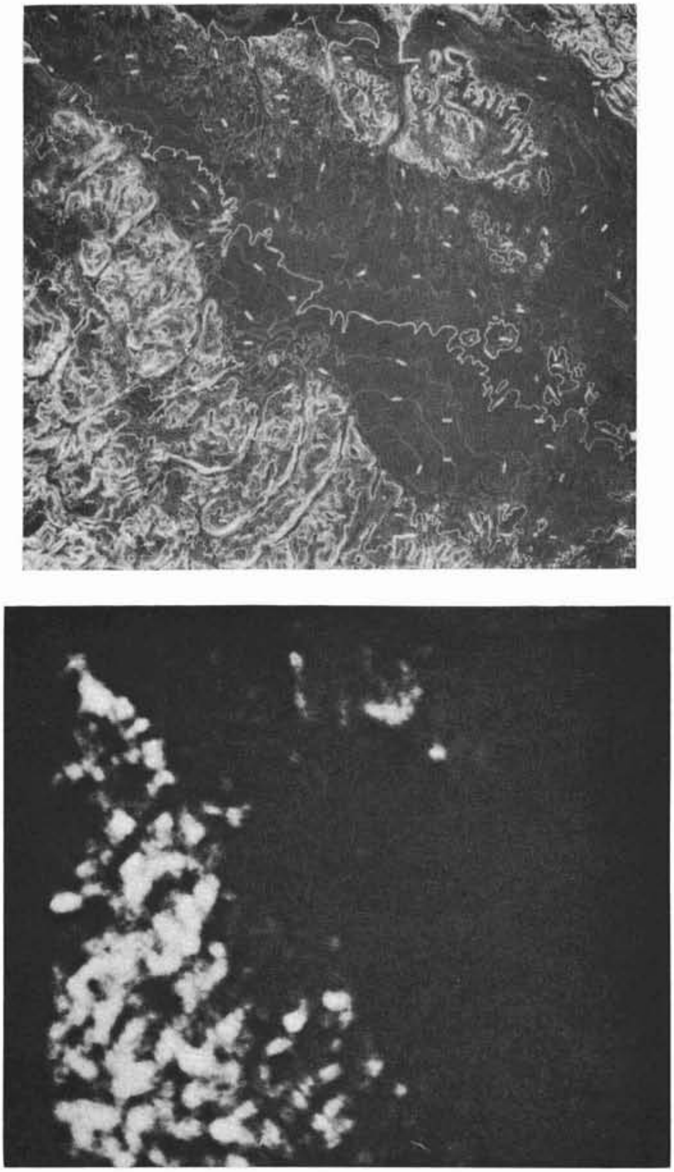

FIG. 1. Fischer's nutation method. Input contour map (*top*) and Output slope map (*bottom*).

Monmonier, Pfaltz and Rosenfeld[2] reported a computer program called SAMP which estimated surface area from a contour map. To arrive at area, the program first computed slope at each point by determining distances to the nearest contours in two orthogonal directions. Different procedures were used depending on whether or not the given point was itself on a contour line, and on whether or not another contour line existed in one or both directions within a specified distance (after which the terrain was regarded as flat). The slope computation also depended on whether the contours reached were the same or different. The methods reported in this paper are simpler in that they consider only distances between contours (whether the same or different), and do not give special treatment to points on contours. On the other hand, the present methods provide map output, which SAMP did not. Figure 2 shows a digital contour map which was used as input to both the SAMP program and the programs reported

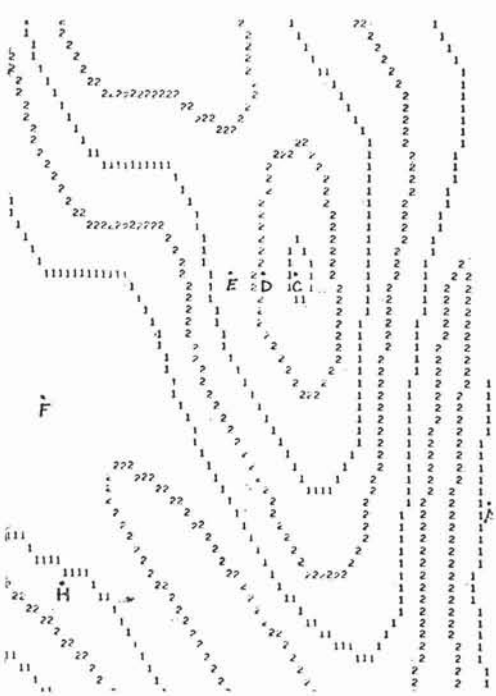

FIG. 2. Digital contour map.

hereinafter: one picture element on this map represents about 100 feet on the terrain.

THE DIGITAL NUTATION METHOD

A simple digital version of Fischer's optical-mechanical method is as follows: The positive and negative transparencies are represented by the digital contour map and its complement. One of these maps is nutated about the other; for each relative shift, they are ANDed, and the results are summed over a complete nutation. Figure 3 shows the resulting sums for a nutation amplitude of 10. As can be seen, the results are too discrete, perhaps because the nutation amplitude is not great enough. Furthermore, there is a tendency toward discontinuity both along and across contours. This may be due to the fact that the diffusing transparency used in the optical process was not simulated in the digital method.

It was decided that a closer equivalent to the optical scheme could be obtained by smoothing or blurring the results of the nutation, i.e., averaging over a fixed neighborhood at each point. The results of such a smoothing, by averaging over a circular neighbor-

FIG. 3. Digital nutation method (no averaging; nutation radius 10)

FIG. 4a. Digital nutation and averaging (blur radius 4), printout.

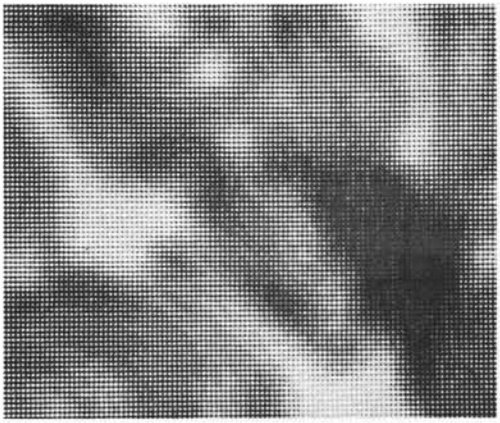

FIG. 4b. Picture output for Figure 4a.

hood of a radius of four at each point, are shown in Figure 4. However, quantitative evaluation for eight test points (Table 1) shows deviations of several percent of slope, which would be too rough an approximation for most purposes. Further study is needed to determine the optimum nutation radius and blur radius for this method.

Qualitatively, a simple smoothing of the original contour map should also yield high values in areas of high slope. The results of such a smoothing, using circular neighborhoods of radius 10, are shown in Figure 5.

THE DIGITAL SLOPE GRADIENT METHOD

The second digital method tested uses straightforward measurement of distance between contours on the digitized map. As

TABLE 1. ROUGH CHECK OF SLOPE GRADIENT VALUES AT EIGHT TEST POINTS

Point	*Result of nutation method* (×3)	*Result of digital slope gradient method*	*Slope gradient computed as square root of sum of squares*
A	36	36	33
B	24	18	19
C	27*	43	39
D	18	24	22
E	15	18	18
F	0	6	5
G	24	27	25
H	18	20	19

* This discrepancy is probably due to interaction between the nutation and the small closed contour just surrounding Point C.

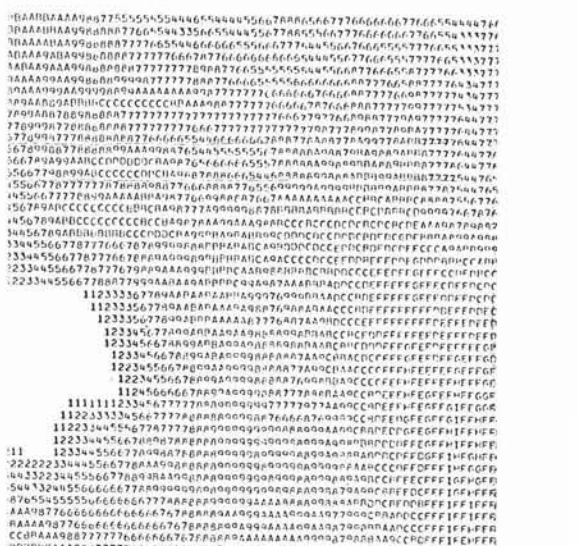

FIG. 5a. Averaging of original contour map (radius 10), printout.

the map scale and contour interval are known, slope can be easily computed from such a distance measurement. This was done in both the x and y directions (Figure 6a–d).*

* It would have been desirable, either as a check or to provide a closer approximation, also to compute slopes in the 45° directions. However, as the contour lines on the digitized map are thin, searches along 45° lines would often cross them without detecting them.

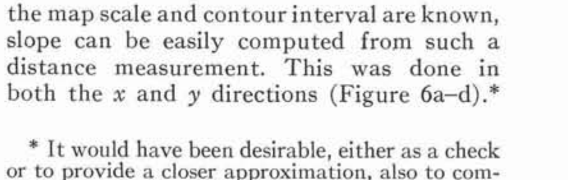

FIG. 5b. Picture output for Figure 5a.

FIG. 6a. Digital slope computation. Distance between contour lines in X-direction

FIG. 6b. X-component of slope for Figure 6a.

FIG. 6c. Similar to Figure 6a but in the Y-direction.

FIG. 6d. Similar to Figure 6b but in the Y-direction.

The slope gradient can then be computed as the square root of the sum of the squares of these x and y slope components. To simplify the computation, an approximation to the square root was used, namely $u+v/2$, where u is the larger and v the smaller of the two components. (Note that for $v=0$ we have

$$\sqrt{(u^2+v^2)} = u + v/2 = u$$

whereas for $v=u$ we have

$$\sqrt{(u^2+v^2)} = u\sqrt{2} = 1.4u$$

and

$$u + v/2 = 1.5u$$

so that this approximation is reasonably good at both ends of the range $0 \leq v \leq u$.) The result is shown in Figure 6e. A quantitative evaluation of the results for the eight test points is given in Table 1.

A slope-class map was constructed by thresholding these results, using the slope intervals 0–2%, 2–5%, 5–10%, 10–20%, and >20% (Figure 7a). The map has a somewhat blocky, grainy appearance. One cause of this is the fact that the estimated slope usually has discontinuities at contour lines. To combat this, the slope map was smoothed by assigning to each point on a contour line the average of the slopes at the four neighboring points, and further by averaging over neighborhoods. The results are shown in Figure 7b. Analysis of the printout shows that scattered errors still survive, but if desired, many of these could be eliminated using standard noise-cleaning techniques.

The errors for the gradient method are of about the same magnitude as in the nutation method, but are differently distributed. An interesting difference between the two methods is that near the edges of the map, the nutation method underestimates the slope, because the map is blank beyond its edge, whereas the gradient method overestimates the slope, because it treats the edge of the map as a contour. In general, the gradient method is more flexible, because it can provide a variety of intermediate products, such as slopes in specific directions.

FIG. 6e. Approximation to the gradient for Figure 6a.

Discussion

The results of these studies indicate that approximate slope computation *in parallel* from a digital contour map is feasible. The computation times required were not excessive (of the order of one minute), and on a parallel computer such as ILLIAC III it should be possible to do the computations for an entire map in times of that order.

An important limitation on such approaches to slope computation is imposed by the fact that the contour lines must be one picture element wide. (In the optical analog application too, the contour lines on the transparencies must have finite width.) Thus the accuracy of slope estimates can never be greater than $1/\bar{D}$, where $\bar{D}$ is the average distance between contours. This limitation is especially serious for steep terrain; thus the input map used in the tests represents a worst case. In spite of this, the results are believed to be encouraging, and the methods warrant serious consideration for use in practical applications.

Acknowledgments

The work reported was initiated by Mr. Scheps in connection with geographic data processing studies under a Secretary of the Army Research and Study Fellowship. Mr. Lee implemented the first method. The guidance of Dr. Azriel Rosenfeld of the University of Maryland, and the support of the National Aeronautics and Space Administration and the Atomic Energy Commission, are gratefully acknowledged.

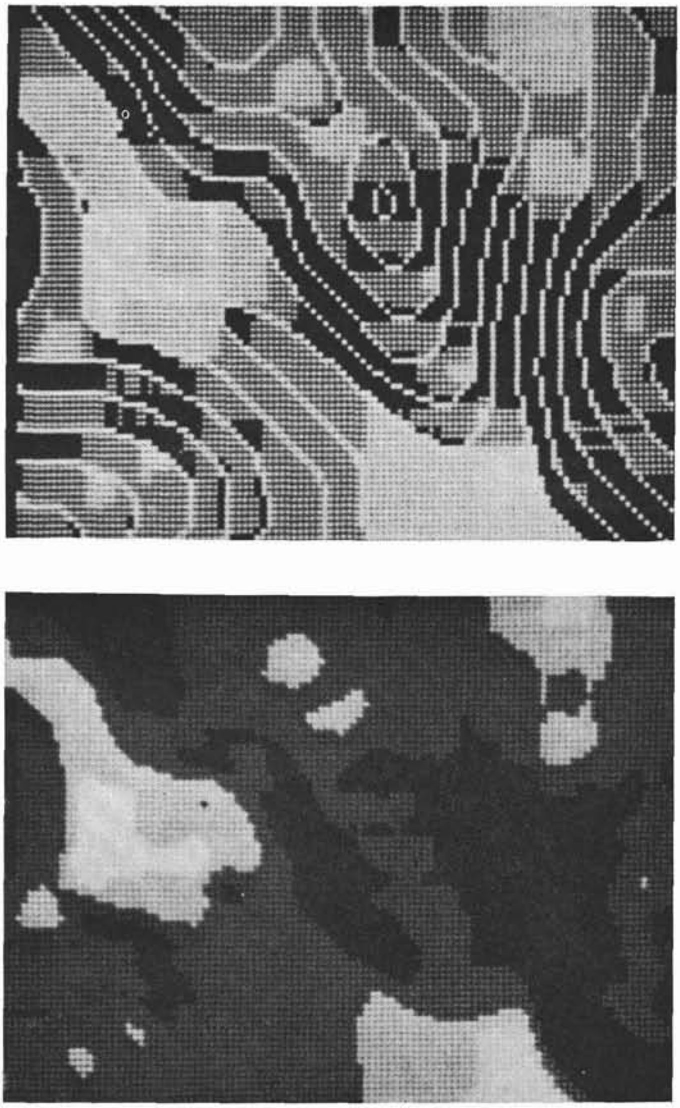

FIG. 7. Digital slope class map. (*a*) Unsmoothed (*top*), and (*b*) Smoothed (*bottom*).

REFERENCES

1. W. A. Fischer, Personal communication to B. B. Scheps, 29 October 1963.
2. M. S. Monmonier, J. L. Pfaltz and A. Rosenfeld, Surface area from contour maps, *Photogram. Eng. 32*, May 1966, 476–482.

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