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Psychophysics

The average-error method is the most valid experimental method for investigating photogrammetric pointing accuracies because of its similarity to observational methods.

ABSTRACT: *Aspects of psychophysics which affect photogrammetric pointing are introduced, including two classical psychophysical methods of testing, i.e., the average-error and constant-stimulus methods. This is followed by an introduction to modern psychophysical scaling methods, and a description of Stevens' psychophysical law. Results of monocular and binocular pointing to blurred targets, by the average-error and constant-stimulus methods, are analyzed by Stevens' law, and an approximate equation for photogrammetric pointing derived. Results of binocular and monocular pointing, and average-error and constant-stimulus methods are compared through Stevens' law.*

INTRODUCTION

THE FIELD OF PSYCHOLOGY that deals with the determination of a relationship between a physical stimulus and the subsequent response, is called psychophysics (Candland, 1968, 83). As an individual's behavior may be measured by his response to stimuli, an understanding of the way in which responses may occur is fundamental to the prediction of behavior to different stimulus conditions.

Psychophysics has an important bearing on everyday life, and in particular on occupations which require fine judgment by one or more of one's senses. Metrology, for instance, may require the careful visual reading of linear scales. Detection of details on radar screens, reading of aircraft dials, and detection of radio signals are other examples of psychophysical tasks in various occupations. Pointing to photogrammetric targets, stereoscopic height measurement, and point transfer are all examples of psychophysical judgments made by photogrammetrists. For an understanding of behavior to stimuli and the associated judgment capabilities in each of these tasks, testing must be carried out over a wide range of conditions. Tasks particularly related to photogrammetry as described above, however, have not been investigated in detail despite the fact that many psychophysical aspects of vision have been investigated very extensively. Considerable research is therefore necessary on psychophysical tasks undertaken in photogrammetry. Examples of investigations which have been carried out are O'Connor (1962), O'Connor (1967), O'Connor (1968), Roger et al. (1969), Trinder (1971), Zorn (1965).

The purpose of this paper is to outline aspects of psychophysics and psychophysical methods which are related to visual tasks in photogrammetry. Observational procedures used during these tasks will be compared with standard psychophysical methods, and measurements obtained by such procedures compared with existing psychophysical laws. Results of investigations on monocular pointing to blurred targets (Trinder, 1971) will be related to Stevens' psychophysical law, demonstrating the significance of the threshold level of blur of the target, on pointing accuracies. Further observations on binocular pointing to blurred targets will be presented and compared with results of monocular pointing, through the application of Stevens' law.

CLASSICAL PSYCHOPHYSICAL THEORY

The classical concept of psychophysics describes the relationship between the physical scale S and the response or psychological scale R as shown in Figure 1 (after Guilford, 1954, 21).

The Stimulus scale extends from zero to a

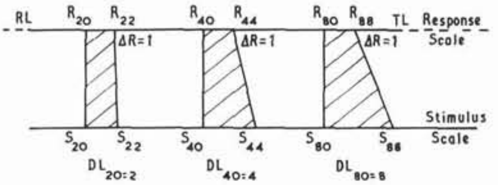

FIG. 1. The relationship between physical and psychophysical scales in classical psychophysics.

very high value beyond the range of the human senses. The Response scale extends from the RL—the *response threshold* or *limen*—to the TL—the *terminal threshold* or *terminal*. Beyond these two points, the scale is marked by broken lines because the RL and TL cannot be well defined. They have a statistical variation depending on the task involved (Swets, 1961), and are generally fixed as points at which a positive response results 50 percent of the time. Although the TL and RL are thresholds on the response scale, they are expressed in terms of measurements on the physical scale.

Differential thresholds or *Difference Limens*—DL—are shown in Figure 1 by the pairs of lines marked by hachuring. At the stimulus value of S_{20} a second stimulus S_i must reach the value S_{22} before it is judged different 50 percent of the time. At S_{40}, S_i must reach S_{44} before it is judged different, and so on. The differences between S_{20} and S_{22}, and S_{40} and S_{44} are described as DL's. On the response scale these differences are shown as unity. That is, these differences give the same impression to the observer, in relation to the level of stimulation, though the magnitudes of the DL's increase as the level of stimulation increases. Experimental results on pointing by O'Connor (1962, 1967), Roger *et al.* (1969), Trinder (1971) are examples of DL's expressed on the stimulus scale.

WEBER'S LAW

The relationship between DL's and level of stimulation, expressed on the stimulus scale, is known as Weber's Ratio or Law:

$$\frac{\Delta S}{S} = K$$

where ΔS is the DL on the stimulus scale corresponding to a change on the R scale of unity, S is the point on the physical scale, and K is the constant depending on the observer and the modality of the sense. K is approximately constant at 1/100 for the discrimination of distances (O'Connor, 1962), 1/30 to 1/40 for lifting of weights and varies for sounds with frequency and intensity. As shown by O'Connor (1967), Weber's Law holds for pointing observations to sharp targets with large annulus widths, but not for observations to sharp targets with small annuli.

FECHNER'S LAW

Weber's Law only describes behavior in terms of the stimulus scale, with no reference to the response scale. Fechner's Law on the other hand expresses a relationship between the stimulus and response scales. The basis of this law is a logarithmic relationship between stimulus and response as follows:

$$R = K \log S$$

where R is the response, S the stimulus above threshold, and K is a constant. In words this law may be stated as: R increases in *equal steps* as S increases in equal *ratio steps*, (*Candland*, 1968, 89). This law makes two assumptions:

i. The DL is a function of all DL's which have occurred previously.
ii. That all DL's are subjectively equal.

Fechner's Law has proved to be an incomplete psychophysical law, although it is satisfactory for some aspects of psychophysics. A completely general law has yet to be found, but more adequate laws than Fechner's Law have been developed (Stevens, 1962).

Before describing two experimental methods used in classical psychophysics, an introduction to some concepts of modern psychophysics will be given.

SOME ASPECTS OF MODERN PSYCHOPHYSICS—THE RESPONSE MATRIX

Modern psychophysics describes the behavior on the response scale of an observer, in terms of the many variables which may be involved. This results in a general behavioral equation.

The response of an organism to a stimulus, is a function of factors in the stimulus, the internal conditions, number of stimulations and time itself (Graham, 1950), i.e.,

$$R = f(a, b, c, d)$$

where a are the aspects of the stimulus, b the number of times the stimulus is presented, c is the time and d are the internal conditions of the observer, e.g., motivation, psychological attitude etc.

Generally in psychophysical investigations, the effects of factor a on the psychological response only are considered. In the results of pointing presented in this paper, stimulus conditions which have been considered are the size of target and degree of blur. The effects of the remaining factors in the behavioral equation were expected to remain constant during the experiment, although it is impossible to say precisely that this was true. In the course of pointing experiments the factor b, for instance, may affect the results if an excessive number of observations cause fatigue. It is not anticipated that the actual time of day would appreciably affect the results, but observations on each target have extended throughout the day and therefore the effect of time tends to be included in the overall results. Internal conditions may perhaps affect the results of observations under certain circumstances. Indeed some psychologists have claimed that even a strategically timed coffee break can improve psychophysical results. The particular purpose of experiments described was to investigate pointing accuracies obtainable in photogrammetric practice. Although the observations have been made with considerable care and concentration, the external laboratory conditions were similar to those which may be expected in photogrammetric practice. In addition it is anticipated that, with a large sample of targets and a large sample of observations on each target, a representative pattern of results has been obtained.

From S_i ($i=1$ to n) stimuli presented to the observer on O_j ($j=1$ to m) occasions, a response matrix R is derived for each observer, where each single response element is R_{ij}. For more than one observer P_k ($k=1$ to 1), an additional dimension is added to the matrix, giving single response elements R_{ijk}. The response R_{jk} by each observer to stimulus S_i has a mean R_k derived from N observations:

$$R_{jk} = R_k + e_{jk}$$

where e_{jk} is the variability of the discrimination of the obseıver for each single observation. The shape of dispersions e_{jk} for psychophysical testing may or may not follow a symmetric pattern.

Over a restricted range on the S-scale, S and R are approximately linearly related. This can be seen by reference, for instance, to Fechner's logarithmic law, which is approximately linear over a restricted range of the S and R scales. It may therefore be assumed that the small dispersions of both S and R follow approximately the same frequency distribution (Guilford, 1954, 28). Guilford states that considerable evidence indicates that discriminal dispersions follow the symmetrical normal distribution, particularly for tasks similar to those studied in this work. O'Connor (1962, 1967) has assumed normality of dispersions on the stimulus scale throughout. There seems ample justification, therefore, for assuming that the observations in this study also follow a normal distribution. Sample tests of normality, however, have been carried out to justify this assumption.

Two classical methods of psychophysical testing which determine the DL on the stimulus scale will be outlined. This will be followed by a brief description of some aspects of the more recently developed scaling procedures, which aim at determining the response matrix R.

Classical Methods of Psychophysics

Method of Average Error

The average-error method has been described as a "free gift to psychophysics from the exact sciences of physics and astronomy" (Guilford, 1954, 86). Indeed, many fields of metrology, including surveying, use this method as a basis for their measurements. The observer is required to produce *equal stimuli* by adjustment of the stimuli himself, e.g., photogrammetric pointing, where the annulus on one side of the measuring mark MM must be equated to the annulus on the other. The central position of the MM can be approached from two mutually opposite directions along each coordinate axis. The mean of the results obtained from each direction along each axis may be used to find the *Point of Subjective Equality* (PSE) (Candland, 1968, 95), (Guilford, 1954, 93), although there does seem to be some doubt over this point. The statistical testing of results obtained by this method is relatively straightforward, but because of the observational technique it does not allow a direct estimation of a difference limen. Candland (1968, 95) states vaguely that the DL can be estimated "from some assumptions about the PSE." Guilford (1954, 93) suggests that the DL can be estimated from the mean of the combined data, if the means from both directions are not statistically different, or alternatively from the standard deviations of the individual directions. The DL in this study has been estimated from the standard deviation from only one direction of approach of the measuring mark MM, because the means from opposite directions prove to be significantly different. Such a value is convenient when referred to photogrammetric practice however, since generally error estimations are made using standard deviations of such observations.

ANALYSIS OF ERRORS IN AVERAGE-ERROR METHOD

Guilford (1954, 93) defines the occasional variable error of each observation as (e_m+e_r), where e_r represents the statistical fluctuation in observations, and e_m the "error of movement" inherent in the method. The extent to which the error of movement e_m is significant must be tested statistically; e_r is most conveniently described by the standard deviation. Results of O'Connor (1967) and Roger et al. (1969) indicate that the so-called movement errors, or systematic errors, are indeed significant, and may be of the order of 3 to 4 times the standard deviation of a single observation. Observations in this study agree with this finding, and in addition prove that such errors may vary if separate sets of observations are carried out several months apart. It therefore becomes necessary to use the standard deviation of one direction of approach as an estimate of the DL, as stated in the previous section. While it cannot be held that such estimates of difference limens would agree with DL's determined by other psychophysical methods, they are suitable for comparing observations of different targets, all observed by the same method. As will be shown later, the method of average error can also be treated as a psychophysical scaling method if certain assumptions are made.

SUITABILITY OF THE METHOD OF AVERAGE ERROR TO PSYCHOPHYSICAL TESTING

The average-error method is considered to be an accepted procedure for psychophysical testing. The active participation by the observer in the experiments increases interest and motivation in the observations, particularly for long series of experiments. However, activation of the equipment (e.g., by handwheels or switch) during observations depends on the transmission of response R through the body muscles to the equipment. The instrument readings are therefore an indirect measure of the observer's response. This leads to the so-called *motor errors* caused for instance, by the observer's inability to stop the instrument at the precise moment when the MM is subjectively central. Motor errors may also be due to the distracting influence of muscular effort which reduces the accuracy of the visual task. Conversely, muscular activity may act as a form of irrelevant information leading to accuracies higher than can be obtained by the visual system alone. Guilford, (1954, 96), quoting Müller, speaks of "uncertainties of the hand" reducing the accuracy of experimental results. However, she maintains that suitable design of equipment should keep such inaccuracies to a minimum.

The method allows the observer to check the observation after stopping the movement of the instrument, and to make a further adjustment in the same direction if he sees fit. Because of the movement error, which apparently depends on the direction of movement of the equipment, a setting must always be made in one direction. If checking shows an overshoot, the observation must be completely repeated.

An advantage of this method is that statistical testing is straightforward, particularly if a normal distribution of observations is assumed. A further important advantage is that the method of observation is similar to that used in photogrammetric observations and though it may include some small inaccuracies due to motor errors, such errors may also be included in any photogrammetric observation. The statistical estimate of the DL derived by these observations is therefore applicable to photogrammetric practice.

CONSTANT-STIMULUS METHOD

Guilford (1954, Ch. 6) describes the constant-stimulus method, or constant method, as the most accurate of the classical psychophysical methods. Referred to the problem of pointing, it is accomplished as follows. Five or seven equally spaced MM positions are selected and the observer must reply whether the MM is *left* or *right* of the center of the target. The two extreme positions are chosen such that replies will be correct about 95 percent of the time, and the middle position will be very close to the center of the target. Generally, psychologists prefer the subject to reply *left* or *right* despite the fact that the MM may be very nearly central. Replies to the middle position should be guesses, and therefore approximately equally distributed between *left* and *right*.

Despite the fact that this method is considered as the most accurate of psychophysical methods, procedures proposed for the treatment of data have varied, particularly for statistical estimation of the accuracy. The most accurate method of computation of the limen and DL involves a least-squares fit to a theoretical curve, using suitable weights for the observations. As mentioned previously, psychophysical tasks of the type involved in these investigations follow closely a normal distribution. If the observations are expressed as percentages of correct replies, (see Table 1), the *right* replies will increase from nearly 0 to nearly 100 percent, as the MM position is moved from left to right across the target. The percentages of *left* replies will be the complement of the *right* replies.

The relationship between S (stimulus or MM position) and p, the probability of *right* replies, will approximately follow a cumulative normal distribution curve or normal ogive. Proportions p can therefore be transformed into ordinates Z on the normal frequency curve. The relationship between S and Z should then be approximately linear.

Five or seven points are determined for the relationship between stimulus S, and the transformed proportions Z. As the observations may not be exactly normal, they may not fall exactly on a straight line. The recommended method for determining a best fit straight line is the least squares technique. A further refinement recommended by Guilford is the assignment of Urban-Müller weights which vary according to the proportions of correct observations. These weights vary from 0.11 for proportions of 99 percent to 1.0 for proportions of 50 percent. A least-squares fit, with Urban-Müller weights, has been used to compute the linear relationship between MM position S and ordinates Z, from observations presented in this paper. The DL is taken as the standard deviation of the normal distribution, i.e., the slope of the straight line, and the subjective center, PSE, the mean of the observations or the position of the MM where $p=50$ percent.

TABLE 1. COMPARISON OF "RIGHT" REPLIES TO TOTAL OBSERVATIONS

Micrometer Setting (MM) S	*No. of "Right" replies*	*Proportion of Correct Replies* (p)
20.00	78	.975
20.40	60	.750
20.80	45	.563
21.20	19	.238
21.60	2	.025

Number of observations to each setting is 80.

SUITABILITY OF THE METHOD

The constant-stimulus method measures the ability to discriminate the position of a motionless MM against the target, and the task is different from the involved in the average-error method. It may be therefore that the DL from this method is significantly different from that determined by the average-error method. As the task of photogrammetric pointing is one of centrally locating a moving MM on a target, rather than of discriminating a stationary MM, it appears that the constant-stimulus method is unsuitable for testing photogrammetric pointing. From an experimental point of view however, both the constant-stimulus and average-error methods are useful and worthy of comparison.

MODERN PSYCHOPHYSICAL METHODS—SCALING METHODS

In modern psychophysical methods, judgments or observations are referred to the response or psychological scale as shown in Figure 1. In all scaling methods, a particular value on a linear scale is assigned to each response. This linear scale may be graduated such that the actual numbers are arbitrarily derived, but each number is correct in relation to the others. Alternatively, the scale may be the stimulus scale itself, and graduations on this scale referred to only as labels.

Scaling methods, involving comparative judgments, may be related to Thurstone's Law of Comparative Judgment which defines the psychological distance or separation on the response scale between two stimuli as:

$$R_j - R_k = z_{jk}\sqrt{(\sigma_j^2 + \sigma_k^2 - 2r_{jk}\sigma_j\sigma_k)}$$

where R_j and R_k are the mean values of responses to stimuli S_j and S_k; z_{jk} is the deviate from the mean of the unit normal distribution, σ_j and σ_k are the standard deviations of each response R_j and R_k to S_j and S_k; and r_{jk} is the coefficient of correlation between R_j and R_k.

Each of the elements in the above formula must be determined to find R_j–R_k. As some of these terms are difficult to derive, Thurstone's Law may take on a number of approximate forms, depending on the assumptions made on σ_j, σ_k and r_{jk}. Approximation referred to as Case V (Guilford, 1954, 156) assumes that $R_j - R_k = z_{jk}\sigma_j\sqrt{2}$; (i.e. $\sigma_j = \sigma_k$ and $r_{jk}=0$)

The methods of average error or constant stimulus may be considered as scaling methods for pointing observations based on Case V above. Each pointing observation by the average-error method requires the discrimination of the annulus on each side of the *MM*, and a decision as to whether the annuli are equal. The response to each annulus on each occasion is R_{ij} and R_{ik}. If $R_{ij}=R_{ik}$ the annuli are subjectively equal despite the fact that the annuli may actually be unequal, i.e., S_{ij} may not equal S_{ik}. R_{ij} and R_{ik} will have dispersions σ_j and σ_k. Any single observation i will therefore result in responses $R_{ij}=R_j+e_{ij}$ and $R_{ik}=R_k+e_{ik}$, where e_{ij} and e_{ik} are the occasional variability in response. Each pointing observation requires that $R_{ik}=R_{ij}$. If the means R_j and R_k must also be equal, then statistically e_j must equal e_k or $\sigma_j=\sigma_k$.

A further assumption made is that $r_{jk}=0$, i.e., the response to the area of the annulus around j (Figure 2) must be uncorrelated with the response to the area of the annulus around k. It is difficult to say positively whether this is indeed the situation. However the significant sections of the target to the observer if the *MM* is moving from left to right, are the areas of the annulus centered around j and k in Figure 2, on opposite sides of the *MM*. These areas are separated by the *MM* diameter, and should cause separate stimulation at, and response by, the observer. It therefore seems reasonable to assume that responses R_{jk} and R_{ik} are uncorrelated.

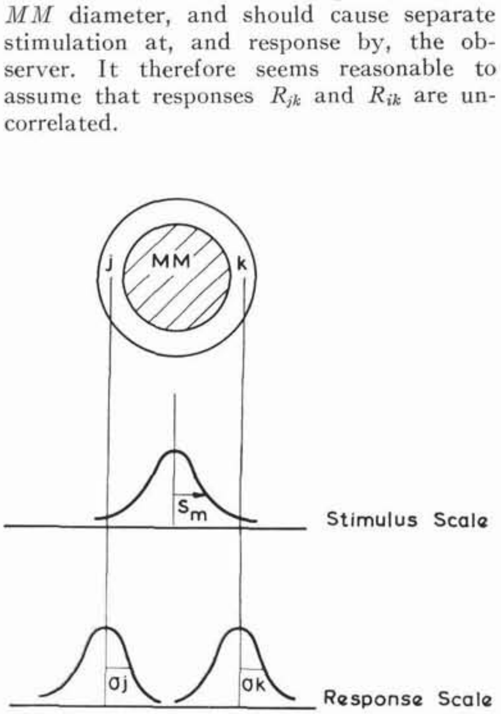

FIG. 2. The relation between stimulus and response scales for the task of pointing.

Based on Case V of Thurstone's Law, provided the above assumptions are valid, the psychological distance is thus $R_j-R_k=z_{jk}(\sqrt{2})\sigma_j$. The execution of pointing by the average-error method, requires the observer to evaluate $R_{ij}-R_{ik}$ continually as the *MM* approaches the subjective central position. At the instant when $R_{ij}=R_{ik}$ (i.e., $R_{ij}-R_{ik}=0$), movement of the *MM* ceases. The assessment of R_j-R_k as seen by the above formula, depends on the dispersion $\sigma_j(=\sigma_k)$. $(R_{ij}-R_{ik})$ for each observation, which is equivalent to the *DL* for pointing, is therefore a statistical quantity proportional to σ_j. The measured value S_m (which represents σ_j) on the stimulus scale hence may be used to derive R_j-R_k if the effects of muscular activity are ignored. That is, S_m on the stimulus scale will be a scaling factor for the *DL* on the response scale. Each individual target, which involves its own task of comparative judgment, will give one scale value against which the scale values of other targets can be judged.

The inclusion of the classical psychophysical methods as scaling methods is supported by Guilford (1954, 260) and Candland (1968, 115). Similar reasoning would also apply to the constant-stimulus method. It is important to realize, however, that since different types of observations are involved for the average-error and constant-stimulus methods, the dispersions may not be the same, resulting in different scale numbers for the two types of observations.

The classical psychophysical methods can therefore be considered as scaling methods provided the necessary assumptions are made on the behaviour of σ_j and σ_k. There are no indications in the literature that these assumptions are invalid for this type of psychophysical task, although very little research has been conducted on such work. Measurements on the stimulus scale are important for this work because results are required for reference to photogrammetric practice. The fact that observations are determined by a valid psychophysical method perhaps allows a direct comparison with psychophysical results and laws derived by other scientists. Whereas it is agreed that results may depend on the method of observation, and to some extent the method of data processing, relative comparisons should still be possible.

Having discussed the various psychophysical methods, the reliability and validity of these methods will be defined. This will be followed by a brief outline of some recently developed psychophysical laws.

Reliability and Validity of Results Obtained by Psychophysical Methods

Blackwell (1953, 4) defines reliability and validity of psychophysical methods as follows:

> *Reliability within a session* is analyzed by statistical processes, which test the goodness-of-fit of the measurements to a theoretical curve, e.g., normal frequency curve.
> *Reliability between sessions* is inversely proportional to the variance obtained with repetitions of measurements by a given procedure.

The reliability of a method may also depend on the variation of means obtained in different sessions. Means have sometimes been found to be unreliable in observations described in this paper. Full statistical tests have been utilized to check the reliability of both means and standard deviations.

A method is valid if it indeed measures the quantity it purports to measure. Regarding psychophysical methods used to determine photogrammetric pointing accuracies, it seems that the method of average error is a more valid procedure than the constant-stimulus method because the observation in the average-error method is very similar to that employed in photogrammetric pointing. This factor was discussed previously. Further, the standard deviation derived by the average-error method is suitable for use in studying propagation of errors in the photogrammetric system, but it may prove to be unsuitable or even invalid if a true estimate of the *DL* is required. The constant-stimulus method, on the other hand, may well prove to be the most suitable method for investigating the true *DL* and the *PSE* unbiased by any movement error.

Recently Developed Psychophysical Laws

Guilford (1954, 41) has quoted variations of Weber's Law in the form of $\Delta S = KS^{\frac{1}{2}}$ after Fullerton and Cattell. A more general form of this equation however is $\Delta S = KS^n$. This is similar to the form of psychophysical equation proposed by Stevens (1962, 30), which refers to the response scale. Stevens' formula is $\psi = K(\phi - \phi_o)^n$ where K is a constant depending on the choice of axis, n is the exponent which depends on the modality or sense investigated, and also external parameters of the experiment (e.g., adaptation level), ψ is the psychological response or magnitude to the stimulus ϕ, and ϕ_0 is the effective threshold measured on the stimulus scale.

Evidence put forward by Stevens includes work based on a number of modalities and even cross-modalities, but this evidence cannot be considered as conclusive. The experimental procedure used by Stevens was the scaling method cf magnitude estimation where the observer must numerically estimate his subjective impression of the stimuli.

Stevens' law will be related to results of experiments presented in the following sections.

Presentation of Experimental Results on Monocular Pointing

Monocular pointing results derived by the Average-Error Method on blurred and sharp targets of various contrasts were presented by Trinder (1971). Results of these experiments, together with further results obatined by binocular observations by him, are summarized in Figures 3, 4 and 5. Figures 3 and 4 present pointing accuracies of two observers, for targets with annulus widths of 0.8 mrad, 2.0 mrad and 5.0 mrad, in terms of the grade of the density profile on the blurred target (expressed in ΔD/mrad). Monocular pointing results in Figures 3 and 4 have been combined in Figure 5 to present accuracies in terms of annulus width, with target blur as a secondary variable.

Experimental results on monocular pointing derived by the Constant-Stimulus Method on the same equipment are given in Table 2 and Figure 6. These have been derived from 100 settings on each of 5 positions of the measuring mark, presented in random order.

Linear regression lines computed for Constant-Stimulus Method results in Figure 6 have been computed from logarithm (Difference Limen) against logarithm (target blur). As annulus width has a very minor effect on pointing accuracies for the targets in Table 2 (Trinder, 1971), all results for each observer have been included in the regression.

Discussion of Results

Trinder (1971) showed that accuracies of pointing to blurred targets depended primarily on target blur, and secondly on target annulus width. Background density had a negligible effect on pointing accuracies. Results derived by the constant-stimulus method follow a similar pattern although the *DL*'s are larger. This also holds for the sharp target with an annulus of 0.5 mrad, where an accuracy of 16.7 μrad was obtained by the average-error method.

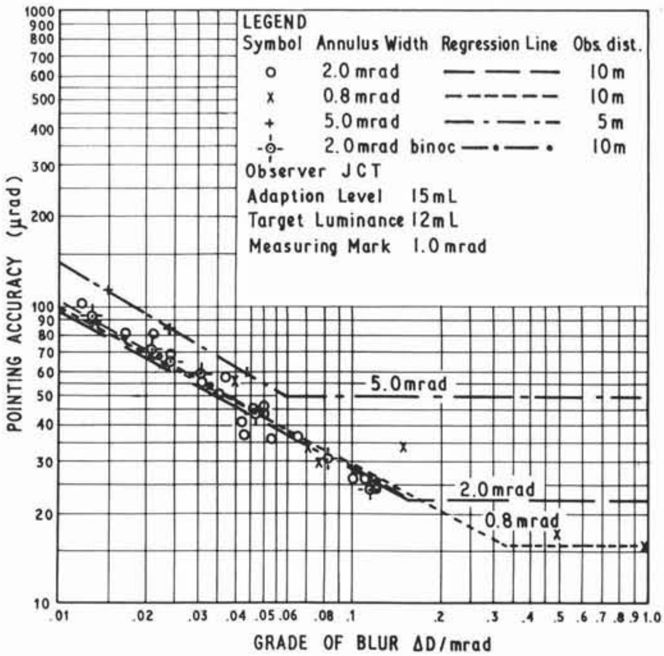

FIG. 3. Pointing standard deviations against target blur (expressed as change in density per milli-radian on the target density profile, ΔD/mrad), for three target annulus widths; observer is *JCT*.

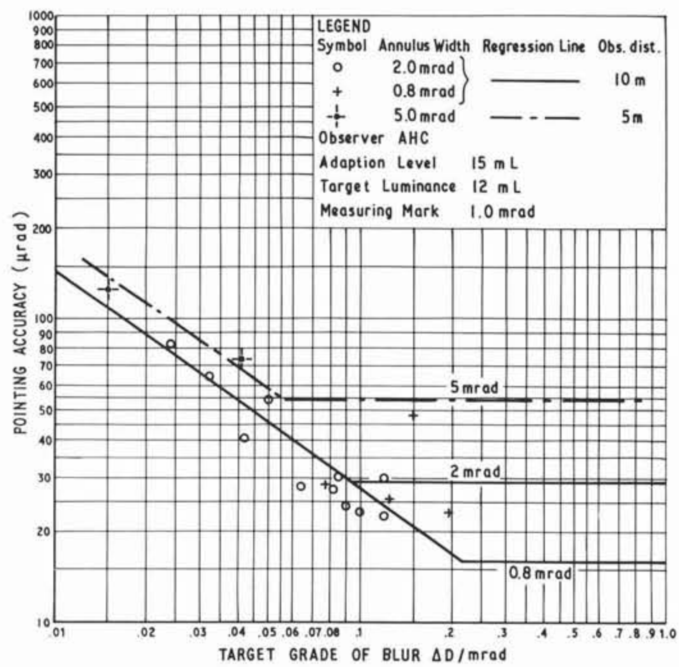

FIG. 4. Pointing standard deviations against target blur (expressed as change in density per milli-radian on the target density profile, ΔD/mrad), for three target annulus widths; observer is *AHC*.

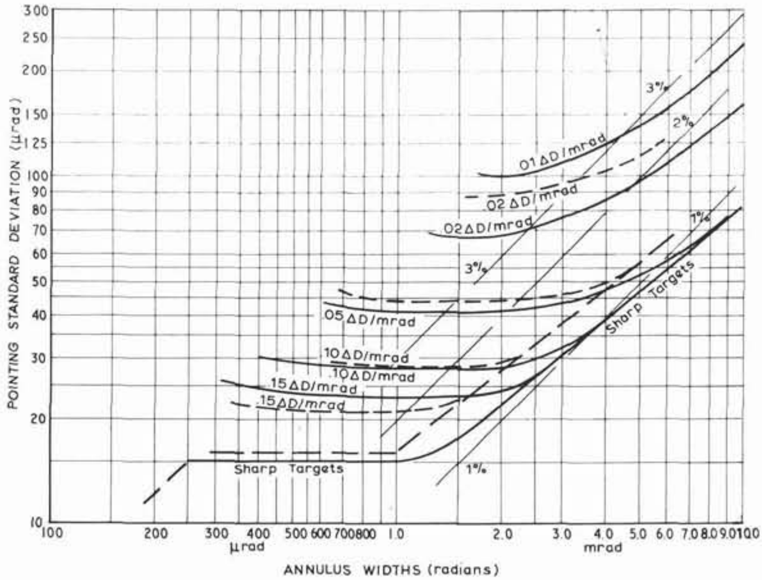

FIG. 5. Pointing standard deviations for *JCT* (full lines) and *AHC* (broken lines) using sharp and blurred circular targets. The grade of blur (in ΔD/mrad) corresponding to each line is shown (measuring mark = 1.0 mrad).

Constant-stimulus method (*CSM*) results however indicate that a different mechanism is apparently involved in discriminating a stationary *MM* than centering a moving *MM*. The accuracies are not only larger, but the differences become proportionally greater as the blur increases, leading to the steeper slope in the relationship in Figure 6. The computational techniques used to derive the standard deviations and *DL*'s in the two methods are different and a direct comparison therefore cannot be made. Normal distributions are assumed for both types of observations, but there is more involved than simply a difference in computation technique. A comparison of the results from the average-error method and constant-stimulus method is shown in Table 3. The ratio of *DL/S*

TABLE 2. CONSTANT-STIMULUS METHOD RESULTS FOR TWO OBSERVERS

Blur ΔD*/mrad*	*Annulus Width mrad*	*Back. Dens.*	*DL. μrad JCT*	*DL. μrad AHC*
0.042	2.0	0.20	79.3	132.5
0.09	1.3	0.40	45.0	83.7
0.12	1.5	0.24	34.0	40.0
0.50	1.4	0.30	19.1	28.4
1.0	0.6	0.30	17.1	29.5
Sharp	0.5	0.30	18.9	—

(the *DL* from the constant-stimulus method, and *S* the standard deviation derived by the average-error method) indicates an increase in *DL* with respect to *S* for increasing blur.

The conclusion may be reached on the basis of observations conducted in this research, that in respect of the standard deviation of observations (or *DL*), an instrument based on the average-error method of observation will give superior results to an instrument based on the constant-stimulus method. This is particularly true with blurred targets, although for sharp targets a smaller, significant difference in accuracies is still present.

Binocular observation results indicate that little or no difference exists between monocular and binocular observation results. A variance analysis based on a factorial design (Moroney, 1951) indicates that these differences are significant at the 92 percent significance level. It may therefore be assumed that such differences are only marginally significant. Comparisons between monocular and binocular results are shown in Table 4.

THE RELATIONSHIP BETWEEN EXPERIMENTAL RESULTS AND PSYCHOPHYSICAL FORMULAS

The results of pointing to sharp targets, given by O'Connor (1962, 1967), Roger et al. (1969), Trinder (1971) clearly indicate the relationship between width and pointing

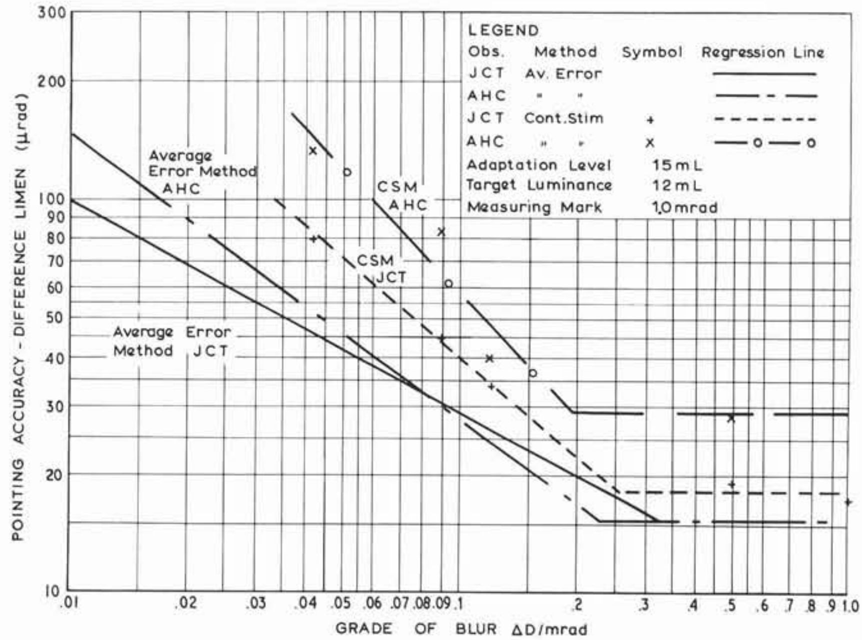

FIG. 6. Pointing standard deviations derived by the average-error method, and DJ's derived by the constant-stimulus method for 2 observers. Annulus width is 2 mrad.

accuracies of 1 to 2 percent, which follows Weber's Law. The section of the curve where annulus widths are between 250 μrad and 1 mrad as shown in Trinder (1965), also follows Weber's Law provided the widths are considered as those seen by the visual system. For annulus widths less than 250 μrad, luminance discrimination appears to be the criterion used for pointing, (Trinder, 1965).

As the average-error and constant-stimulus methods may be considered as psychophysical scaling methods, it is valid to apply the results of pointing investigations to existing psychophysical laws. In this instance pointing accuracies on the response scale are represented by measurements on the stimulus scale, purely as scale numbers.

Stevens' law given previously is a suitable psychophysical law which may be applied to blurred target results.

The general form of Stevens' psychophysical law is $\psi = K(\phi - \phi_o)^n$ where K is a constant depending on the choice of axes, n is the exponent depending on the modality or sense investigated and also the parameters of the experiment, ψ is the psychophysical reaction to the stimulus ϕ, and ϕ_o is the effective threshold measured on the stimulus scale. This relation is linear if ϕ and ψ are plotted on logarithmic scales.

The linear regressions in Figures 3, 4 and 6 are computed in terms of grade of blur of the target. To compute similar regression lines

TABLE 3. COMPARISON OF RESULTS OBTAINED FROM THE CONSTANT-STIMULUS AND AVERAGE-ERROR METHODS

Grade of Blur ΔI / mrad	DL Const. Stim. Meth. (μrad)		S (μrad) Aver. Err. Meth.		DL/S	
	JCT	AHC	JCT	AHC	JCT	AHC
0.042	79.3	132.5	41.0	40.5	1.94	3.28
0.09	45.0	83.7	30.6	24.2	1.47	3.33
0.12	34.0	40.0	24.7	22.7	1.37	1.75
0.50	19.1	28.4	17.1	16.0	1.11	1.77
1.0	17.1	29.5	15.6	17.3	1.10	1.89

TABLE 4. COMPARISON BETWEEN MONOCULAR AND BINOCULAR RESULTS

Observer: JCT

Grade of Blur ΔD/mrad	Stand. Dev. Monoc. μrad	Stand. Dev. Binoc. μrad
0.012	104.0	93.2
0.021	80.3	72.0
0.024	69.8	64.8
0.031	56.5	59.0
0.046	45.8	44.0
0.082	30.0	31.9
0.12	24.7	24.0

based on Stevens' formula, the equations must read:

$$S = K(\Delta D - \Delta D_o)^n \quad (1)$$

where S is a accuracy in μrad, ΔD is the grade of blur (ΔD/mrad), ΔD_o is the visibility threshold grade of blur for pointing for a particular annulus width, K and n are constants.

If the equation of the linear regression is presented as $n = a$ and K = antilog b. The ΔD_o

$$\log S = a \log (\Delta D - \Delta D_o) + b$$

term must be estimated before the above regression can be computed. Table 5 gives the visibility thresholds ΔD_o for the three annulus sizes for the two observers *JCT* and *AHC*. These thresholds have been estimated from experience gained during the observations and from values given by Hempenius (Trinder, 1965, 60) who states that the necessary density changes for visibility range from 0.004 to .01ΔD/mrad, with no reference to size of object. The smaller annulus widths, however, definitely required a greater change in density to be visible. According to Stevens' formula, pointing accuracies become infinite as the threshold is approached, because the exponent n is less than zero. Thresholds may therefore also be estimated from the pattern of the near threshold targets for annulus sizes of 0.8 and 2.0 mrad. A plot of the monocular thresholds in Table 4, against annulus width on logarithm scales gives approximately a linear relationship. The computed regression lines using the values of $(\Delta D - \Delta D_o)$ led to the versions of Formula 1 given in Table 6 and plotted in Figures 7 and 8.

The exponents in these equations for monocular observations by *JCT* decrease for increasing annulus width, whereas the coefficients of the equations are approximately constant. The relationship between width and exponent is linear on logarithmic scales, giving an equation for the exponent for *JCT*, viz:

$$\text{exponent} = -0.322 \text{ (width) } 0.282$$

A general expression of the equation for monocular pointing accuracy by the average error method for *JCT* based on Equation 1 therefore may be written as

$$S = 12(\Delta D - \Delta D_o)[-0.322 \text{ (width) } 0.282] \quad (2)$$

This formula, although complicated, indicates the general form of the equation that describes pointing accuracies in terms of grade of blur and width of the annulus. The thresholds in the above results are clearly an important factor, particularly for the smaller annuli. A similar formula for *AHC* cannot be derived accurately. Observations by *AHC* were always more erratic than those of *JCT*, and therefore considerably more observations would be necessary to derive an accurate formula. This was not possible because of *AHC*'s limited time.

For binocular vision, a slightly lower threshold of 0.008 ΔD/mrad for targets with 2 mrad annulus width was adopted. Stevens' formula gave a relationship which is very similar to the corresponding formula for monocular vision. Index n which depends on the type of psychophysical task and the sense being investigated, should be the term to change if visual performance were to show a fundamental variation between monocular

TABLE 5. VISIBILITY THRESHOLDS FOR THREE ANNULUS SIZES

Width mrad	*JCT* ΔD/*mrad*	*AHC* ΔD/*mrad*	*Comments*
0.8	0.03	0.04	Monocular
2.0	0.01	0.015	Monocular
5.0	0.0035	0.005	Monocular
2.0	0.008	—	Binocular
2.0	0.015	0.025	Monocular, CSM

TABLE 6. VERSIONS OF FORMULA 1

Width	*Observer*	*Equation*	
0.8 mrad	*JCT*	13.5 (D − .03) − .297	(Figure 7)
2.0 mrad	*JCT*	11.2 (D − .01) − .403	(Figure 7)
5.0 mrad	*JCT*	12.0 (D − .0035) − .499	(Figure 7)
0.8 mrad	*AHC*	18.8 (D − 0.04) − 0.123	(Figure 8)
2.0 mrad	*AHC*	7.3 (D − .015) − 0.52	(Figure 8)
5.0 mrad	*AHC*	12.3 (D − .005) − 0.52	(Figure 8)
2.0 mrad	*JCT CSM*	9.0 (D − 0.015) − 0.607	(Figure 7)
2.0 mrad	*AHC CSM*	13.7 (D − .025) − .555	(Figure 7)
2.0 mrad	*JCT Binoc.*	11.0 (D − 0.008) − .415	(Figure 7)

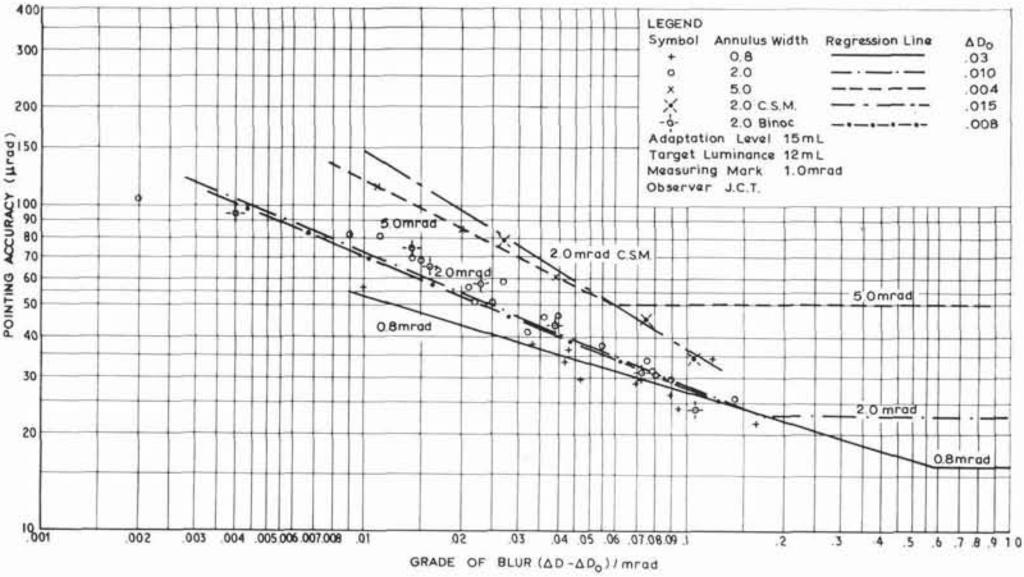

FIG. 7. Pointing standard deviations (from average-error method) and DL (from constant-stimulus method) against (grade of blur − threshold), i.e., $(\Delta D - \Delta D_o)$, based on Stevens' Formula. ΔD_o values are shown in legend. Observer is JCT.

and binocular vision. That this is not so is indeed a logical conclusion, considering the physiology of the usual system, and the inherent reliability of visual performance. The slight improvement in results obtained for very blurred targets viewed binocularly seems to be due solely to the marginally lower threshold associated with observations with two eyes.

From results derived by the constant

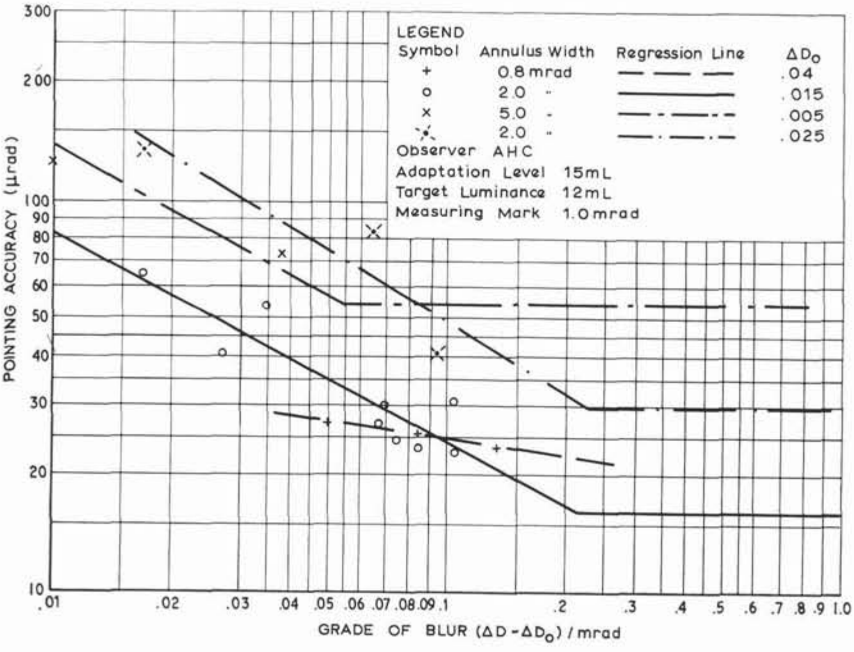

FIG. 8. Pointing standard deviations (from average-error method) and DL (from constant-stimulus method) against (grade of blur − threshold), i.e., $(\Delta D - \Delta D_o)$, based on Stevens' Formula. ΔD_o values are shown in legend. Observer is AHC.

stimulus method, the coefficients of equations in Table 6 are substantially the same but the exponents n on the other hand are smaller. This seems to reflect the greater difficulty experienced by the observer in discriminating the position of the motionless MM. One of the problems associated with constant stimulus method observations, particularly with blurred targets, is that the target is always partially obscured by the MM. With the average-error method on the other hand, the MM is moved away from the target center before observations commence, and therefore a better impression of the true center can be gained.

Summary

From a psychophysical point of view, the transition between the two straight lines in Figures 7 and 8 probably should be gradual rather than abrupt. Figures 7 and 8 indicate that Stevens' formula is only an approximate description of pointing accuracies over the full range of target blur. A more complete formula embracing both straight line sections, together with the transition, would seem to be more appropriate. Such an equation would no doubt vary for different observers in magnitude, and in respect of the interactions between variables. The description of the equation however, is not necessary for photogrammetry, if indeed it could be formulated. Figure 5 is more useful for the practical application of results.

The successful application of Stevens' law to results on pointing to blurred targets is important for two reasons. Firstly, because it increases knowledge on the behavior of the visual system, and secondly, because it adds validity to the experimental methods and approach adopted in this research. The important points derived from the application of Stevens' law are as follows:

- Visibility thresholds are a significant factor affecting pointing accuracies to blurred targets.
- The similarity between results derived for monocular and binocular pointing can be explained through Stevens' law.
- Lower accuracies of observations by the constant-stimulus method are apparently due to greater difficulties associated with discriminating a stationary measuring mark, and a higher visibility threshold level.

The average-error method is the most valid experimental method for investigating photogrammetric pointing accuracies because of its similarity to observational methods in photogrammetry. The constant-stimulus method on the other hand, is unsatisfactory for this purpose unless a sensitivity or difference limen is required (Zorn, 1965). It is doubtful if results derived by the constant-stimulus method can be used to obtain corresponding average error method results, and therefore such results are useful only as a means of relative comparison between observers.

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