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Errors in Point Marking

Photomicrographs and a microdensitometer are applied in the analysis of the transfer of points used in aerotriangulation.

Introduction

Transfer or pass points and tie points are required in aerial triangulation to connect models in a strip, and to connect adjoining strips. For a high accuracy of the triangulation it is necessary that the transfer points be identified and measured without ambiguity in the models in which they appear. They may later serve as minor control points for absolute orientation of the photographs for map compilation.

The ideal locations of the transfer points are on the triple overlap of photographs having at least 60 percent end-lap. Three points are so selected that one is as close as possible to the photo center and the others are on the side-laps—usually 25 to 30 percent—one on either side of the flight line.

The transfer points are selected on the diapositives within a certain tolerance circle. Thus, two types of points are available for precise aerial triangulation, namely, (*a*) natural points and targetted points which are distinct ground details, and (*b*) artificial points which are marked and transferred with the help of a point-marking and transferring device.

Several point-transferring devices have been developed to aid the operator in precise identification and measurement of pass points and tie points in both analog and analytical methods of aerial triangulation. Three such devices were used for the study—Wild PUG2, Wild PUG3, and the Multiscale Stereo Point Marker Device designed and built by Bausch and Lomb, Incorporated.

Purpose and Scope of Study

Preparation of photographs for aerial triangulation involves two main cases of point marking. Case A deals with points in the end-lap where marking of only one photograph at a time is suggested (unless for monocomparator observations). Case B is concerned with points in the side-lap where simultaneous marking of the two photographs is recommended. To these will be added Case C, in which it becomes necessary to transfer an already marked point to an adjacent photo. It will be expected that Case C will create a critical situation, particularly if different sizes of measuring mark and marker are encountered.

Abstract: *Several point-marking and transferring devices have been developed to help the operator in precise identification and measurement of points on photographs for aerial triangulation. Three different instruments are used to study the errors that arise from the point-marking and transferring on two overlapping photographs. Photomicrographs and microdensitometer traces of the marked points are analyzed for possible errors caused by imperfections in the shapes and sizes of the points. Statistical tests are utilized to compare precisions between natural and artificial points, stereoscopic and pseudoscopic observations, and between the relative merits of the instruments used.*

The purpose of the study was to examine the precision with which points could be marked and transferred on overlapping photographs. Cases B and C were examined. Statistical tests were utilized to compare the attainable precisions between natural and artificial points, and between stereoscopic and pseudoscopic observations.

Microdensitometer traces were drawn for a sample of the marked points to help in analyzing possible errors arising from imperfections in the shape and size of the marks, and from the photo tones of the terrain on which a point is marked.

Experimentation

Point Marking

Two photographs with a 60 percent overlap were taken with a Wild RC8 Camera at an altitude of 6,000 feet above mean ground. Three identical copies of each photograph were prepared from Kodalith Ortho Film Type 3 with Estar base designed for high contrast. The emulsion thickness was 0.004 inch.

Eighty points were located on the overlapping area of the photographs. These were made up of (*a*) 37 natural points, such as street marks and intersections, isolated short trees and corners of man-made features, and (*b*) 43 artificial points. The points were scattered over the photographs at various topographical locations on both built-up and rural types of terrain with varying photo tones. The artificial points, in particular, were selected on flat terrain insofar as possible.

The Wild PUG3 was used for the simultaneous marking of the points (Case B). The Wild PUG2 was used for the transfer of points from one photograph to the other (Case C). All the points were drilled first on the left photograph using the right photograph of Case B as a guide. The natural points were the same points; the artificial points were selected as closely as possible to the original points. The new photograph to be marked was then placed on the right picture carrier, and each point drilled under stereoscopic viewing. The drill sizes in both cases were 100 micrometers.

The Multiscale Stereo Point Marker Device was used, as with the PUG2, to mark and transfer points from one photograph to the other. The dot size was 40 micrometers with a 200-micrometer circle around it. For quick identification of the points, a 2,000-micrometer ring was marked around all the points on the right hand photograph. The transferring was done at 12× magnification which is the closest to the 10× fixed magnification of the PUG2 and PUG3.

Coordinate Measurement

Measurement of the coordinates was performed on the Wild A7 Autograph. The ratio of model scale to picture scale was 3:1. The elevation counter and gears were set to read in millimeters.

Numerical relative orientation was performed using the Dependent-Method method. The precision of the relative orientation, m_{py}, was determined from 15 points in each model and expressed in terms of standard error in residual *y*-parallaxes at picture scale. The results were:

PUG2: $m_{P_y} = 7.1\mu m$

PUG3: $m_{P_y} = 5.2\mu m$

MULTISCALE: $m_{P_y} = 6.3\mu m$

If no errors occur in the point marking, and the relative orientation is perfect, points on one picture will fuse into their conjugate points. By measuring the coordinates of the *double* points it was possible to determine the separation between them. This was achieved by stereoscopically setting the measuring mark on the surface of the point, and monocularly setting each half mark on its corresponding point and recording the X, Y and Z coordinates.

It must be mentioned here that correlation exists between relative orientation and the Y-component of the separation between the points. The X-component is correlated with height measurement, that is, how well the measuring mark is placed on the surface of the point, which calls for experience in the operation of stereoscopic instruments.

The dove prisms were rotated to create a pseudoscopic view, and the coordinates were again recorded as described above.

Coordinate differences were obtained by subtracting left-hand picture readings from those of the right-hand picture. The difference between stereoscopic mean height and pseudoscopic mean height of each point was also determined.

Error Analysis

Physical Analysis

Photomicrographs of three of the PUG marks, at a magnification of 250 diameters, are shown in Figure 1. Their corresponding microdensitometer traces are presented in Figure 2. The dark band appearing in Figure 1(a) is the emulsion deposited on the periphery of the hole by the drill. In Figures 1(b) and 1(c), the holes are partially filled with emulsion and green ink which was used in labelling the marks (see their traces in Figure 2). However, the photo tone of the surroundings of 1(b) permitted the centering of a measuring mark more easily than on Figure 1(c).

Statistical Analysis

In the model space, the two drilled points

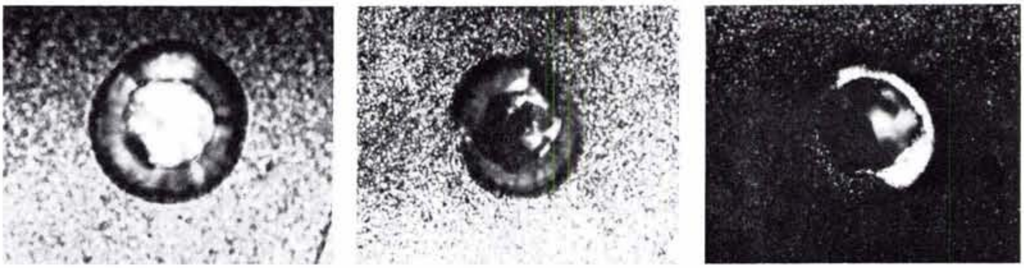

FIG. 1. Photomicrographs of PUG marks.

fuse together if transfer and instrumental errors are not present.

Let X_1 and X_2 be the x-coordinates of the same point in the left hand and right hand pictures, respectively, as read in the model space.

The mean reading, $\overline{X}$, is given by

$$\overline{X} = \frac{X_1 + X_2}{2}. \tag{1}$$

If d denotes the x-difference between the points, and v_1 and v_2 are the deviations from the mean,

Then

$$|v_1| = |v_2| = \frac{|d|}{2} \tag{2}$$

and

$$v_1^2 = \frac{d^2}{4}. \tag{3}$$

By definition, the Sample Variance, m^2, is given by

$$m^2 = \frac{[vv]}{n-1} \tag{4}$$

where n is the number of observations, and [vv] is the sum of the squares of the deviations from the mean. Therefore,

$$m_x^2 = \frac{v_1^2 + v_2^2}{2-1}.$$

From Equations 2 and 3,

$$m_x^2 = d^2/2 \tag{5}$$

The Standard Error of One Measurement is

$$m_x = \sqrt{(d^2/2)} \tag{6}$$

For N points in the overlap,

$$m_x^2 = \frac{m_{x_1}^2 + m_{x_2}^2 + m_{x_3}^2 + \cdots + m_{x_N}^2}{N}$$

$$= \frac{1}{N}\left[\frac{d_1^2}{2} + \frac{d_2^2}{2} + \frac{d_3^2}{2} + \cdots + \frac{d_N^2}{2}\right] \tag{7}$$

and

$$m_x = \left[\frac{\sum_{i=1}^{N} d_i^2}{2N}\right]^{1/2}. \tag{8}$$

Tables 1 and 2 give the mean coordinate differences with due regard to sign, sample variances and standard errors of unit weight of the observations in stereo and pseudo. The values have been reduced from model scale to picture scale, and are in units of micrometers. The mean differences in y are of the same magnitude and sign in both stereoscopic and pseudoscopic measurements. A change

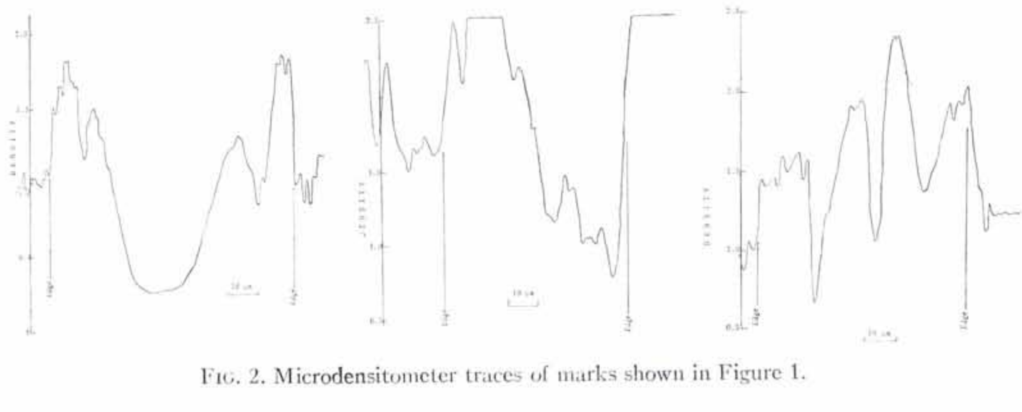

FIG. 2. Microdensitometer traces of marks shown in Figure 1.

TABLE 1. MEANS, VARIANCES AND STANDARD ERRORS OF ALL POINTS—STEREO—IN UNITS OF MICROMETERS AT PICTURE SCALE

Instrument	$\Delta x_{ave.}$	m_x^2	m_x	$\Delta y_{ave.}$	m_y^2	m_y
PUG2	1.3	152.8	12.4	− 0.3	383.3	19.6
PUG3	8.7	174.2	13.2	−10.0	364.2	19.1
MULTISCALE	−11.3	209.2	14.5	−4.7	181.0	13.4

in sign in the x-differences is caused by the dove prisms which laterally interchange the pictures in the optics. The larger values of the pseudoscopic mean x-differences are a function of the height measurement. In this study, height measurement of the terrain in the vicinity of the drilled holes showed that the pseudo heights were greater than the stereo heights by an average of about 50 micrometers in each model.

The residuals in y for PUG2 (Stereo) are shown in the histogram in Figure 3. The class interval, approximately equal to $m/4$ is in millimeters at model scale. A goodness-of-fit test performed on all the residuals shows that they are normally distributed at the 5 percent significance level.

The F-Statistic was used to test a number of hypotheses at the 5 percent significance level. The results of the tests are:

- In all the instruments, the stereo and pseudo observations were found to belong to the same population.
- There was no significant difference between the natural and artificial points.
- There was no significant difference between PUG2 and PUG3. The Multiscale was significantly better than both PUG2 and PUG3 insofar as the y-errors are concerned, but there was no significant difference in x.
- The determination of errors of point transfer in y depends on how good the relative orientation is. If the errors of both are about the same in magnitude, it may be argued that the observed differences in the points are, in fact, caused by inaccurate relative orientation. The result of the test showed that, in all the models, the relative orientation was significantly better than the y-components of the point transfer errors.

SOURCES OF ERROR

The differences in the marks for PUG2 (Stereo) are shown vectorially in Figure 4. The head of the arrow represents the position of the point on the right hand picture, and the foot represents the position on the left hand picture.

In general, large differences appear in the upper right hand and lower left hand sections of the models, and also at the right hand and left hand edges. Although these differences do not follow any distinct pattern, there is some similarity between the three models.

The following errors may have affected the accuracy of the point transfer; they may be systematic, random or a combination of both.

ERRORS IN PHOTOGRAPHY

Distortions due to film, lens and atmosphere. Radial distortion errors are corrected in the Wild A7 with the appropriate correction plates. In the PUG there is no provision for correcting these distortions which have their maximum effect at the corners of the model.

Variations in emulsion thickness. These are irregular errors.

INSTRUMENTAL ERRORS

Instrument errors considered are those attributed to:

- The eccentricity of drill and measuring mark in the PUG.
- Imperfections in the shape of the drill.
- Mechanical and optical errors in the aerial camera, the point-transfer instruments and the coordinate measuring instrument.

OPERATIONAL ERRORS

Positioning the Photographs. In the PUG, film-base pictures are fixed on the plate carriers with adhesive tape such that the emulsion side is upward. Absolute flatness of

TABLE 2. MEANS, VARIANCES AND STANDARD ERRORS OF ALL POINTS—PSEUDO—IN UNITS OF MICROMETERS AT PICTURES SCALE

Instrument	$\Delta x_{ave.}$	m_x^2	m_x	$\Delta y_{ave.}$	m_y^2	m_y
PUG2	−17.3	259.2	16.1	− 1.7	409.9	20.2
PUG3	−14.0	203.2	14.3	−10.0	375.6	19.4
MULTISCALE	12.2	148.0	12.2	− 5.0	141.9	11.9

the film is necessary if micrometers are considered significant. In the A7 the pictures are placed on the plate carriers with emulsion side down, and plain glass is used to press the film down flat. Differences are, therefore, most likely to show up in the positions of the drilled holes. In the Multiscale, the film is held down by vacuum and acetate masks.

Variable pressure on the drilling lever of the PUG. This error has the effect of producing various shapes of the drilled hole.

Poor handling of photographs. Clearing of the picture surfaces during drilling results in particles of emulsion being deposited in the holes. This is not the case in the Multiscale where the mark is molded with a heated die.

Relative orientation errors. This type of error causes deformation of the model. Numerical methods can improve the quality of relative orientation and the precision can be determined.

Conclusions

In general, the Multiscale gave higher precision than the PUGs, especially in the y-direction. Large errors in the PUGs may have been caused by the film-base photographs which could not be fastened perfectly flat on the plate carriers. The justification for using film-base photographs was to ensure their safety, as all stages of the experiment were done at different places, involving in some cases, mailing of the photographs.

The marking system of the Multiscale results in a clear mark with well defined edges. Although microdensitometer traces of the Multiscale marks were not available, the difference between its marks and those of the PUGs was conspicuous in the measuring instrument.

The diameter of the Multiscale mark was

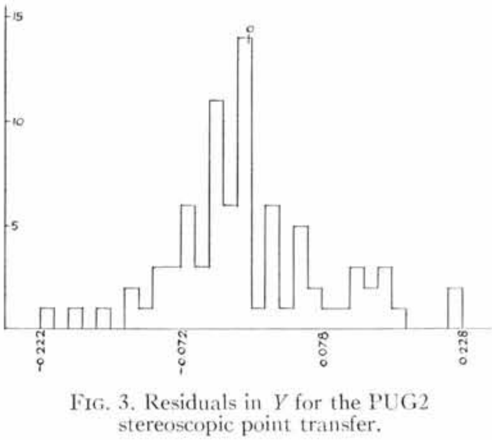

Fig. 3. Residuals in Y for the PUG2 stereoscopic point transfer.

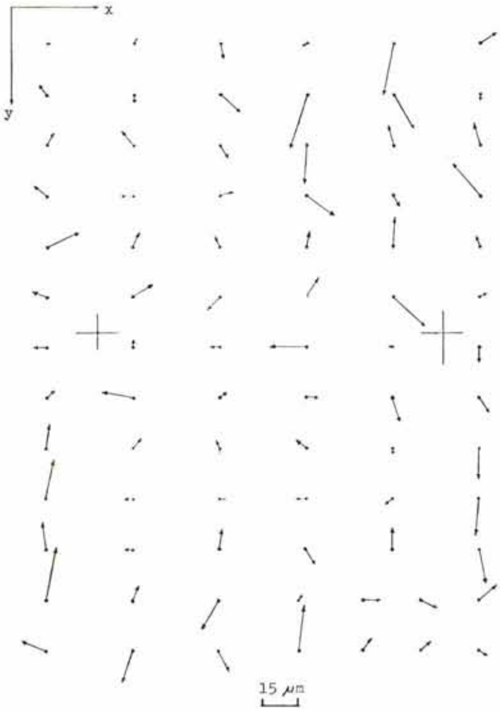

Fig. 4. Vector representation of the differences for the PUG2 stereoscopic point transfer.

40 micrometers, which was the same as the measuring mark available on the Wild A7. The accuracy of coordinate measurement might be affected. The precision of pointing in this case is low, according to Hempenius[5].

As found from the PUG2 and PUG3, the precision of simultaneous marking on two photographs is of the same order of magnitude as that of the transfer of an already marked point to its conjugate position.

There is no significant difference between natural and artificial points.

Pseudoscopic measurements gave the same order of accuracy as stereoscopic measurements. However, as found from the height observations, there seems to be a constant difference in height between the two. This be due to backlash of the instrument, and to physiological and psychological effects of changing from normal to pseudoscopic viewing.

The purpose of statistical hypothesis tests is to guide the experimenter in his conclusions. For an effective analysis, a large number of observations is required, such as 500 to 1,000. Eighty points may be considered too small a sample on which the behavior of an entire population should be based.

In general, the confidence limits for the y-

errors were greater than those for the x's. This is due to the fact that the human eye can more efficiently resolve x-parallax than it can resolve y-parallax. For this reason, in order to remove all the parallaxes at a point, it is advisable to change y-parallax into x-parallax.

In practice, stereoscopic observation of an overlap is restricted to the neat model. Figure 4 shows that, in this study, most of the large errors occurred outside the neat model. These errors may have been caused by film distortion, poor resolution at the edges, or by the obliquity of the epipolar plane which affects the accuracy of setting of the measuring mark.

Recommendations

The critical part of the preparation of photographs for aerial triangulation is the selection, transferring and marking of control and pass points. This stage of the work should be handled by an experienced and reliable operator.

Marking of natural points is faster and may be preferred if available. The disadvantage of marking natural points is that the mark often obscures the detail, but in stereoscopic work only one photograph need be marked and the detail still appears on the adjacent photographs. For production work, therefore, stereocomparators, rather than monocomparators, are recommended.

To avoid backlash in the stereoscopic instruments, the measuring mark should be directed on to the point from the same direction in both stereoscopic and pseudoscopic observations.

Extreme care should be taken in the handling of marked photographs, particularly as regards labelling, emulsion dust and storage.

The use of photomicrography and microdensitometry is recommended for the study of the physical and operational conditions of point marking and transferring instruments, especially when drills are used. The quality of the drilled marks is readily seen from the photomicrographs and the microdensitometer traces.

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