DR. WOLFGANG FAIG
University of Illinois
Urbana, Illinois 61801

Shapes of Thin Soap Membranes

A fluorescent chemical is mixed into the soap solution and a special pair of cameras is constructed.

ABSTRACT: *In various technical fields, e.g., architecture and mechanical engineering, thin soap membranes are used as analogue models to solve certain problems. Due to the fact that photogrammetry requires neither direct contact with the object nor long-time stability of the model it seems to offer an attractive means of surveying such an unstable membrane. Until now the optical characteristics as well as the small size of thin soap membranes have limited its use only to a few experiments however. In order to achieve a reasonable accuracy, the membrane has to be prepared for stereophotography, and close-up camera equipment must be available. The author solved the first part by making use of fluorescent light. He constructed a short-range stereo camera and performed a thorough calibration of it. This yielded large lens distortions which forced him to perform an analytical evaluation of the photographs. For an accuracy analysis a well known shape—the sphere—was determined, using this method and equipment. Although only a Jena 1818 comparator was available for the evaluation, it proved suitable for the purpose, as the determined errors amounted to less than 0.12 mm in the subject. The method used has therefore fulfilled the requirements for the measurements which were of a satisfactory standard.*

INTRODUCTION

IN MODERN ARCHITECTURE a variety of shapes and surfaces are used which are extremely difficult to express either by closed mathematical equations or in numerical form. As a result a model structure is needed from which the necessary data can be obtained for use in the course of eventual construction.

The author first faced this problem during the measurements on experimental models for the roof of the German Pavilion at Expo 1967 in Montreal[6]. The original shape created by a thin soap membrane had been transformed into a wire model only by application of the knowledge of the physical characteristics of the membrane and by some imagination. The construction data was then determined from this large-scale wire model. Direct measurements of the soap membrane would have saved time and money and could also have provided a more accurate presentation of the shape of the membrane.

Inasmuch as the object is very unstable, photogrammetry seems to offer the best method of approaching the problem. The desire to measure the shape of a soap membrane directly is not new, as soap films have been used for some decades to solve torsion problems.[9,11] Apart from mechanical methods,[1] several experiments have been conducted[16,17] to determine the inclination of the surface by optical means. There have also been a few photogrammetric attempts. Anthes[2] used rectification for flat membranes which yielded more qualitative results than quantitative values.

Thiel[22] was the first to use stereo photogrammetry. He prepared the membrane for photography by putting light seeds (lycopodium) onto the surface. Sitek[20] used cosmetic powder to solve this problem and Bonanno[3] added titanium dioxide to the soap solution.

As all these methods failed with very thin membranes, the author concluded that the membranes used either had to be thicker or that rather flat membranes as in Thiel's case had to be used. In order to obtain a more general method two main problems had to be solved: (1) how to render the membrane visible for photography; and (2) how to obtain a suitable camera for close-up photography. The evaluation of the photographs could then follow well-known procedures.

Preparation of the Object for Photography

The use of soap membranes has been restricted mainly to the consideration of very thin films in order that the influence of gravity can be neglected. Having a thickness of a few molecules only, i.e., about the magnitude of the wave length of light, the membranes are completely transparent and practically invisible except for the outline. The membranes are very smooth, and a small fraction of the light coming upon them will be reflected from the surface in a direction depending on the incident angle. For stereo-photos any projected grid might be shown in each picture, but the same point of the grid will be reflected from a different part of the surface in either picture and, if a stereo model is created in this way, it does not represent the shape of the membrane.

Even with changes in the direction and intensity of lighting, and with the use of different photographic emulsions or backgrounds, the membrane remains monotonous and transparent. Therefore other means were to prepare the membrane for photography.

Optical Means

Several mechanical methods for using soap bubbles were considered to be unsatisfactory, such as cosmetic powder, Lycopodium powder, thin nylon threads and hairs, and needles.

As the outline of the membrane is always visible, it seemed to be possible to use this characteristic by putting the membrane into an array of parallel light. The projected image could then be photographed on a screen placed perpendicular to the direction of the light. After rectification with the aid of a fine grid drawn onto the screen, the pictures showed the outline of the membrane perpendicular to a certain direction. By rotation of the object, numerous pictures would represent the surface. The author tried to perform this operation with the aid of several students and satisfactory results were obtained[18]. However a problem was created because the projection of the outline of the membrane onto the horizontal plane was represented generally by an unknown curve. Only for simple figures, such as a sphere, is it a straight line, as shown in Figure 1. Furthermore this method requires a more stable and therefore thicker membrane due to the time involved in turning the object and taking the necessary photographs.

After considering the light scattering properties of soap particles and the use of smoke or fog, a suitable solution to the problem was achieved by mixing a fluorescent chemical into the soap solution. This chemical must produce fluorescent light in the visible section of the spectrum, most favorably in the green-blue area, in order that orthochromatic emulsions may be used for the photography. Further, it must neither react chemically with the soap solution nor change its physical characteristics. A chemical which meets these requirements is *Fluoresceinnatrium*, which was used by the author in later experiments. As high energy radiation is needed for the emission of low energy fluorescent radiation, ultra violet light had to be applied, thereby creating a shiny green membrane surface.

In order to obtain a texture on this surface, the ultraviolet radiation was partially screened, resulting in a light-and-shade reaction on the surface. Parallel radiation through a grid made up of bars of approximately 1/2 cm thickness and separated by 1-cm spaces proved to be most satisfactory for this purpose. Mainly because of diffraction, a smaller grid could not be used. Although the border lines between light and dark were not sharp, the pattern was sufficient for stereo-photogrammetric purposes (Figure 2).

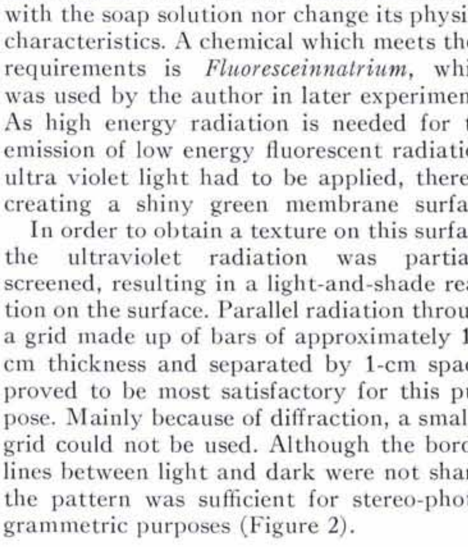

FIG. 1. Horizontal and vertical views of a simple (spherical) and a complex (tent-shaped) membrane illuminated with light parallel to the x-axis. The center views are projections of the outlines onto the yz-plane. The lower views show the projections onto the xy-plane.

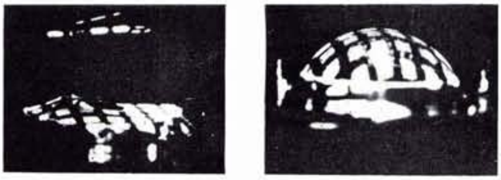

FIG. 2. Soap membranes with fluorescent light.

CONSTRUCTION AND CALIBRATION OF A SHORT-RANGE STEREO CAMERA

SURVEY OF AVAILABLE EQUIPMENT

In order to obtain a reasonable photoscale, neither a common photogrammetric camera focussed on infinity nor a stereometric camera with fixed focus (such as the ones built by Wild[4], Zeiss, Oberkochen[26] or Zeiss, Jena[27] can be used because the minimum object distances are several meters. For this special problem, the camera should meet the following specifications:

- It should have a variable focus setting.
- It should be possible to use high quality wide angle lenses with changable diaphragm.
- The backframe should accept glass plates.
- It should be possible to make fine adjustments of the interior characteristics of the camera.
- It should be possible to establish a base perpendicular to the direction of photography.

After considering the camera equipment discussed in References 14, 15, 21, 23 and 25, the author decided to construct a new camera.

CONSTRUCTION OF A SHORT-RANGE STEREO CAMERA

Using the elements of a "SINAR-Norma" 13×18 cm camera as a base, a new camera was built according to the specifications of the author. The variable exterior settings of the camera, such as the shift and tilt were zeroed and the equipment fitted with fine adjustment screws. The focus setting was also combined with a measuring screw, allowing readings of 0.01 mm.

The back frame (Figure 3), together with a device for pressing the photographic plates against the fiducials was the main subject of the reconstruction. Because a direct contact between plate and frame with fiducials was desired, either the plates had to be removed from the cassettes during exposure, or the frame had to press into the cassettes as in common photo theodolites. As this construction would have been too elaborate, it was decided to introduce the plates from the top using cloth bags as for the TAF (Finsterwalder's light phototheodolite). Upon release, a metal plate, supported by springs, presses the photographic plate onto the frame against the fiducials. After exposure, the metal plate can be pulled back and the photoplate pushed back up into the bag by a scissor-like lifting device (*Nürnberg Scissors*) because the mount of the camera does not allow the plate to fall vertically downwards (Figure 4).

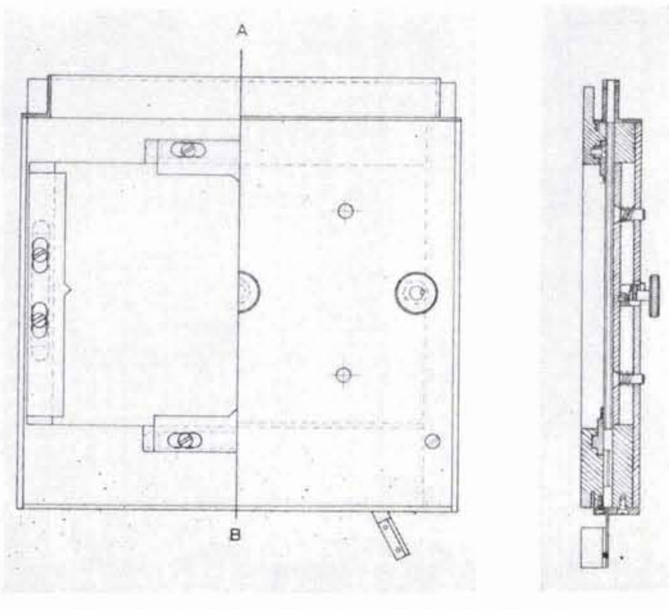

FIG. 3. Workshop drawings of the backframe.

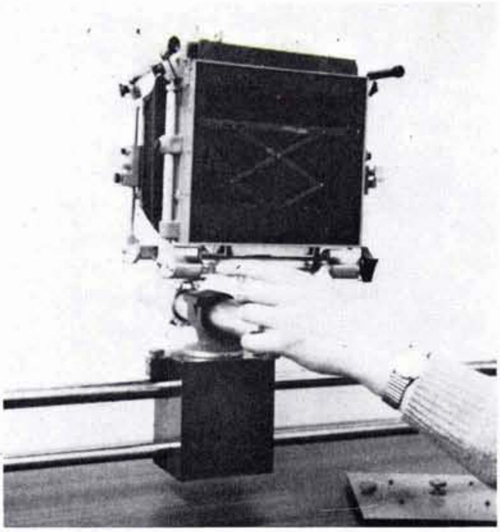

FIG. 4. Nürnberg Scissors.

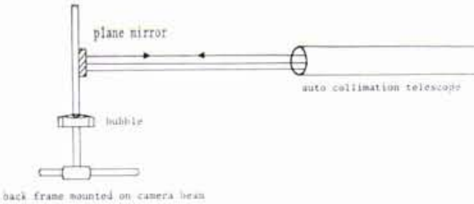

FIG. 6. Arrangement for adjusting in the image plane in the vertical position.

For stereo-work a second camera was built in the same manner, and also a metal bridge of 1.20 m length was constructed onto which one camera can be fixed rigidly (Figure 5). The other can be moved along and set at intervals of 10 cm, thus allowing different base settings for different object distances. A centering device for the Kern DKM-1 theodolite has been established on each end of the bridge, which can be exactly levelled. With the known base length, control points around the object can be determined by intersection. A built-in thermometer shows temperature changes of the bridge so that corrections to the base length can be made if necessary.

ADJUSTMENTS AND CALIBRATION

Before the equipment could be used, several adjustments had to be performed on each camera.

(1). The image plane had to be adjusted so that it was exactly vertical when the bubble on the back frame was level. This was achieved through the application of autocollimation (Figure 6). With the same arrangement the camera-beam along which the back and front frames can be moved was checked for straightness.

(2). In order to determine the infinity position of the objective lens, the front frame carrying it was installed between mirror and telescope (Figure 7). Through-a-lens autocollimation can only occur if it is located exactly at the focal length in front of the mirror and if its axis is exactly parallel to the telescope axis. It was quite time-consuming to find this position, adjust the frame, and then take the reading on the focussing screw.

(3). With the frames adjusted so that they can only be moved along the beam without change in the orientation relative to each other, the motion of the lens relative to the front frame during changes in the focus setting must then be parallel to the axis of the lens. This condition was established by inserting lenses of different focal length into the front frame and adjusting the moving part so that autocollimation occurred by variation of the focus setting.

(4). Fiducial marks had to be adjusted in such a position that the intersection of the lines connecting them would coincide with the principal point. For this task the camera was mounted on a lathe which allowed motions to be measured to ± 5 μm in two perpendicular directions.

The length of the stereobase was measured with a theodolite, similar to optical distance measurement with the subtense bar, by en-

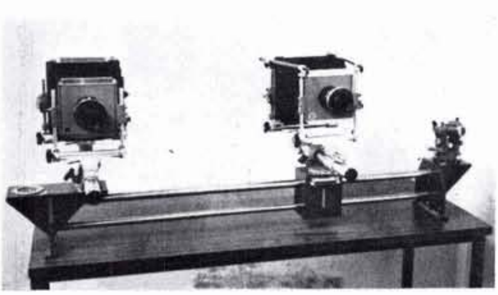

FIG. 5. Stereo camera.

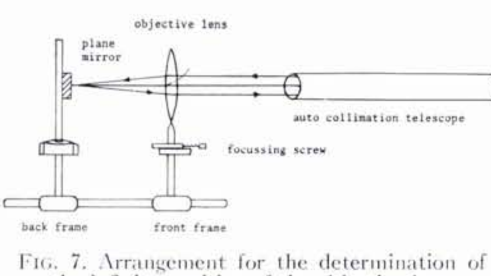

FIG. 7. Arrangement for the determination of the infinity position of the objective lens.

larging an extremely accurate shorter distance. These measurements were performed at various temperatures in order to get an idea on the expansion of the steel due to temperature changes.

The interior orientation data was determined in two steps. First the radial distortion was determined together with affinity and asymmetry. This was done for the focus setting at infinity at the Zeiss factory in Oberkochen. As the results obtained differed only by a few micrometers with the theoretical values given by the lens producer, it was a proof that the construction of the lens followed very closely the theoretical optical calculations on which the theoretical distortion is based. Therefore the theoretical distortion values for various focus settings as provided from the factory were used.

Due to the fact that the distortion was quite large (up to 0.7 mm at the corners of the picture), the image coordinates were corrected for distortion before performing any further steps. With these *new* image coordinates, the computational effort was reduced considerably.

A calibration was performed by a method proposed by Harley[12] who uses a three-dimensional array of control points and calculates a space resection thereby including the origin of the image coordinate system as well as the camera constant as unknowns. For this a minimum of 5 control points are needed.

As a test field the author used a large wooden plate onto which metal blocks with bolts of different lengths were mounted. A number of points were also marked directly on the plate, which was set up about 1 m in front of the camera as it was planned to use this distance for the membrane photography. For the same reason the bolts ranged up to 20 cm in height.

With these simple control points and a

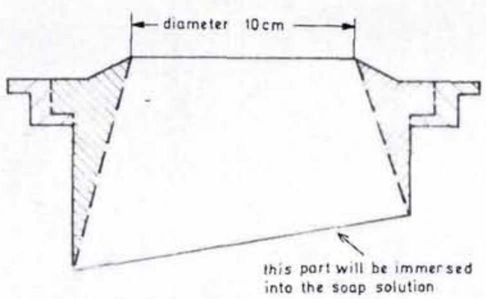

FIG. 8. Support for the soap membranes.

FIG. 9. The "eye".

computer program based on Harley's method, the camera constant and the principal point could be determined to an accuracy of ± 0.01 mm which was sufficient for this project.

EVALUATION OF THE PHOTOGRAPHS

Due to the large distortion of the objective lenses it was impossible to use an analogue plotter for the evaluation of the photographs. An investigation showed that a level plane photographed with this stereo camera at a distance of 1 m would be deformed by more than 1/2 mm, which means that contour lines of any object would have to be referred to such a deformed base.

Therefore an analytical approach was required which could take care of the influence of distortion. In this instance the object needs to be represented by numerous points which have to be measured with a stereo comparator. Afterwards the image coordinates have to be corrected for distortion with the aid of a small computer program and the resulting *corrected* image coordinates can then be used in an analytical orientation program. This program was written in similar steps to the analogue method and consists of the three parts: relative orientation, computation of model coordinates and absolute orientation.

The first two parts follow mainly the well known formulas derived by Schut[19] for aerial photogrammetry. For terrestrial application the y- and z-axes can be interchanged. The absolute orientation is performed according to the formulae published by Kubik[13], who solves the adjustment for the rotational matrix by the method of conjugate gradients. With the known elements of orientation any number of points can be transformed.

PRACTICAL EXAMPLE AND RESULTS

EXPERIMENTAL PROCEDURE

For the practical test of the method, a theoretically well-known soap membrane was chosen: the segment of a sphere (Figure 8). Also a more complicated open surface, a so called *eye* was measured (see Figure 9). Both

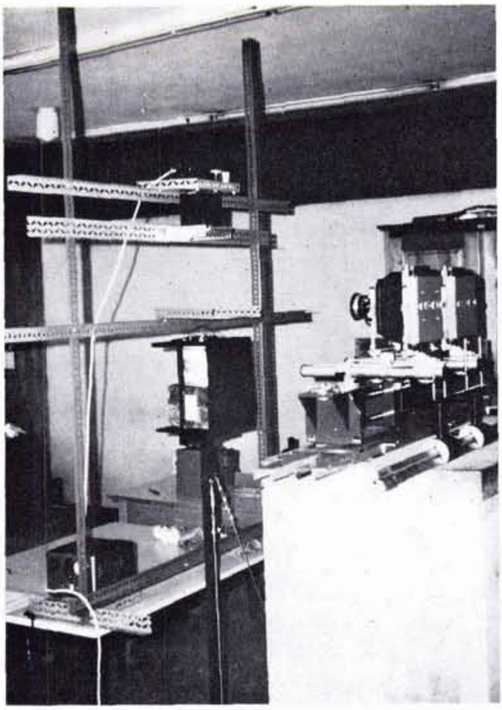

FIG. 10. Experimental arrangement.

membranes were based on a circular cross-section of 10 cm in diameter.

The support is of a conical shape. Upon immersion in the soap solution a membrane forms, which ascends to the upper limit of the support similar to a foam bubble which rises in a beer bottle. After the base is closed off at the bottom the membrane can be set under light pressure and forms part of a sphere. (Figure 10).

In order to be able to see the fiducial marks in the pictures, sheets of white cardboard were fixed in the background and illuminated by candles which were screened off from the camera (see Figure 11). To avoid disturbing reflections, the whole experimental arrangement was painted matt black or covered with black matt paper. For the photography an object distance of about 1 m and a base of 30 cm was chosen.

The membrane was created by dipping the base into a special soap solution according to a recipe of Boys[5], into which a few grains of the fluorescent chemical were added. After illumination through a grid from above by parallel ultra violet radiation, emitted by a coal-arc lamp through an ultra-violet filter, the membrane appeared with a green and black pattern. For reasons of symmetry the back half could be shaded off. The small amount of light required long time-exposures which were possible as the membranes had a stability of several minutes.

RESULTS

After analytical evaluation of the photographs measured with a Jena 1818 comparator, contour maps were drawn (see Figures 12 and 13). No irregularities could be detected for the sphere. The evaluation of several cross-sections proved that, with the exception of very few points, the theoretical surface was obtained with an error of less than 0.1 mm. Additional measurements were performed to obtain an impression of the accuracy with which points can be measured on the surface. Several points on different slopes with different images were measured at the comparator, each 20 times.

For the determination of the surface, the error in direction of the photography was of main interest. This error depends directly on the error in x-parallax and amounted after the full analytical treatment to ± 0.11 mm in the object space. The errors in the other two directions were 30 to 50 percent smaller.

Although it was expected for geometrical reasons that the positioning error would be smaller at the steeper parts of the membrane, there was practically no difference. The author believes that the small amount of the fluorescent light emitted at these parts because of the extremely thin concentration of the solution cancelled the advantage of a better intersection of rays at these points.

CONCLUSIONS

The results obtained by this method of

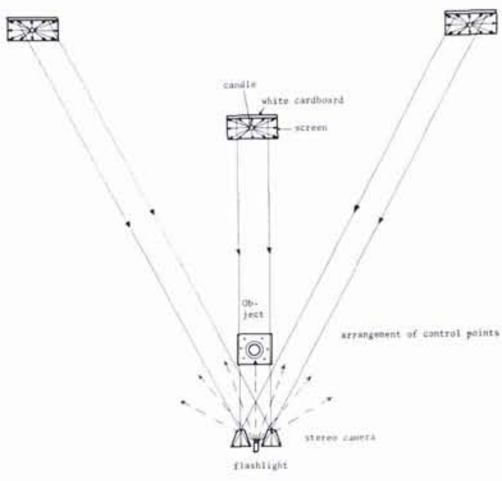

FIG. 11. Experimental arrangement showing the illumination of control points and of fiducials.

determination of the shape of thin soap membranes were very satisfactory. With the use of fluorescent light there is no restriction insofar as thickness or inclination of the membrane is concerned. The stereo camera as well as the analytical evaluation met the specifications laid down beforehand. However, through the use of a precision stereo comparator better results can probably be obtained.

For the production of a plan with contour lines, it would be more economical to evaluate the photographs directly on an analogue plotter. Although this was not possible in this instance due to the distortion, it can very well be done in the future, because in the meantime two wide angle lenses ($f=115$ cm) with practically no distortion ($<2\mu$m) could be purchased from Wild Heerbrugg, Switzerland, for the stereo camera.

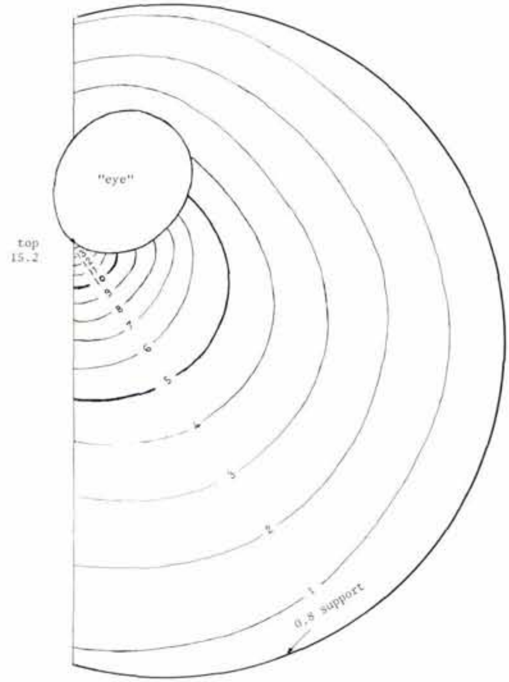

FIG. 13. Contour map of the front surface of an excentrical "eye" based on a circular support.

ACKNOWLEDGEMENTS

The author wishes to express his thanks to Dr. K. Linkwitz, professor and head of the Institute for Applications of Surveying in Civil Engineering at the University of Stuttgart, W. Germany, whose helpful advice enabled the author to perform this research in partial fulfillment of the requirements of the degree of Doctor of Engineering while studying at his institute.

Credit is also given to Mr. M. Mann, who built the camera, to Mr. W. Stranger and Mr. W. Drescher who performed the calibration of the camera, and to Mr. V. Stahl and Mr. H. Wittlinger who conducted some of the experiments as part of their diploma theses under the guidance of the author.

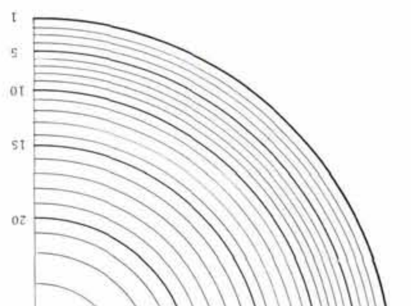

FIG. 12. Contour map of a closed membrane based on a circular support (segment of a sphere).

REFERENCES

1. Albrecht, U., *Bestimmung der Schubspannungsverteilung für ein Profil mit zweifach zusammenhängendem Querschnitt mit Hilfe der Prandtl'schen Membrananalogie*, Studienarbeit beim Institut für Statik und Dynamik der Luft- and Raumfahrt-konstruktionen, T.H. Stuttgart, 1965.
2. Anthes, H., *Versuchsmethode zur Ermittlung der Spannungsverteilung bei Torsion prismatischer Stäbe*. R. Dietze Verlag, Berlin, 1906.
3. Bonanno, F. R., "Torsion Constants of Certain Cross Sections by Non-Topographic Photogrammetry," *Photogr. Eng.* 1958, pp. 803–809.
4. Bormann, G., "Neue Wild-Geräte für die photogrammetrische Nahbereichsmessung," *Presented paper*, Comm. II, Xth Int. Congress for Photogrammetry, Lisbon 1964.
5. Boys, C. V., "Soap Bubbles, their Colours and the Forces which Mould them." German: *Natur und Wissen Verlag*, R. Dresch, München, 1960.

6. Bundesbauverwaltung, u.a., *Expo 67 Montreal—Deutscher Pavillon*. Werner Verlag GmbH., Düsseldorf, 1967.
7. Faig, W., "Vermessung dünner Seifenlamellen mit Hilfe der Nahbereichsphotogrammetrie," Deutsche Geodätische Kommission, München, Reihe C, Heft Nr. 144, 1969.
8. Finsterwalder, R. and Hofmann, W., *Photogrammetrie* 3. Auflage Walter de Gruyter und Co., Berlin 1968.
9. Föppl. L., "Eine Ergänzung des Prandtl'schen Seifenhautgleichnisses zur Torsion," *Zeitschrift f. Angew. Mathematik*, Band 15, Heft 1/2, 1935, pp. 37–40.
10. Gelhaus, R., "Analytisch Photogrammetrische Nahbildmessung," *Wiss. Arbeiten der Lehrstühle für Geodäsie Photogrammetrie und Kartographie* an der T.H. Hannover, Dissertation 1966.
11. Griffith, A. A. and Taylor, G. D., "The Use of Soap Films in Solving Torsion Problems," *Engineering*, Dec. 1917, pp. 652–655 and 699–701.
12. Harley, I. A., "The Calibration of Cameras for Non-Topographical Photogrammetry." Paper presented at the Symposium of the International Society of Photogrammetry, Comm. V, Tokyo, Oct. 13–18, 1966.
13. Kubik, K., "Ein Programm zur absoluten Orientierung von photogrammetrischen Einzelmodellen," *Bildmessung und Luftbildwesen*, 1968, pp. 30–35.
14. Maruyasu, T. and Oshima, T., "Some Applications of Photogrammetry to Precision Measurements of Sculptures," Report of the Inst. of Industrial Science, The University of Tokyo, Vol. 13, No. 7, 1964.
15. Officine Galileo, *Galileo-Santoni Photogrammetric Equipment*, Officine Galileo Florence, 1968.
16. Quest, H., "Eine experimentelle Lösung des Torsionproblems," *Ing. Archiv*, 1933, Band IV, pp. 510–520.
17. Reichenbächer, H., "Selbständige Ausmessung von Seifenhautmodellen," *Ing. Archiv*, 1936, Band VII, pp. 257–272.
18. Reinhardt, W. and Waldraff, S., *Bestimmung der Geometrie eines Minimalflächen-Seifenfilms*, Studienarbeit am Inst. für Leichte Flächentragwerke, Universität Stuttgart, 1967/68.
19. Schut, G. H., *Analytical Aerial Triangulation*. Report of the National Research Council of Canada, Ottawa 1957.
20. Sitek, Z., "Application of Photographic Cameras and the Photogrammetric Method in Some Engineering Measurement Problems." Dissertation at the University of Mining and Metallurgy, Krakow 1962 (written in Polish; correspondence with author).
21. Sutor, J., "Photogrammetrische Baupläne," *Bildmessung und Luftbildwesen*, 1957, pp. 12–18.
22. Thiel, A., "Photogrammetrisches Verfahren zur versuchsmässigen Lösung von Torsions-aufgaben," *Ing. Archiv*, 1934, Band V, pp. 417–429.
23. Thompson, M. M., "Photogrammetric Mapping of Sand Banks in a Hydrolic Test Flume," *Photogr. Eng.*, 1958, pp. 468–475.
24. Torlegard, R. and Wilson, I., "Fiducial Marks and Vaccuum Back for the Hasselblad SWC," *Särtryck uv Svensk Lantmäteritidskrift*, nr. 2, 2, 1968, pp. 241–245.
25. Weimann, G., "Die Askania Doppelmesskammer," *Askania Warte*, Heft 60/1962 und Heft 61/1963.
26. Zeiss, *Instrumente für Photogrammetrie und Luftbildinterpretation*, Carl Zeiss, Oberkochen, 1967.
27. Zeiss, *Stereomesskammer SMK 5/0808*. VEB Carl Zeiss, Jena, 1968.

ASP Solicits Officer Nominations

The Nominating Committee for the American Society of Photogrammetry's new officers to be installed in March 1972 earnestly solicits members' suggestions for names of persons to be considered as candidates.

Because of the changes that have been duly made to the Bylaws and Constitution of the Society, it is now necessary to select nominees for *only* the office of Second Vice President. The incumbent First Vice President and Second Vice President become by precedent single candidates through succession to be elected to the offices of President and First Vice President, respectively, unless for good reason an eligible officer cannot accept his new post. (The Directors of the Society are now elected by the Regions.)

Under the precedent being followed, the candidates for Second Vice President who are proposed this year should be selected from persons in government service (rather than from industry). Barring unforeseen circumstances, the successful candidate will assume the office of President in March 1974.

To have your suggestions considered, the names of your choices must be received before October 1, 1971, either by ASP Executive Director Lawrence P. Jacobs (105 N. Virginia Ave., Falls Church, Va. 22046) or Mr. Pliny Gale, Chairman of the Nominating Committee (P.O. Box 13089, Houston, Texas 77019).