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Focal Plane Images For Film Deformation

Irregularities in mask serve as fiducial marks for film correction.

INTRODUCTION

THE NEED TO UPDATE the system of correcting film deformation by the Carman Method (1946), utilizing the comparison of fiducial marks in the focal plane on a glass flash plate relative to the aerial film, was expanded to include any number of cognizant images around the focal plane. This was used as a system to expand a four side fiducial equipped camera to eight *fiducials* by utilizing the computer program for correcting film deformation as developed by the United States Coast and Geodetic Survey, US C&GS[1,2] (now the National Ocean Survey, NOAA). This US C&GS program can include high-resolution focal plane images in lieu of any fiducial marks allowing one to choose four, eight, or even 80 images which may be used as a *reseau* to correct for film deformation.

STUDY

Early in our documentation of analytical aerotriangulation systems we optioned for the US C&GS system due to its superior methodology and treatment of most apparent errors. However, the US C&GS method is applicable only to cameras with either four corner fiducials, or eight fiducials if any are present along the mid-points of the sides.[3] Several early projects proved the validity of both our photogrammetric and computer systems, one of which would necessitate reworking when we encountered a four-side-fiducial camera. The computer program has subsequently been modified to allow a subroutine for solving four simultaneous linear equations when given only the four side fiducial marks. The following data are confined to a photogrammetric systems study, to realize what may be accomplished by including relevant image data around the focal plane in an analysis of image deformation compensation.

If a four side fiducial camera was utilized on an analytical project, our first reaction was to attempt a solution two ways:

1. Utilize the four side fiducials, using the corner fiducial routine which would skew the plate by 45°.
2. Bypass the film deformation routine by inputing the fiducial mark readings for the flash plate before each photographic pass point and image deck.

Neither solution gave satisfactory results, with the first method coming to a rapid termination with huge parallax messages being printed out from the computer run. The second yielded root-mean-square (*rms*) parallax statements in excess of 100 micrometers by the fifth triplet, indicating film deformation throughout the plate.

Utilizing a Wild Pug-4 point-transfer device at 24× magnification, the flash plate was examined along the focal plane, and various minute images were resolved, identified, and sketches were drawn for each one. These images were also clearly evidenced on the glass plates produced from the aerial film. In each instance they were within 10 mm of the corners of the focal plane.

The flash plate and glass diapositives were measured on a Space Optic monocomparator with a least count of one micrometer. All glass diapositives were measured five times on each fiducial mark and focal plane images. This particular instrument was equipped with 10× viewing optics, and all the various images were readily identified and measured.

The existing side fiducial marks were used to produce machine coordinates of the principal point X_p, Y_p calculated with two simultaneous equations, allowing determination of coordinates for the corner points which were previously identified as focal plane images on the flash plate:

$$(Y_1 - Y_3)X_p + (X_3 - X_1)Y_p = (X_3Y_1 - X_1Y_3)$$
$$(Y_4 - Y_2)X_p + (X_2 - X_4)Y_p = (X_2Y_4 - X_4Y_2).$$

Each point X_j, Y_j, where $j=1 \cdots 8$, was rotated about the axis of the principal point $X=Y=0$ so that $Y_2=Y_3$. In determining the quantities Δx and Δy that represent the correction due to this rotation, r (the radial distance to each point) retained its value and the slope of the X-axis became zero. As r is a constant only for each corner, the equation for a circle was realized for each point:

$$(X_j \pm \Delta x)^2 + (Y_j \pm \Delta y)^2 = r.$$

The angle of rotation θ was a constant for each point, allowing $r=1 \cdots 4$ as variances, and the length of arc S on a circle is proportional to the central angle θ:

$$\Delta x^2 + \Delta y^2 \sim s^2$$

or explicitly

$$\Delta x_j^2 \pm \Delta y_j^2 = C^2$$

where C is the chord length of the arc S solved by the law of cosines.

The four side fiducials and four focal-plane images were subsequently measured on each plate, along with control and pass points, and the US C&GS program yielded root-mean-square parallax statements between 4 and 7 micrometers. The resulting second degree strip tie points had a root-mean-square error of less than 1.5 feet horizontally at a flying height of 12,000 feet above mean terrain, which is acceptable for the type of mapping to meet contract requirements,[6] (planimetric, 1 inch to 400 feet map scale).

The project was later re-run using images along the focal plane sides as well as the images in the corners. The results showed no apparent loss of accuracy, but no apparent gain was evidenced.

Conclusions

- Acceptable images along the limits of the focal plane do exist and are part of the camera frame, which can be used in lieu of fiducial marks.
- Any pattern of the marks is acceptable.
- Four fiducials are necessary to define the principal point.
- The radii are assumed to be vectors which are solely rotated.
- Although the image deformation of points along the focal plane, which may be over 157 mm from the center of the photo, may not account for local deformations within the format,[7] the results with this technique were better than utilizing a four side fiducial routine.
- The curl S is a vector which is invariant under a rotation within the confines of the given coordinate system defining $X_p=Y_p=0$, controlled by the angle of rotation and the actual radial distance as constants.
- Application of a priori weight factors[4,5] for the different reliabilities of the side fiducial marks should hopefully be an included part of the program updating. This weighting could be inversely proportional to the squares of the standard errors (e) as pertains to the deviation of a number of film samples as compared to the absolute values determined from several flash plates.

If we arrive at a standard error of unit weight e_o for the four fiducials, we have:

$$e_o = [\sum v^2 p/n]^{\frac{1}{2}}$$

where v^2 is the sum of squares of the residuals, p the weight of unity (arbitrary value), and n is the number of observation equations.

This will determine an average standard error of unit weight and this can be used as a ratio of e to e_o to determine a weight number $\sqrt{p}$ to represent the relative reliability, said weight number being inversely proportional to the squares of the standard errors.

However, this weighting can be done if only the side middle fiducials are considered for the transformations.

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