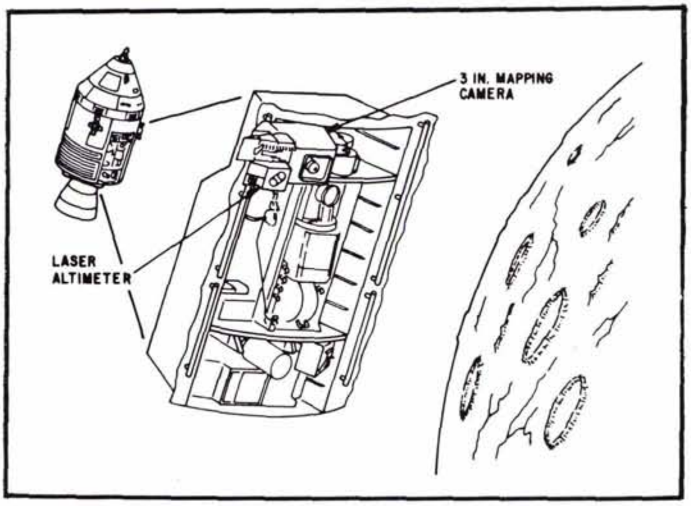

FRONTISPIECE. Apollo mapping camera system.

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Altimeter Observations as Orbital Constraints

Astro-photogrammetric triangulation is expected to improve significantly in accuracy.

(*Abstract on page 340*)

NASA's APOLLO Program plans to orbit an advanced *M*apping *C*amera *S*ystem (MCS) on Missions 15, 16 & 17 to the moon. Basically, the MCS is composed of a stellar camera, a metric mapping camera and a laser altimeter to provide a distance D to the surface where the optical axis of the mapping camera is pointing. The altimeter observation is syncronized to fire simultaneously with the exposure of both the stellar and mapping camera. The result is a stellar and metric camera exposure and a D to the ground for each time t_i. The purpose of this paper is: to show a mathematical approach for utilizing the altimeter observation D in a photogrammetric triangulation program which also employs orbital constraints (it is proposed to modify the well known Lunar Orbiter Strip/Block Triangulation Programs (LOSAT/LOBAT) to accept this observation according to the following concept), to derive the equations for determining latitude, longitude and height (ϕ, λ, h) on the ellipsoid from the altimeter distance and its associated orientation as reduced from the stellar photographs, and to show the utility of utilizing the derived position on the ellipsoid as an additional constraint for multiple ray direction cosine observations. In a geometrical sense, this second purpose will permit the entry of this derived ϕ, λ, h as a ground tracking station on the moon. Therefore, the precision of the laser altimeter (± 2 meters) can be

ABSTRACT: *Lunar satellite geodesy utilizing the Apollo Mapping Camera System can be expected to make significant improvements over previous lunar control point networks. It seems widely recognized that satellite altimetry will contribute significantly to these improvements. However, the literature on the subject seems sparse and also few of the well-known computer programs accept altimetry data and mapping camera data such as that to be collected on the Apollo Missions. This paper introduces some concepts for employing these data in programs which do not accept altimeter observations directly, and also it illustrates a rigorous, direct approach. The concept of constraining several common photo image-point measurements from different camera stations to intersect at the point measured by the altimeter is expected to be a very strong constraint to the overall triangulation and orbit determination applications.*

utilized in the overall orbit determination problem. Although we must recognize the error propagated to the ground position from the spacecraft, this can be an especially effective way to utilize the altimeter information where the computer program does not accept a Pythagorean type altimeter observation equation directly.

Let the altimeter distance D, as illustrated by Figure 1, be represented by a Pythagorean relationship as

$$D^2 = (X^c - X^a)^2 + (Y^c - Y^a)^2 + (Z^c - Z^a)^2 \tag{1}$$

where X^a, Y^a, Z^a are the moon-centered rectangular coordinates of the *point on the ground* where the altimeter distance D intersects the surface, X^c, Y^c, Z^c are the moon-centered rectangular coordinates of exposure station position in space, and D is the measured slant distance of altimeter to the lunar surface, aligned with mapping camera's optical axis.

Transposing and taking the square root of Equation 1, we have

$$F = D - [(X^c - X^a)^2 + (Y^c - Y^a)^2 + (Z^c - Z^a)^2]^{\frac{1}{2}} = 0. \tag{2}$$

For the purpose of including observations of Equation 2 in the least-squares

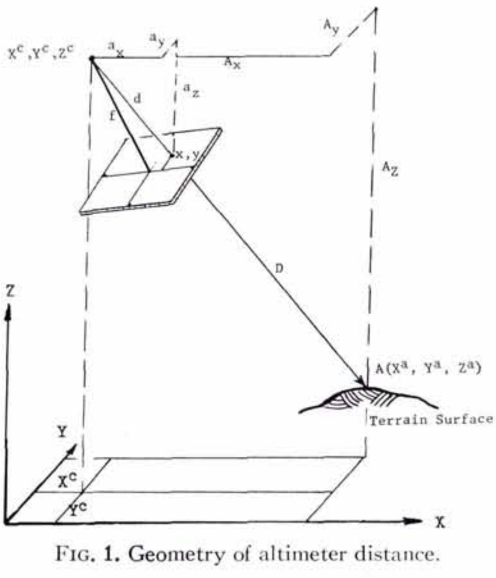

FIG. 1. Geometry of altimeter distance.

adjustment, it is necessary to expand by a Taylor's series about approximate values of its seven parameters as

$$F^0 + \frac{\partial F}{\partial D} dD + \frac{\partial F}{\partial X^a} dX^a + \frac{\partial F}{\partial Y^a} dY^a + \frac{\partial F}{\partial Z^a} dZ^a + \frac{\partial F}{\partial X^c} dX^c + \frac{\partial F}{\partial Y^c} dY^c + \frac{\partial F}{\partial Z^c} dZ^c = 0$$

where, $\partial F/\partial D = 1$. In the absence of ground control, D_i could be considered as a constant; then $dD = 0$, and D would not contribute another unknown per photo. In general, the linearized altimeter observation equation has the form

$$F^0 + dD + \frac{(X^c - X^a)}{D} dX^a + \frac{(Y^c - Y^a)}{D} dY^a + \frac{(Z^c - Z^a)}{D} dZ^a - \frac{(X^c - X^a)}{D} dX^c - \frac{(Y^c - Y^a)}{D} dY^c - \frac{(Z^c - Z^a)}{D} dZ^c = V_D \quad (3)$$

F^0 is the measured D minus the approximated D or, in other words, it is Equation 2 evaluated with the best estimates. This difference is the constant term in the linear form of each observation equation. V_D is the residual error in D after adjustment because, in the method of least squares, observation equations are never satisfied exactly due to random error in the measurement. The altimeter distance D in Equation 2 is an observed value subject to an error. It offers some interesting possibilities which can be shown by the following equation which relates the true distance to the measured distance:

$$D_i = D^0{}_i + dD + V_{D_i}$$

where D_i is the true distance, $D^0{}_i$ is the measured altimeter distance for each i-th photo, dD is a bias factor (one per each mission) and V_{D_i} is the residual for each $D^0{}_i$. If the photo object space contains accurate control points and/or if time t for each D is known precisely, then dD can be considered as a bias factor and one dD can be determined for each mission. In addition, a residual V_{D_i} for each $D^0{}_i$ can be determined. This technique is often called *self calibration*. On the other hand, $D^0{}_i$ may be considered as an observed distance subject only to random error V_{D_i}. The choice depends on the geometric quality and distribution of the total set of observations contained in the simultaneous adjustment scheme. The latter application seems most appropriate for Apollo largely because the moon's surface has no surveyed control points to help define dD, then $dD = 0$.

From astrodynamics,[1] the exposure stations X^c, Y^c, Z^c can be given as functions of the six Keplerian orbital elements a, e, i, Ω, ω, t_i or as position and velocity components. This is where orbital constraints are of potential value, for if X_0, Y_0, Z_0, $\dot{X}_0$, $\dot{Y}_0$, $\dot{Z}_0$ denote the position and velocity (state vector) of the spacecraft (exposure station) at some arbitrary epoch t_0, we may write in principle

$$\begin{bmatrix} X^c \\ Y^c \\ Z^c \end{bmatrix}_i = g_1[X_0, Y_0, Z_0, \dot{X}_0, \dot{Y}_0, \dot{Z}_0, t_i, C_{nm}, S_{nm}]. \quad (4)$$

This is by virtue of a knowledge of the differential equations of motion. From this, one can compute X^c, Y^c, Z^c coordinates at any time t_i provided the initial conditions X_0, Y_0, Z_0, $\dot{X}_0$, $\dot{Y}_0$, $\dot{Z}_0$ at time t_0 are also known. In the LOSAT/LOBAT Programs,[2] the initial state vector must have been previously determined by an orbit reduction and ephemeris computation such as that provided by NASA from the Apollo tracking data. The C_{nm} and S_{nm} terms of the gravity potential function are considered as constants in the LOSAT/LOBAT program.

The projective equations of photogrammetry are fundamental to the formulation of the general triangulation problem. They are given by Schmid[3] and Doyle[4] and can be functionally represented as follows.

$$\begin{bmatrix} x \\ y \end{bmatrix}_{i,j} = F_1[x_{pi}, y_{pi}, f_i, (\phi, \omega, K)_i, (X^c, Y^c, Z^c)_i, (X, Y, Z)_j] \tag{5a}$$

where $(x, y)_{ij}$ is the distortion-corrected j-th photo image point measurement on i-th photo, x_{pi}, y_{pi}, f_i are the calibrated photo coordinates of the principal point and principal distance of the i-th photo, usually considered constant for one camera, ϕ_i, ω_i, K_i are the orientation angles of i-th photo in the moon-fixed system, X_i^c, Y_i^c, Z_i^c are the object space coordinates of each i-th photo on the orbit, and X_j, Y_j, Z_j are the object space coordinates of the j-th point on the lunar surface.

The linearized form of Equation 5 contains in the matrix of partial derivatives $[\boldsymbol{B}]$, the three attitude angles and each of the partials for the six coordinates of Equation 2. Therefore it is expedient to substitute Equation 4 into Equation 5a and rewrite 5a as

$$\begin{bmatrix} x \\ y \end{bmatrix}_{i,j} = F_2[(\phi, \omega, K, t)_i, (X_0, Y_0, Z_0, X_0, Y_0, Z_0, C_{nm}, S_{nm}), (X, Y, Z)_j]. \tag{5b}$$

The concept here constrains the photo position X^c, Y^c, Z^c to be on the orbit as defined by the state vector. Note that by the chain rule from the calculus, one can derive the partials of each unknown parameter to obtain the matrix of partial derivatives (called $\boldsymbol{B}$ in Ref. 4) for Equation 5b. Six orbital elements and t_i for each photo replace $3n$ points of X^c, Y^c, Z^c. The term n is the number of photo stations.

From Equations 5a and 5b we have

$$[x, y]^T{}_{i,j} = F_1 = F_2$$

and then $d(F_2) = d(F_1)$. As F_1 is a function of X^c, Y^c, Z^c, the chain rule gives the partial derivative of F_2 as:

$$d(F_2) = \frac{\partial\{F_1\}}{\partial\{X^c, Y^c, Z^c\}} \times \frac{\partial\{X^c, Y^c, Z^c\}}{\partial\{X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}, t\}} [dX, dY, dZ, dX, dY, dZ, dt]^T \tag{5c}$$

If Equation 5c is evaluated, we have the partials for Equation 5b as

$$\frac{\partial F_2}{\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}, t)}.$$

If Equations 3 and 5b are used in a simultaneous adjustment, then the parameters $(dX^c, dY^c, dZ^c)^T$ in Equation 3 must be replaced by Equation 5C. Note that the time of each exposure t_i locates the exposure station position on the orbit.

The 6×6 matrix in the normal equations representing the state vector will provide linear corrections to the state vector for each iteration. Another row and column should be added for time t_i. Since we have a t_i for each photo, it is convenient to associate time with the set of orientation angles as in Equation 5b. This matrix is located in the border of the normal-equation matrix so the terms may be partitioned efficiently. The typical form of the normals is shown in the LOSAT/LOBAT report.[2] With D as a constant, the six parameters of Equation 1 do not add any additional unknowns to the normals; moreover, the origin of D is constrained to the orbit trajec-

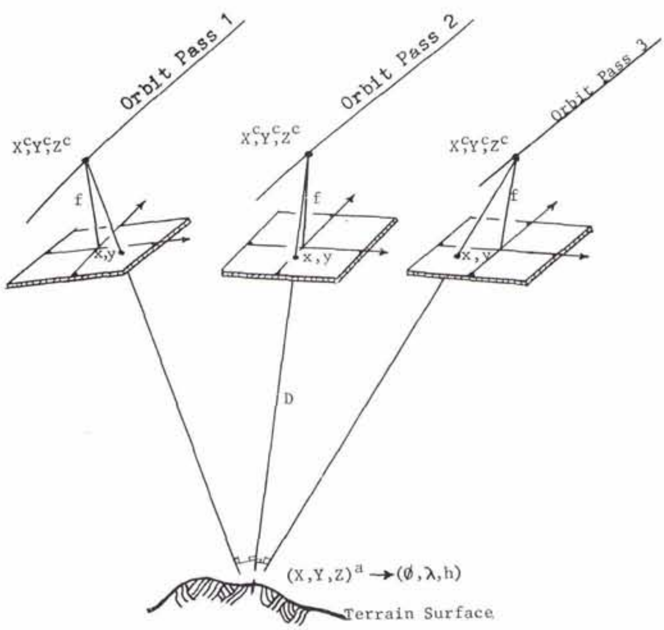

FIG. 2. Altimeter distance and multi-ray observations.

tory. This concept should be associated with conjugate image points to be of maximum effectiveness as a constraint. Figure 2 illustrates conjugate x, y image measurements.

Concerning the second purpose of this paper, the item of particular importance is how do we obtain object space ground coordinates for the terminus of the altimeter distance at the surface. Referring to Figure 1, this point is called X^a, Y^a, Z^a and the geometry of the altimeter is given. The image point x, y near the optical axis is furnished by boresight calibration for each camera and is x_a, y_a in the remaining equations.

If X^c, Y^c, Z^c are given from the ephemeris for each photo, x_a, y_a, f are plate coordinates of the altimeter point image as given by previous calibration and the principal distance of the camera. The matrix

$$[\boldsymbol{T}_M] = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

is the attitude matrix, "object space-to-photo" for the i-th terrain photo. Determined from Stellar photo reduction for angles such as ω, ϕ, K. $\boldsymbol{T}_M$ is in the True Moon-Centered Coordinate System of date.

The terms X^a, Y^a, Z^a (which are ground coordinates of the altimeter distance where the altimeter distance intersects the terrain surface at point A) are required.

Let

$$\boldsymbol{a} = \begin{bmatrix} x_a \\ y_a \\ -f \end{bmatrix}_{3,1}$$

with respect to camera system coordinates, (Figure 1). Then

$$\boldsymbol{a}' = [\boldsymbol{T}_M]^t \boldsymbol{a} = \begin{bmatrix} a_x' \\ a_y' \\ a_z' \end{bmatrix}_{3,1}$$

The length of $\boldsymbol{a}$ is:

$$\boldsymbol{a}' = (a_x'^2 + a_y'^2 + a_z'^2)^{\frac{1}{2}} = (x_a^2 + y_a^2 + f^2)^{\frac{1}{2}} = d.$$

Now the components of $\boldsymbol{A}$ as illustrated in Figure 1 are:

$$A_X = \frac{D}{d} a_x' = X^a - X^c$$

$$A_Y = \frac{D}{d} a_y' = Y^a - Y^c$$

$$A_Z = \frac{D}{d} a_z' = Z^a - Z^c$$

or

$$\begin{bmatrix} X^a \\ Y^a \\ Z^a \end{bmatrix} = \begin{bmatrix} X^c \\ Y^c \\ Z^c \end{bmatrix} + \frac{D}{d} \begin{bmatrix} a_x' \\ a_y' \\ a_z' \end{bmatrix} \tag{6}$$

This completes the derivation of the X^a, Y^a, Z^a ground position of the altimeter terminus in moon-centered rectangular coordinates. It may be necessary to determine a selenographic position ϕ, λ, h of this point on the ellipsoid. By reversing our thinking, such a point on the ellipsoid is analogous to a tracking station located on the body being orbited. It also contains the error propagated through the equations from the spacecraft. It is conventional to enter tracking station positions with azimuth, elevation and range data. The X^a, Y^a, Z^a point-data could be entered, if necessary, in this manner, if the available program would not accept a specific equation such as the Pythagorean relation. However, this is probably not desirable in most programs because the tracking stations are usually part of the parameter set to be differentially corrected and are therefore limited in the number available for useage.

Functionally, the transformation from selenographic to rectangular coordinates is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_a = F_3(\phi, \lambda, h),$$

where a and b are given as the semi major and semi minor axes of the ellipsoid. Specifically,

$$X = (N + h) \cos \phi \cos \lambda$$

$$Y = (N + h) \cos \phi \sin \lambda$$

$$Z = \left(\frac{b^2}{a^2} N + h\right) \sin \phi$$

where N is the radius of curvature normal to the ellipsoid [4,5]. The inverse that we seek is:

$$\begin{bmatrix} \phi \\ \lambda \\ h \end{bmatrix}_a = F_4(X,\ Y,\ Z)_a \qquad (7)$$

(Usually two iterations are required.) The selenographic to selenocentric transformation and its inverse are well documented in the Manual of Photogrammetry by Doyle, p 466, and in Heiskanen[5] and will not be repeated here. When Equation 7 is completed, the altimeter's terminus is positioned on the ellipsoid. This is shown in Figure 3.

The final item in this paper is illustrated in Figure 2. in which the altimeter's terminus is the position where the camera from adjacent orbit passes also photographs the same common ground point. The TRACE Orbit Determination Program, written by the Aerospace Corporation[6] accepts direction cosines from such geometry. The TRACE Program minimizes the miss distance between the various direction cosines from each camera ray. In the planned Apollo application, about 6 to 12 rays can be expected to such a point. These conjugate images will be marked and measured and transferred by conventional photogrammetric procedures. Utilizing these multiple ray direction cosines as observations and having the ϕ, λ, h as determined from Equation 7, it will be possible to tie the adjacent Apollo orbits together and determine a unified control point network. By virtue of the stellar camera and orbit constraints, the network will be related to the right ascension, declination coordinates of the stars and the moon's center of mass. In addition, experimenters who are attempting to make improvements in the gravity and libration models may also benefit accordingly.

In reference to Figure 2, and the photogrammetric concept in general, it should be clearly seen that the image point of the altimeter from each particular exposure, and all exposures, is to be transferred to all overlapping photographs. These points are to be used as common image pass points. The same image appears in several different photographs, but each of them must go to the same place on the ground. The distance D should be constrained in the intersection formulation. Then, the intersection of all rays to a point will occur on or near the ground surface which is the altimeter's terminus position. All parameters, including intersection, are solved simultaneously. This exploits the altimeter's measuring accuracy while tying the different photos together into a unified photogrammetric block. The photogrammetric adjustment will force the conjugate image points to intersect, or in the case of the TRACE program, it will minimize the miss distance from intersection of the several rays. This ties the photos together in a unit block. By virtue of all the photos in the block now being one unit and recreated in space as they were at the time of exposure, the X^c, Y^c, Z^c position of each photo finds its most probable position also. In principle, this movement

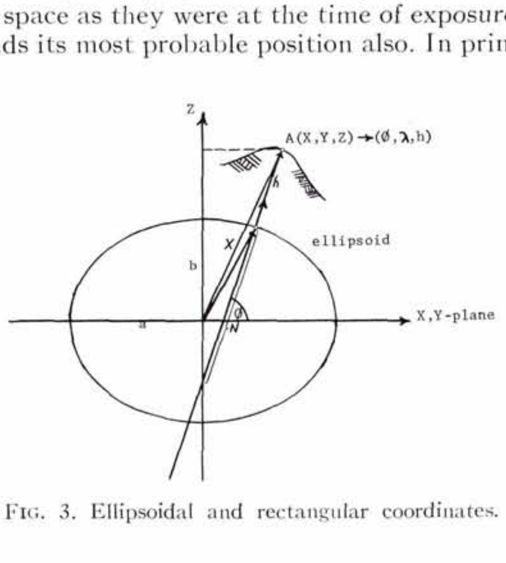

FIG. 3. Ellipsoidal and rectangular coordinates.

can be reflected in small corrections to the gravity coefficients. Clearly, a simultaneous solution, or iteration with improved gravity and libration models, will likewise improve the control point network.

Summary

In summary, three concepts have been treated here. The first shows the usage of the rigorous Pythagorean relationship for representing altimeter observations while forcing the exposure stations to fall on the orbit. The second derives the altimeter's terminus ellipsoidal position which can be used as a control point or tracking station. And, thirdly, shows the usage of the derived position as an additional constraint for surface points which will also have direction cosine rays from several different cameras from the same or adjacent orbits. The concept illustrated by Figure 2 is considered to be a very strong constraint for the problem. It seems obvious that such applications of altimetry will result in more accurate adjustments during the 1970s. Simulations should be conducted to verify and validate these possibilities.

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