PROF. JOSEPH V. CORNACCHIO
State University of New York
Binghamton, New York 13901

Holographic Resolution

Two-point analysis of the effects of phase, coherence and emulsion response.

ABSTRACT: *Some problems involved in the use of classical resolution theory are analyzed to determine resolution limits of the holographic imaging process. Principal emphasis is placed upon the effects of object-phase distribution for the situation where resolution is limited by either emulsion response or finite hologram size. A brief two-point resolution analysis is developed based upon Kelley's model of the photographic process and utilizing the emulsion impulse response determined by R. C. Jones. Attention is drawn to some fundamental differences between the resolution problem for the two cases. Graphs show calculated intensity values for both the two-point distribution and the individual point-images. Included as parameters are the relative phase difference and the separation of the sources, the latter expressed in terms of a reference distance called the Rayleigh separation. The results show that the nonlinear relation between object and image intensity, which is characteristic of a coherent imaging system, has a significant effect on the actual state of resolution, independent of the particular factor limiting the resolution. In particular, the resolution limit derived by use of the Rayleigh criterion may be seriously in error.*

INTRODUCTION

ALTHOUGH PROBLEMS associated with the classical theory of resolving power as applied to conventional optical imagery have been extensively discussed in the literature,[1] some recent studies[2] have employed the classical approach to analyze the resolution limits of holography. However, conclusions drawn from the use of the classical approach raise questions concerning their domain of validity due to the effects of the phase distribution over the test object and the shape of the emulsion impulse response curve.

In view of these considerations (and because expressions for the resolution of a system may survive the assumptions made in the analysis used to obtain them[3]) a more critical study of the two-point resolution problem in holography is presented here. In this work we wish to illustrate the effect of these factors on the analysis of the resolving power of the holographic imaging process. We will be concerned only with the two-point resolution. The more complicated and less well-defined problem of utilizing the results of such two-point analyses in the determination of the resolution of detail of an arbitrary object will not be discussed. See Perrin[4] for a discussion of the factors involved in these considerations.

Two fundamental aspects of holographic imagery enter into the determination of the resolving power of the process and which significantly affect the analysis:

- The image-object intensity relationship is nonlinear and thus the superposition of image intensities, due to separate sources in the test object, is not valid except in certain special cases involving specific object phase distributions.
- The impulse response of the photographic emulsion, which enters the process directly[5] rather than as a final detector for example, yields an impulse response for the imaging process which has a fundamentally different behavior than that of a diffraction-limited conventional optical system, i.e., it is a monotone decreasing function as opposed to the oscillatory behavior characteristic of the latter.

Thus, in the development to follow, we will determine the effect of the phase distribution of the test object on resolution where the holographic imaging system is resolution-limited by the emulsion impulse response and

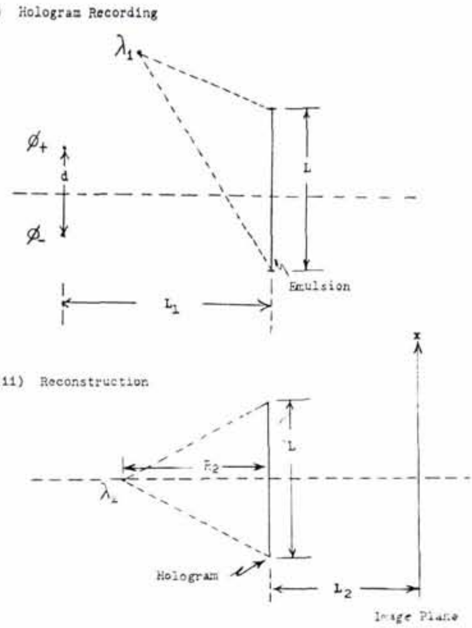

FIG. 1a. The one-dimensional problem of the holographic imaging of two line sources.

finite hologram size. Calculated intensity values will be graphically displayed in terms of parameters defining object spacing and phase difference.

RESOLUTION LIMITED BY EMULSION RESPONSE

THE TWO-POINT RESPONSE

Consider the one-dimensional problem (Figure 1a) of the holographic imaging of two line sources where the construction and reconstruction reference beams are cylindrical waves with arbitrarily located centers and the two sources have equal intensity but different phases given by ϕ_+ and ϕ_-. Let the emulsion have a line-spread function, or impulse response, given by $h(x)$ in Kelley's model[6] for the photographic process as used by Van Ligten[5] and Diamond.[2] If L is the length of the hologram and d is the separation between lines symmetrically located with respect to the hologram center, then the real-image amplitude is given as a function of position x in the image plane by

$$F(x, d) = Ce^{i\phi(x)}\left[e^{i\phi_+}(y_+) + e^{i\phi_-}h(y_-)\right] \quad (1)$$

where

$$y_\pm = \frac{x \pm (d/2)M}{M_2} + Y$$

$$M_2 = 1 + L_2/R_2$$

$$M = M_2/(1 - L_1/R_1).$$

L_1 and L_2 are the object- and image-plane distances, respectively, and R_1 and R_2 are perpendicular distances to the construction and reconstruction point sources respectively. All distances are measured relative to the hologram plane. C, Y and ϕ are independent of d and the phases ϕ_+ and ϕ_-, and are fixed for a given transverse position of the construction and reconstruction sources, and for fixed construction and reconstruction wavelengths λ_1 and λ_2.

This expression is valid if the impulse response of the emulsion is the limiting factor in the imaging process and is obtained by taking the solution for the amplitude of the line-image,[2] including an arbitrary source phase term and then applying superposition to the *amplitudes* to determine the amplitude reconstructed from the two line sources.

Note that the problem is linear in the amplitude because of a further assumption that the overall photographic response is linear—i.e., that the amplitude-transmittance of the emulsion is a linear functional of the incident exposure[5]—and thus the superposition of amplitudes is justified. However, unless this additional assumption is made, the amplitudes cannot be superposed because, as has been pointed out[5,6], the overall photographic problem involves the point-wise nonlinearity of the emulsion sensitometric response.

The real-image intensity $I(x, d)$ is given in terms of $F(x, d)$, Equation 1, by

$$I(x, d) = F(x, d)F^*(x, d) \quad (2)$$

where F^* indicates the complex conjugate. Thus

$$I(x, d) = |C|^2[h^2(y_+) + h^2(y_-) + 2h(y_+)h(y_-) \cos \Delta\phi] \quad (3)$$

where $\Delta\phi \equiv \phi_+ - \phi_-$ and we have made use of the fact that $h(y_\pm)$ is real.

The resolution of the process is now determined by applying the resolution criterion of interest to the behavior of $I(x, d)$ as given by Equation 3.

Before we discuss the resolution analysis in detail, notice that there are three terms on the right-hand side of Equation 3: the first term represents the image intensity due to a single point source* located at $x = d/2$; the second is a similar result for $x = -d/2$; the third term, a cross-product, represents the effect of the phase *difference* between the two sources. The last term, which is the source of ambiguity in the classical analyses, differs in a significant way from the other two terms as it can be positive, zero, or negative depending on the phase difference $\Delta\phi$. If one assumes the process is linear in intensity and computes the real-image intensity accordingly, as is done in the classical or incoherent solution, then the result $I_L(x, d)$ given by

$$I_L(x, d) = |C|^2[h^2(y_+) + h^2(y_-)] \quad (4)$$

and is equivalent to assuming that the phase difference $\Delta\phi$ in Equation 3 is equal to $\pi/2$, $3\pi/2$, etc. The actual image intensity for values of $\Delta\phi$ other than $\pi/2$ will differ from Equation 4 by a contribution which may be either positive or negative as a function of x, and it is clear that this may have a nontrivial effect on the resolution limit if the distance d is small.

Although Equation 3 describes the situation if the impulse response of the film is the dominant resolution limiting factor, it is characteristic of the general form in which the two-point image intensity can be written, independent of the precise nature of the basic resolution limiting factors. Therefore the two-point image intensity can be written as the superposition of the intensity corresponding to each point considered alone with an additional term involving the cross-product of the magnitude of the fields, multiplied by the cosine of the phase difference between the sources. This general property is, of course, a direct consequence of the linearity of the process with respect to the fields. From the preceding discussion it is clear that in terms of the linear systems description of imaging systems[7,8], h is the amplitude impulse response, and h^2 is the intensity impulse response of the two-step holographic imaging process. Caution must be exercised in the use of the term *intensity impulse response* to describe the point-image intensity because the process is obviously not linear in intensity.[9]

To analyze in detail the resolution properties of the process, we take as a model for the point-spread function h, the expression given by Jones,[10], and discussed by Gilmore[11]:

$$h(x) = \frac{2}{\pi a} \frac{1}{1 + 4(x/a)^2} \quad (5)$$

The quantity a is the *width* of the spread function defined as the distance between the points where the response is equal to one-half the value at the center where $x = 0$ (See Figure 1b).

Substituting the functional form of h in Equation 3, we obtain the normalized results shown in Figure 2 plotted as a function of the variable

$$X = \frac{x}{M_2(a/2)} + N$$

where

$$N = 2Y/a.$$

CHARACTERISTICS OF THE POINT-IMAGES

Figure 2a shows the normalized intensities I_+ and I_- due to each of the point sources (located at $d/2$ and $-d/2$, respectively) considered independently, i.e., the first and second terms (normalized) of Equation 3. A parameter F_R is now introduced which specifies the separation d between the sources in terms of a reference distance d_R which is determined as follows: the *width* of the point-image is defined as the distance between locations where the point-image intensity distribution has decreased to one-half its value at the center.† In units of X, as plotted in Figure 2a, this width is given by the constant $2[2^{1/2} - 1]^{1/2}$.

Now d_R is defined to be that value of d such that the separation between the sources is one-half the preceding quantity, i.e.,

$$d_R = (1/2)[2^{1/2} - 1]^{1/2}(M_2/M_1)$$

The significance of this value of the separation, to be called the *Rayleigh Separation*, is as follows. Having defined the width of the

* We shall now use the noun *point* for *line* yielding more usual terminology, such as *two-point resolution* although the problem is one-dimensional and thus the adjective *line* is more precise in referring to the sources.

† Because the point-image intensity distribution involves the *square* of the impulse-response h, the width of the point-image is *less* than the width of h measured between the same levels; this fact has evidently been overlooked in previous studies. Compare W_1 and W_2 in Figure 1a.

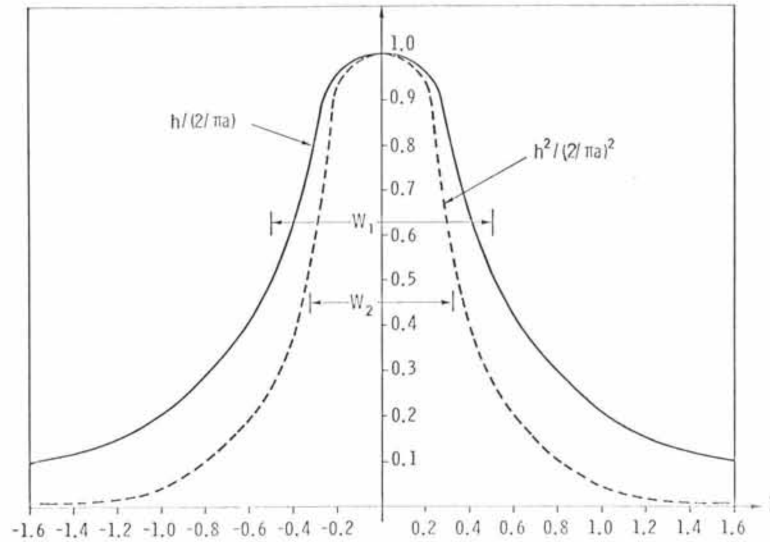

FIG. 1b. Emulsion impulse-response, or line-spread function, and its square for the model given by R. C. Jones. The graph represents the normalized value of h and its square plotted against the normalized distance x/a. W_1 and W_2 are the half-widths of h and h^2, respectively. For the case of holographic imaging with resolution limited by the emulsion, h is also the field impulse-respose of the two-step imaging process and h^2 is the intensity impulse-response.

point-image intensity distribution, d_R represents the resolution limit if one applies the classical theory in the form described by Rayleigh's criterion[12] to this problem. The result for d_R given result differs from $d_{\min}$ of Diamond's work[2] by: (1) the factor of 1/2, which accounts for the fact that the Rayleigh criterion involves one-half of the width of the point-image[12,‡], and (2) the factor $(\sqrt{2}-1)^{1/2}$ which accounts for different values of the width of the point-image and the width of the impulse response h. The parameter F_R is then defined for a fixed d by $d=F_R d_R$, i.e., the distance between the sources expressed in units of the Rayleigh Separation—we shall call the quantity F_R the *Rayleigh Factor*. If $F_R=1$, we have $d=d_R$, i.e., the two sources are separated a distance equal to the Rayleigh Separation and the sources are classically considered resolved. We shall see that, depending on $\Delta\phi$, this may not be the case.

PHASE EFFECTS

Returning to consideration of the real-image intensity distribution (Equation 3), attention will now be directed to characteristics of the total intensity $I(x, d)$, as a function of two parameters: $\Delta\phi$, the phase difference between the two sources, and F_R, the Rayleigh Factor (Figures 2b–2c).

Figure 2b illustrates $I(x, d)$ and $I_{\pm}(x)$ for two sources separated by the Rayleigh Separation d_R, i.e., $F_R=1$. Focusing attention on $I(x, d)$, for values of $\Delta\phi$ equal to 0, $\pi/2$ and π, it is apparent that the curves for $\Delta\phi=0$ and $\pi/2$ exhibit a behavior which is quite different from that for $\Delta\phi=\pi$ because they are monotone decreasing from the center, and therefore represent a situation where the sources are *unresolved*. For $\Delta\phi=\pi$, however, the center of the pattern is the location of a local minimum, i.e., the two sources are *resolved*! Before entering into the details concerning Figure 2b, we would like to stress the significance of the previous observation. At a fixex distance d, equal to the minimum resolvable distance given by the Rayleigh Criterion, the holographic image of two point-sources if limited by the impulse response of the emulsion exhibits different *states* of resolution depending upon the phase difference between them. Thus, the straight-forward application of the classical Rayleigh Criterion to this problem leads to a conclusion which represents an inherently ambiguous situation concerning the actual resolution limit of the system.

The source of this ambiguity can be traced as follows: the Rayleigh criterion involves

‡ The original form of Rayleigh's criterion is actually stated in terns of maxima and minima of the point-imate intensity distribution; if converted to *width* of the image, the reference distance is one-half the width if this is chosen as the distance between the first relative minima on each side of the central maximum.

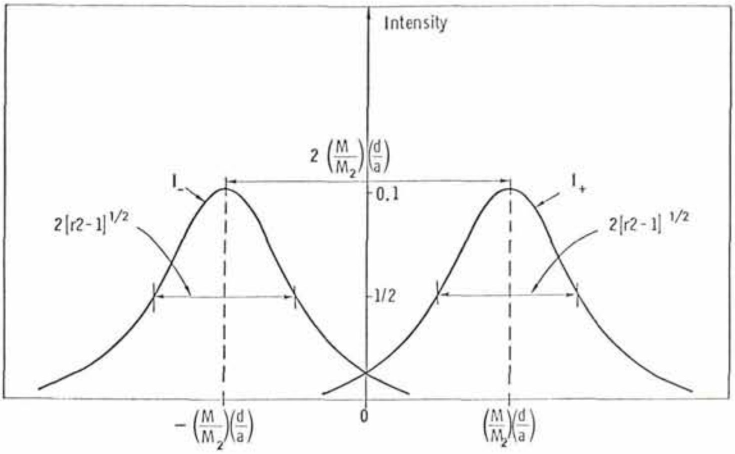

FIG. 2a. Characteristics of normalized point-image intensity distributions I_+ and I_- for the impulse-response limited reconstruction using the model of Jones for the line-spread function h. The location of maxima, separation between maxima, and half-width are indicated.

properties of only the individual point-images $I_\pm$ (such as location of central maxima, relative minima, width, etc.) and is not directly concerned with the total intensity $I(x, d)$. The criterion tacitly assumes that if the point-images, considered independently, are appropriately located relative to each other, then the resultant total intensity will exhibit a central *dip*. This assumption is valid for incoherent fields and classical diffraction-limited optics.[12] In a coherent problem where the phase[13] of the sources becomes important, it is no longer sufficient to consider the individual responses alone because the phase difference will determine the total intensity although it does not affect the point-image intensity distribution (Equation 3). In the coherent case the presence of a central *dip* may be lacking even though the point-images are properly spaced. The problem may be complicated further in the case of holographic imaging because the point-image, if determined primarily by emulsion effects, is monotone, contrary to the situation existing

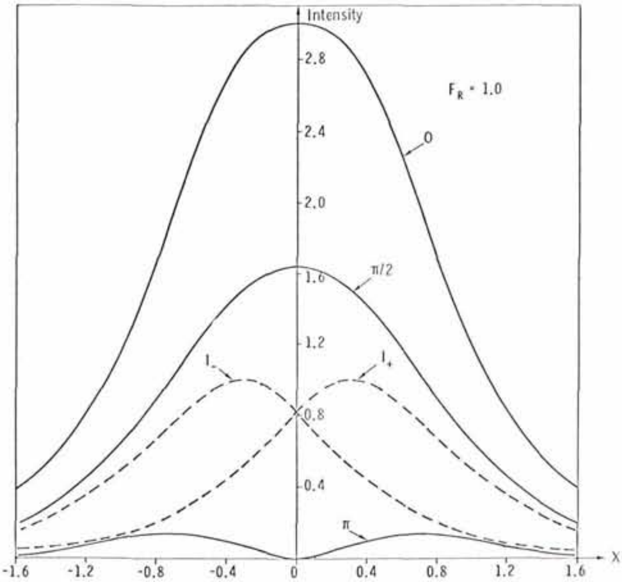

FIG. 2b. Plot of the normalized two-point intensity distribution (See Equation 3) with phase difference $\Delta\phi$, and Rayleigh Factor F_R, as parameters. Values of $\Delta\phi$ label the curves. The dotted curves are the individual point-image distributions.

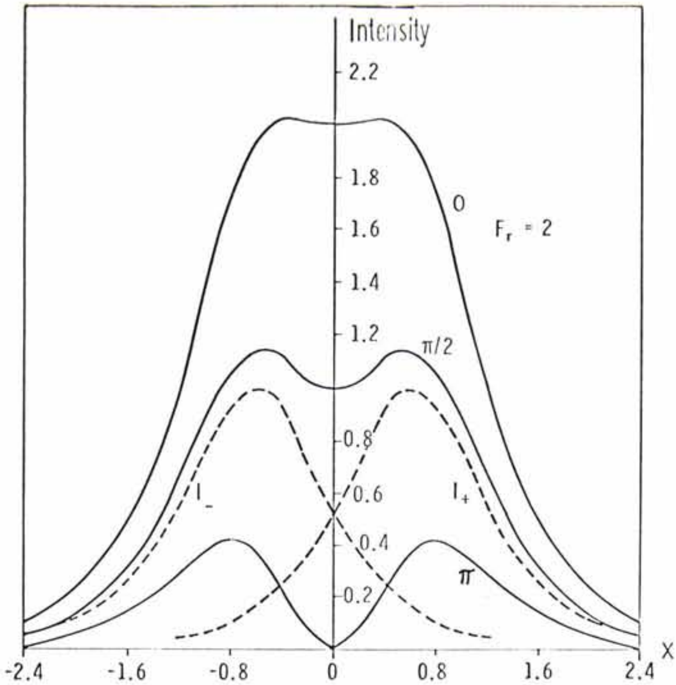

FIG. 2c. Similar to Figure 2b except that $F_R=2$.

in classical diffraction-limited optics.[12]

With respect to some details of the results displayed in Figure 2b, notice that the curves for $\Delta\phi=\pi/2$ and 0 illustrate the fact that the *a priori* choice of the point image is not arbitrary as might be assumed. For instance, consider the case where $\Delta\phi=\pi/2$ where the cross-product term in Equation 3 is zero for all values of x and, as mentioned in a previous section, corresponds to the assumption that linearity prevails, and thus would be equivalent to the incoherent or classical version of the problem. It might then be expected that, at least for this case, the two sources should be resolved; however, they are not. This situation can be attributed to the choice of the point-image width here. If the width is chosen twice the value previously assumed, then the sources would be resolved because, in that case, the resultant intensity distribution is formally equivalent to that shown in Figure 2b. This effect of the *definition* of point-image width on the resolution limit as derived by application of the classical Rayleigh Criterion has apparently been overlooked in those analyses of holographic resolution where the impulse response of the film is the limiting factor.

One can single out a unique phase difference $\Delta\phi_s$ corresponding to the situation where the intensity *just begins* to dip by determining that value of $\Delta\phi$ for which

$$\left.\frac{\partial^2 I(x,d)}{\partial x^2}\right|_{x=0}=0,$$

i.e., by application of the Sparrow criterion.[14] The existence of such a value can be inferred by observing that all values of the response are included in the range $0\leq\Delta\phi\leq\pi$, that at π there is a pronounced dip whereas at 0 there is a central maximum, and that I is a continuous function of $\Delta\phi$.

Notice, however, that it would be incorrect to assert that two sources (characterized only by having equal intensities and separated a distance d_R) would be resolved, because for values of $\Delta\phi<\Delta\phi_s$ the distribution $I(x, d)$ would not display a central relative minimum. The intensity distribution for $\Delta\phi=\pi$ has a zero at the origin. This occurs for any separation d independent of h (assuming symmetry of pattern) due to the assumption that the two sources have equal intensity.

Figures 2c, 2d and 2e (plotted on a compressed scale) illustrate the effect of increasing F_R to 2, 4, and 10 respectively. Observe that at $F_R=2$, $I(x, d)$ shows a local minimum at the origin in all cases. The dip increases as the phase difference increases, and is just discernible (on this scale) for $\Delta\phi=0$, increas-

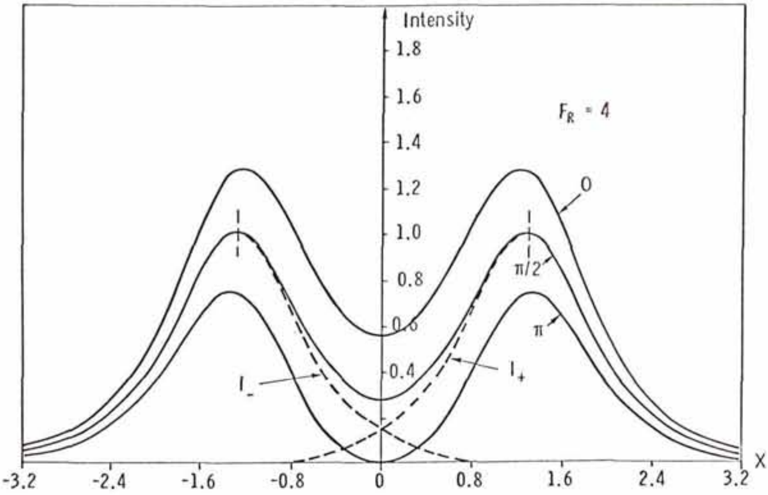

FIG. 2d. Similar to Figure 2b except that $F_R=4$.

ing to a maximum of slightly more than 0.4 at $\Delta\phi=\pi$. If one were to redefine the width of the point-image intensity to be twice that chosen previously, then the separation d corresponding to Figure 2b would be one (new) Rayleigh Separation, say $d'_{R'}$, i.e., the value generated by the classical procedure using d'_R and F_R equal to one, and for this value of d the two sources would be resolved. (See previous discussion concerning the influence of point-image width on resolution determination.) As F_R gets very large, it can be seen that all the curves approach each other, i.e., the effect of phase difference on the reconstructed image becomes negligible and therefore superposition of the individual point-image responses to obtain the total intensity is valid for any phase difference. However, for small values of F_R, which would be the significant ones from a resolution viewpoint, it has been shown that a similar conclusion does not follow.

Finally, notice that for all values of F_R the intensity $I(x, d)$ increases monotonically as $\Delta\phi$ decreases from π to 0 for any fixed value of x; this is due to the fact that h is positive for all values of x (see Equation 3). (This behavior does not occur in the case where resolution is limited by finite size of the hologram as shown previously.) Thus, in effect, although it is possible to resolve the sources at smaller separations where the phase difference is large, the resultant intensity values are low, thereby requiring more sensitivity in detection.

RESOLUTION LIMITED BY FINITE HOLOGRAM SIZE

As stated previously, the analysis of resolution limitation due to finite hologram size shares many features which are present in the resolution problem involving the emulsion impulse response, although the point-image intensity distribution is not monotone decreasing in this case. Consequently, only a brief treatment of the resolution problem for this case will be given in this section.

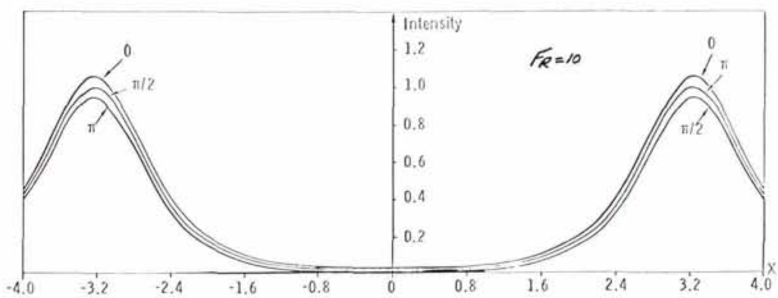

FIG. 2e. Similar to Figure 2b except that $F_R=10$.

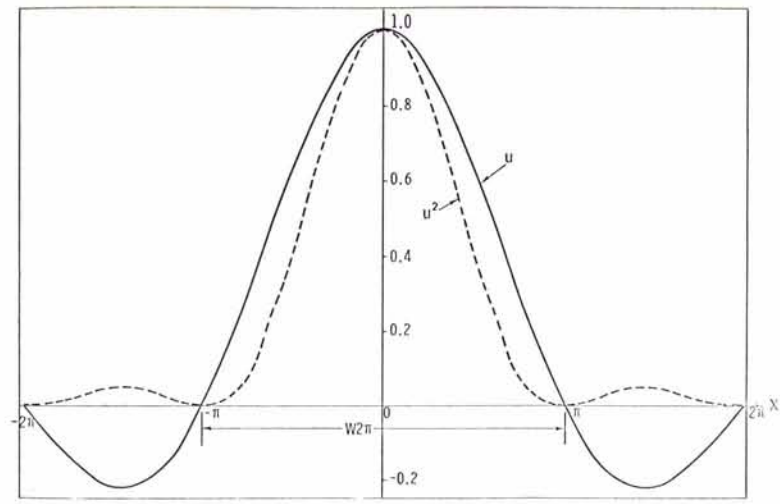

FIG. 3. The field impulse-response $u(z)$ and the intensity impulse-response $u^2(z)$ for the case where resolution is limited by finite hologram size. W is the width of both responses.

THE TWO-POINT RESPONSE

Consider the previously outlined holographic imaging problem under the following assumptions: let $h(x)=\delta(x)$, i.e., assume the modulation transfer function (MTF) of the emulsion is constant for all frequencies; let the hologram be of finite length L, and set $\lambda_2=\lambda_1=\lambda$ for simplicity. Furthermore, let the construction and reconstruction sources recede to infinity in a manner such that their angular displacement, relative to the hologram normal is kept constant, i.e., consider the case where plane waves (at oblique) incidence are used in the construction and reconstruction stages. Also, let L_1 and L_2 become large so that the object and image planes are located at sufficiently large distances for Fraunhofer diffraction to govern the image formation process. Under these assumptions, the reconstructed intensity distribution corresponding to Equations 2 and 3 (normalized here) is given by

$$I(x, d) = u^2(z_+) + u^2(z_-) + 2u(z_+)u(z_-) \cos \Delta\phi \quad (3')$$

with

$$z_\pm \equiv \pi x' \mp \frac{\pi}{2}\left(\frac{L}{L_1}\right)\left(\frac{d}{\lambda}\right), \quad x' \equiv \frac{L}{L_2}\left(\frac{x}{\lambda}\right),$$

where $u^2(z)$ is the point-image intensity distribution and $u(z)$ is given by

$$u(z) = \frac{\sin z}{z}.$$

Equation 3′ is essentially a generalization of the classical, or incoherent, result which forms the basis for the quoted[2] resolution limit of the equivalent diffraction-limited conventional optical system as derived by application of the Rayleigh criterion to Equation 3′, with $\Delta\phi=\pi/2$.[12] The behavior of $u^2(z)$ and $u(z)$ (Figure 3) is to be contrasted with $h^2(x)$ and $h(x)$ (Figure 1b). Notice that u^2 does not decrease monotonically from its central maximum and that u changes sign as a function of z, Equation 3′ is seen to have the same general form as Equation 3: there are three terms, two representing the intensity distribution due to each source considered acting alone, and a cross-term dependent on the phase difference between the two sources and involving the product of the field impulse-responses.

CHARACTERISTICS OF THE POINT-IMAGES

The influence of the parameters L, L_1, L_2, d and λ on the first two terms of Equation 3′ (i.e., I_+ and I_- where $I_\pm=u^2(z_\pm)$) is schematically illustrated in Figure 4a where the responses I_+ and I_- are plotted. In units of x', the distance between the central maxima is given by $(L/L_1)(d/\lambda)$. Notice that the local minima are zeros of intensity and that the distance between the first local minima on each side of the central maximum, defined as the *width* of the point-image, is of constant value 2 (also in units of x'). The width defined here corresponds to that used in the holographic resolution studies of Reference 2, and is the equivalent width suggested by the original form of the Rayleigh criterion[12] applied to diffraction patterns of this type.

It is interesting to observe that the expres-

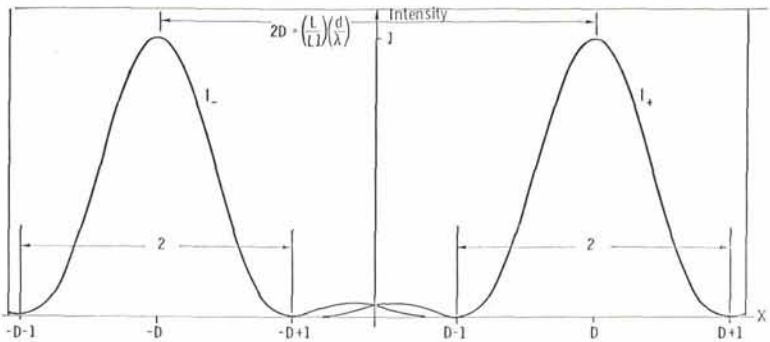

FIG. 4a. Characteristics of the normalized point-image intensity distributions I_+ and I_- for the case where resolution is limited by finite hologram size. For each distribution, the locations of the central maximum, and the adjacent minima are shown, together with the width of the image. Also illustrated is the distance between the central maxima of the individual images $D \equiv (L/L_1)(d/2\lambda)$.

sion for $u^2(z)$ is precisely the same as the one-dimensional, diffraction-limited point-image which is derived in classical diffraction theory, and this fact has probably motivated the (hazardous) use of classical resolution criteria in this problem involving coherent optics. The form of $u^2(z)$ also suggests a *natural* definition of the point-image width such as defined above. We can now introduce the distance d_R, the Rayleigh Separation, as before, i.e., we define d_R to be the value of d at which the separation between the two images, considered independently, is equal to one-half the width as defined above, thus

$$d_R = \lambda L_1/L,$$

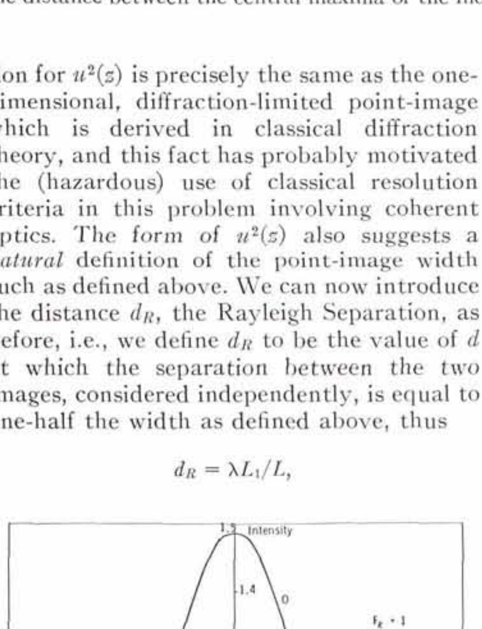

FIG. 4b. Plot of the normalized two-point intensity distribution (*Equation 3′*) with phase difference $\Delta\phi$ and Rayleigh Factor F_R as parameters. Dotted curves are point-image distributions.

and the distance d will be given in terms of d_R by $d = F_R d_R$ such that when $F_R = 1$, the sources are at the Rayleigh Separation.

PHASE EFFECTS

We now consider the intensity distribution described by Equation 3′ for fixed F_R with the phase difference $\Delta\phi$ a parameter as shown in Figures 4b–4e.†

In Fig 4*b*, $F_R = 1$, i.e., the sources are at the Rayleigh Separation d_R. Notice that for $\Delta\phi = \pi/2$, the resultant intensity distribution shows a dip or local minimum at the origin with the ratio of minimum to adjacent maximum intensity given by approximately 0.8. Thus the assumption that the sources are resolved when the Rayleigh Criterion is satisfied[2] is justified in this case. However, for the case where $\Delta\phi = 0$ it is seen that the two-point image distribution does not display a dip at the center, i.e., the two lines are not resolved for $F_R = 1$. For $\Delta\phi = \pi$ the pattern shows the characteristic previously discussed i.e., the local minimum at the center is a zero of the intensity pattern. This is true for all values of F_R and again occurs because the two sources have equal intensity, yield symmetric impulse responses and are symmetrically located with respect to the center of the hologram; thus the analysis of the preceding section also holds here. Because the central intensity minimum is zero, we again find that for $\Delta\phi = \pi$ the two lines are resolved for any finite F_R.

The results displayed in Figure 4b, i.e., for $F_R = 1$, suggest then that the value

† Note that by defining d as above, the behavior of I as a function of x' is determined completely if F_R and $\Delta\phi$ are specified. In this case $(L/L_1)(d/\lambda) = 2D = F_R$.

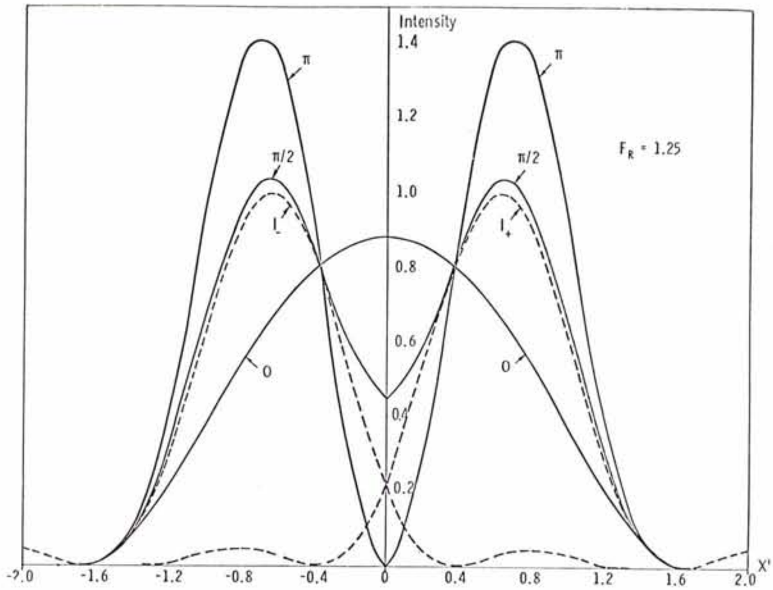

FIG. 4c. Similar to Figure 4b except that $F_R = 1.25$.

$$d = d_R = \lambda L_1/L$$

for the holographic resolution limit due to finite hologram size as derived by the classical approach[2] is not valid unless a further assumption is made concerning the phase difference between the sources.

The results given in Figures 4c–4e show that for phase differences of $\pi/2$ and π radians, the sources are resolved for all $F_R > 1$, whereas for $\Delta\phi = 0$, resolution occurs somewhere in the range

$$1.2 < F_R < 1.5.$$

Finally, it can be seen from Figure 4e that for F_R large, the curves for all values of $\Delta\phi$ approach each other and, as in the previous case of impulse-response limited resolution, the phase difference is not a significant factor.

The curves shown in this section are similar to results given by Carswell and Richard[15] for the microwave region concerning the resolving power of ideal circularly symmetric imaging systems. The similarity of the results

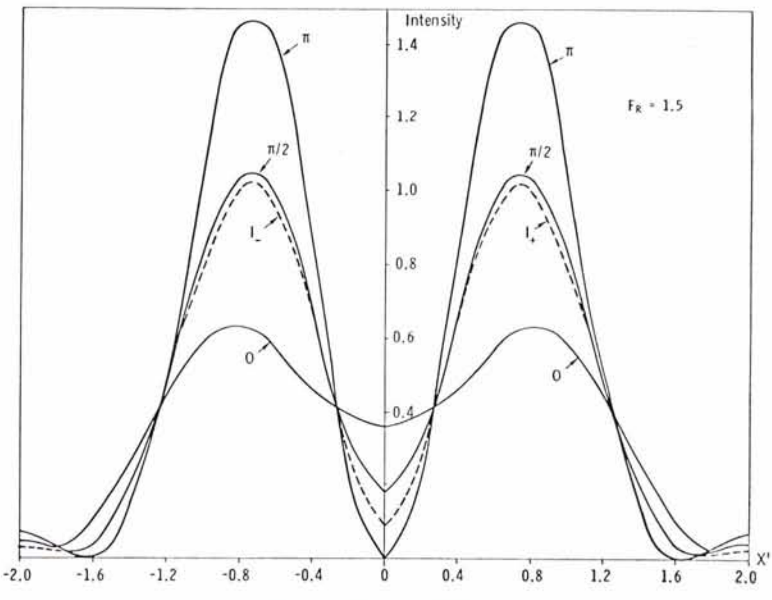

FIG. 4d. Similar to Figure 4b except that $F_R = 1.5$.

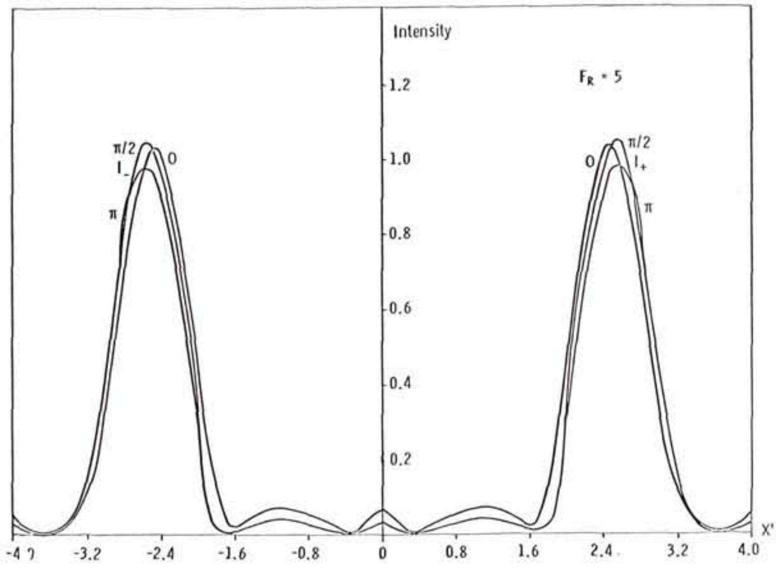

FIG. 4e. Similar to Figure 4b except that $F_R = 5$.

is due to the assumption that the emulsion spread-function in the linear photographic response model[6] is a delta function. Under this condition, the expression yielding the real image field in the case of holographic imaging is equivalent to that obtained for a diffraction-limited conventional optical imaging system. In the problem discussed here, we have a one-dimensional example of this fact. Some of the interpretations and conclusions reported in this section are also discussed by Carswell and Richard.[15]

CONCLUSION

Due to the influence of the shape of the point-image intensity distribution and the phase difference between the sources on the actual state of resolution in a holographic imaging system, the indiscriminate use of classical two-point resolution criteria, e.g., the Rayleigh Criterion, renders the results obtained by application of such criteria subject to questions of validity. The resolution limits so derived, depending on the phase difference involved, can be in error in either direction, i.e., they may be extremely conservative and thus underestimate the potential of the imaging system or may be physically unattainable.

REFERENCES

1. H. Osterberg, *J. Opt. Soc. Am.*, 40, 295 (1950); F. H. Perrin, *J. Society of Motion Picture and Television Engineers*, *69*, 151 (1960); R. Barakat, *J. Opt. Soc. Am.*, *52*, 276 (1962); G. diFrancia, *J. Opt. Soc. Am.*, *45*, 497 (1955); M. Beran and G. B. Parrent, Jr., *Theory of Partial Coherence*, Chap. 8 (1964), Prentice-Hall, Inc., New Jersey.
2. For example, J. A. Armstrong, *IBM Journal of R&D*, *9*, 171 (1965); F. I. Diamond, *J. Opt. Soc. Am.*, *57*, 503 (1967).
3. K. Preston, Jr., *Phot. Sci. Eng.*, *11*, 190 (1967).
4. Perrin, *op. cit.*
5. R. F. VanLigten, *J. Opt. Soc. Am.*, *56*, 1 (1966).
6. D. Kelley, *J. Opt. Soc. Am.* *50*, 269 (1960).
7. E. L. O'Neill, *Introduction of Statistical Optics* (Addison-Wesley Publishing Co. Inc., Reading, Mass., 1963).
8. J. V. Cornacchio, *J. Opt. Soc. Am.*, *57*, 1325 (1967).
9. M. J. Beran and G. B. Parrent, Jr., *Theory of Partial Coherence*, (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1964).
10. R. C. Jones, *J. Opt. Soc. Am.*, *48*, 934 (1958).
11. H. F. Gilmore, *J. Opt. Soc. Am.*, *57*, 75 (1966).
12. M. Born and E. Wolf, Principles of Optics, 3rd Edition (Pergamon Press, New York, 1965), Section 7.6.3.
13. J. V. Cornacchio and R. P. Soni, 11 *Nuovo Cimento*, *38*, 1169 (1965).
14. R. Barakat, *J. Opt. Soc. Am.*, *52*, 276 (1960).
15. A. I. Carswell and C. Richard, *Appl. Opt.* *4*, 1329 (1965).

Erratum

In connection with the article, "Underwater Mapping with Photography and Sonar" by Joseph Pollio (page 955, September 1971), it should have been noted that the paper had been published previously in the U. S. Naval Oceanographic Office *Special Publication 153* under the title, "Manned Submersibles and Underwater Surveying."