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Economics of Image Deformation Correction

A reseau is more effective in reducing image deformation if each image point is corrected individually using the nearest reseau point or, better still, the four surrounding reseau points.

INTRODUCTION

RESEAU CAMERAS have been used for aerial photography for more than two decades. However, only a few investigations comparing results obtained both with and without the use of the reseau for image deformation correction have been published.[2,3] These results indicate only a small increase in the accuracy due to the use of reseau corrections of adjusted coordinates as judged by the *rms* positional error.

ABSTRACT: *Based on a review of the effectiveness of various methods for the correction of image deformation, an analysis of mensuration and computational effort has been undertaken in view of the use of reseau corrections. Reseau corrections performed individually for each point are more effective than methods correcting all points of one photograph simultaneously. These methods are also less demanding in computations. Because a dense reseau permits a control over the maximum image deformation, the photographic scale can be selected under the assumption that the residual image deformation is approximately normally distributed. Hence, one can select a significantly smaller photoscale which will result in a reduction of the number of points to be measured for a project.*

A reseau which is part of the camera lens is superimposed on the image, therefore, at the moment of the exposure of the original negative. Such a reseau fulfills an additional function which often is not emphasized enough: it not only makes it possible to check the functioning of the aerial camera and all processing and copying procedures, i.e. all sources of image deformation,[6] but it can also be used in analytical procedures to reduce the image deformation to an extent determined by its point density. Thus, a dense centrally projected reseau guarantees the usability for analytical procedures of any image, regardless of the extent of its deformation, and it allows the prediction of *rms* and maximum errors with a very high probability.

In Canada, specifications for aerial triangulation give a maximum rather than *rms* positional error. It is this fact which has led to the considerations presented in this paper.

THE CORRECTION OF IMAGE DEFORMATION

A review of a number of methods for the correction of image deformation is given in Reference 5. Methods based on four or eight fiducial marks were further discussed in Reference 4. In this article the photographs used in Reference 4 will be used to present average values for the effectiveness of methods for the correction of the image deformation. Also, the same Transformations 1 to 6 are used again[4]:

$$X = a_0 + a_1x - b_1y \qquad Y = b_0 + b_1x + a_1y \tag{1}$$

$$X = a_0 + a_1x + a_2y \qquad Y = b_0 + b_1x + b_2y \tag{2}$$

$$X = (a_0 + a_1x + a_2y) / (1 + c_1x + c_2y) \qquad Y = (b_0 + b_1x + b_2y) / (1 + c_1x + c_2y) \tag{3}$$

$$X = a_0 + a_1x + a_2y \qquad + a_4xy \tag{4}$$

* Presented at the Image Deformation Conference, June 1971, Ottawa, Canada.

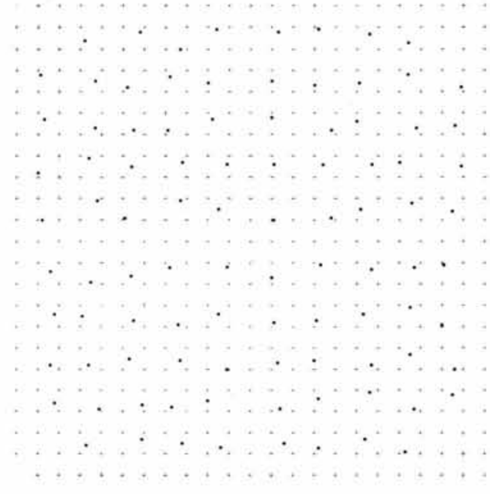

FIG. 1 Distribution of the 96 fictitious image points

$$Y=b_0+b_1x+b_2y \qquad +b_4xy$$

$$\begin{aligned} X&=a_0+a_1x+a_2y+a_3x^2+a_4xy+a_5y^2\\ Y&=b_0+b_1x+b_2y+b_3x^2+b_4xy+b_5y^2 \end{aligned} \tag{5}$$

$$\begin{aligned} X&=a_0+a_1x+a_2y+a_3x^2+a_4xy+a_5y^2+a_7x^2y+a_8xy^2\\ Y&=b_0+b_1x+b_2y+b_3x^2+b_4xy+b_5y^2+b_7x^2y+b_8xy^2, \end{aligned} \tag{6}$$

and supplemented by:

$$\begin{aligned} X&=a_0+a_1x+a_2y+a_3x^2+a_4xy+a_5y^2+a_6x^3+a_7x^2y\\ &\quad+x_8xy^2+a_9y^3\\ Y&=b_0+b_1x+b_2y+b_3x^2+b_4xy+b_5y^2+a_6x^3+b_7x^2y\\ &\quad+b_8xy^2+b_9y^3 \end{aligned} \tag{7}$$

In order to have image points not coincident with reseau points, it was decided to add 96 fictitious image points with positions indicated in Figure 1 to each aerial photograph using Transformation 4 and the four reseau points surrounding the image point. The computed coordinates of these image points were used as their measured coordinates. These coordinates do not contain any additional error so that an individual point correction based on the four surrounding reseau points of the 1-cm reseau and Equation 4 seems to eliminate all image deformation on these 96 image points (see Figure 2, Transformation 4 based on 1-cm reseau). At present, the material is reprocessed using computed image-point coordinates containing in addition normally distributed errors.

The effectiveness of the Transformations 1 to 7 applied simultaneously to all points of reseaus of different density is indicated in Figure 2, where average values for all photographs, computed with the same procedure as in Reference 4, but using the 96 image points indicated in Figure 1, are presented. These figures show that a significant reduction in the image deformation is obtained if additional terms are added to the transformation. The addition of further terms to Equation 7 will lead to only insignificant further improvement according to a preliminary, so far unpublished, investigation. Figure 2 also shows that the *rms* positional error is reduced by less than 10 percent if one uses the 2-cm reseau instead of the 5-cm reseau, and the 1-cm reseau instead of the 2-cm reseau.

A reseau can be used more effectively if for each image point an individual transformation is performed, based on only a few reseau points, or on only one reseau point. Results obtained using reseaus of different density and Transformations 1 to 4 based on the four reseau points surrounding an image point are presented in Figure 3. In addition, the image points were also corrected using always the reseau point located to the left and below the image point *BL* and the reseau point nearest to the image point *N*. In these situations, the image deformation present at the reseau point was corrected at the image point.

Other than in Figure 2, the denser reseau leads to significant improvement of the correction of the image deformation in Figure 3. This statement is likely to remain true after the introduction of random errors to the image coordinates, because the difference is caused by the fact that the individually derived corrections are significantly more effec-

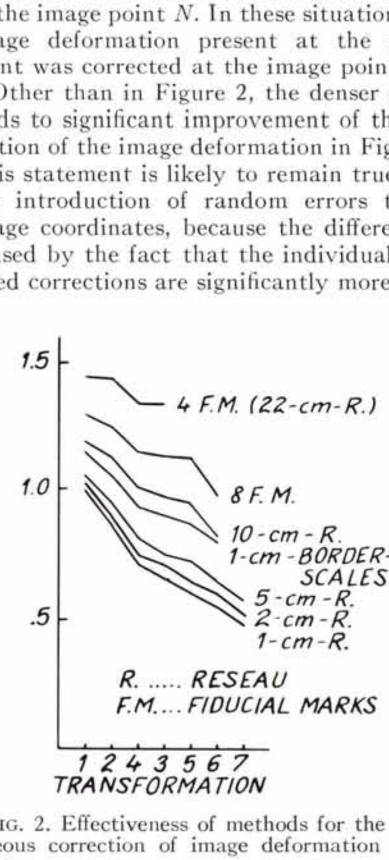

FIG. 2. Effectiveness of methods for the simultaneous correction of image deformation on all points.

tive in reducing local large deformations than are corrections derived from simultaneous transformations based on a large number of points. Figure 3 also indicates that Transformations 1 to 4 do not significantly differ in their suitability. All four are, as had to be expected, more effective than the correction based on one point only. However, the difference in effectiveness is reduced as the reseau becomes denser.

Test Block for Evaluation of the Economy

After comparing the effectiveness of different methods for the correction of image deformation, the effort necessary to accomplish corrections will now be investigated. Considering the fact that accuracy in photogrammetric operations is particularly critical in large-scale work, the author approached the National Capital Commission (NCC) in Ottawa (which undertakes extensive mapping in the national capital area) for specifications regarding analytical aerotriangulation for the determination of the orientation points for each model. The NCC specifications are:

Map Scale	1:2500
Size of map sheet	80 cm×80 cm
Scale of Photography	1:15000
Maximum positional error	±70 cm
Spot heights	90% within 25% of the contour-line interval.

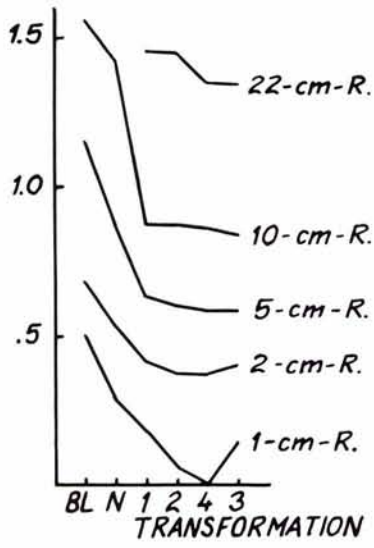

Fig. 3. Effectiveness of methods for the correction of image deformation for each point individually.

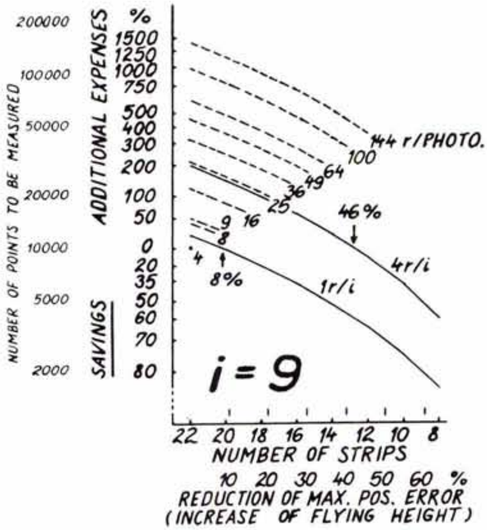

Fig. 4. Relationship between the number of points to be measured in the test block, the reduction of the maximum positional error, and the changes in mensuration costs if only 9 orientation points per photograph are to be measured and corrected for image deformation using r fiducial marks or reseau points.

In addition, the following values will be assumed:

Forward lap	60%
Sidelap	20%
Size of area	3685 sq km (1424 sq miles) =60.7 km by 60.7 km.

It is also assumed that the flight lines are parallel to the map-sheet dividing lines.

As the investigation results in comparative values, the block size is of secondary importance only. The block is considered to be surrounded by 20 control points. Two cases will be considered here: Case 1, 6 orientation points per model are required and, Case 2, 6 pairs of orientation points per model are required. It is further assumed that the strips are flown perfectly in view of minimizing the number of orientation points, i.e., each orientation point located in a model corner can be used for four models.

Expenses for Mensuration

The number of points to be measured in each photograph amounts to:

—for simultaneous correction of all i points,

4 fiducial marks	$4+i$
8 fiducial marks	$8+i$
10-cm reseau	$9+i$
5-cm reseau	$25+i$
2-cm reseau	$144+i$
1-cm reseau	$529+i$

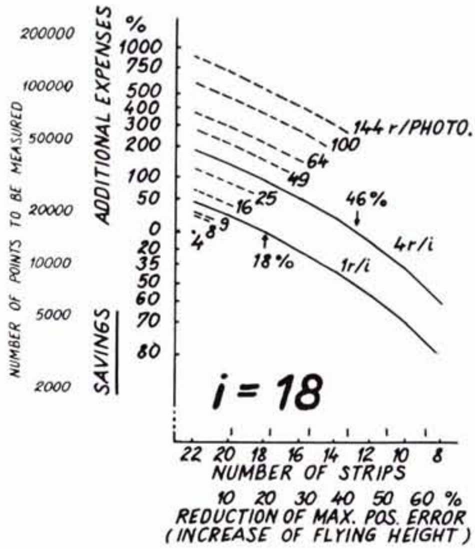

FIG. 5. As Figure 4 but for 18 orientation points per photograph.

—for individual correction using one reseau point per image,

4 fiducial marks	$2i$, but not more than	$4+i$	for $i \leq 25$
8 fiducial marks		$8+i$	
10-cm reseau		$9+i$	
5-cm reseau		$2i$	
2-cm reseau		$2i$	
1-cm reseau		$2i$	

—for individual correction using 4 reseau points per image point,

4 fiducial marks	$5i$, but not more than	$4+i$	for $i \leq 36$
8 fiducial marks		$8+i$	
10-cm reseau		$9+i$	
5-cm reseau		$25+i$	
2-cm reseau		$5i$	
1-cm reseau		$5i$	

It should be noted, however, that in using 4 reseau points per image point, a number of reseau points will be usable for more than one image point, thus reducing the number of points to be measured. In Case 2 of our investigations, i.e., the use of six pairs of orientation points, it is, for example, assumed that 6 reseau points instead of 8 are needed to correct the 2 image points.

The number of measurements needed for both instances is indicated in Figures 4 and 5. The ordinate gives the number of points to be measured within the block (using a logarithmic scale) and the percentages of additional expenses or savings, respectively, compared to photography as specified above in which the 9 or 18 image points, respectively, and the 4 fiducial marks are measured. The number of strips flown to cover the area is given on the abscissa.

Assuming that the specified maximum error (70 cm on the ground) is maintained, a reduction of the maximum image deformation is translated into a reduction of the scale of photographs, which is equivalent to an increase of the flying height. Also given at the abscissa in Figures 4 and 5 are the percentages of reduction of the maximum image deformation. The two solid lines indicate the assumed use of one and four reseau points per image point and individual correction of that point whereas the dashed lines indicate the simultaneous correction of all image points based on a certain number of reseau points or other reference marks. The dashed lines are not extended as far as the solid lines in order to indicate that the individual point correction is far more effective in the elimination of large local deformations than any of the methods correcting all image points with one higher order transformation.

A reduction of the maximum image deformation by 8 percent (if translated into a scale change so that the maximum positional error remains unchanged) would equal the number of points to be measured for Case 1 (9 orientation points) if instead of the traditional four fiducial marks nine reseau points were measured. In Case 2 (18 orientation points) and replacement of the traditional four fiducial marks by 18 reseau points, the reduction of the maximum image deformation would have to amount to 18 percent in order to equal the number of points to be measured. If each image point is to be corrected with the four surrounding reseau points rather than just one reseau point, the reduction of the maximum image deformation would have to be 42 percent in order to have the same amount of measurements as in the traditional case.

How realistic are the foregoing considerations? The maximum image deformation in the original specifications of 47 μm seems to be large if one considers that a positional rms error in image scale of 8 μm[1] can be expected. Although the measuring errors are approximately normally distributed,[6] the image deformation is not, and large local disturbances can occur.[6] As image deformation correction for each point individually, using only reseau points located near that point, proves very effective in reducing any deformation regardless of its size, it becomes possible to predict the *rms* as well as maximum errors with a high probability. Hence, disregarding for the moment other considerations, an image scale can be chosen which will just yield the specified accuracy.

Extent of Computations

An analysis of the programs used for computing corrections based on the Transformations 1 to 7 under the assumption that, in view of computer time,

$$1 \text{ multiplication} = 1 \text{ division} = 20 \text{ additions} = 20 \text{ subtractions},$$

revealed the following required number of multiplications for the correction of i image points based on r reseau points or other reference marks:

Transformation 1,	$25.9+19.4\ r+\ 4.2\ i$
2,	$25.8+\ 7.6\ r+\ 4.2\ i$
3,	$183.0+34.0\ r+\ 8.3\ i$
4,	$49.7+13.9\ r+\ 7.3\ i$
5,	$126.8+29.8\ r+13.5\ i$
6,	$249.8+49.6\ r+19.7\ i$
7,	$427.8+73.8\ r+25.9\ i.$

The number of multiplications for Case 1 (9 image points, i.e., $i=9$) as function of the number of reseau points r is given in Figure 6 which indicates Transformations 2 and 4 are the least demanding. These two transformations are, according to Figure 3, best suited for the correction of image points based on four surrounding reseau points. Figure 6 also indicates that the computational effort increases rapidly, not only with the number of reseau points if these are all used simultaneously to define one transformation, but also with the number of terms in the transformation.

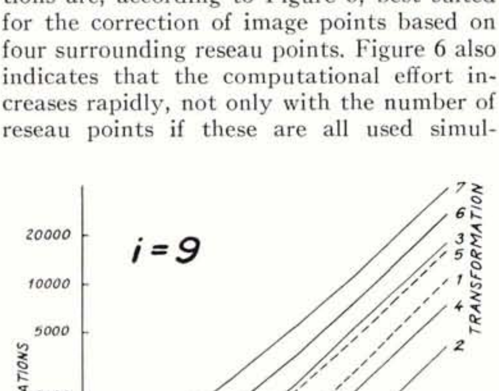

Fig. 6. Comparison of the computational effort needed for the seven transformations, if a varying number r of fiducial marks or reseau points is used to determine corrections for nine orientation points per photograph.

Using Transformation 2 and four surrounding reseau points, the number of multiplications needed for individual image point correction, is

$$(25.8 + 7.6.4 + 4.2.1)i = 60.4 \cdot i$$

which compares quite unfavorably with the number

$$0.2 \cdot i$$

for the transfer of the correction from a reseau point to an image point. For $i=9$, one attains 543.6 and 1.8 multiplications respectively. The first of these two values is given in Figure 6, the second is negligibly small.

Figure 6 also reveals that Transformation 6 (and any more complex transformation) require more computational effort for $i=9$ even if used with the least number of points needed to define all the coefficients, than a correction with Transformation 2 using four reseau points per image point.

Side Benefits Resulting from the Use of the Reseau

The presence of the reseau in an aerial camera, and the possibility to reduce any image deformation so that only approximately normally distributed residuals with an estimated *rms* error of ± 3 to $\pm 8\ \mu$m (depending on the density of the reseau) remain, enable several solutions which can no doubt further add to the economy of reseau photography. For example, a significantly improved lens distortion correction, a reduction in the field work for the determination of the elevations of the orientation points and a better flight plan can be achieved.

Lens Distortion Correction

The presence of a large number of reseau points in an aerial camera permits the definition of a large number of imaging rays. The points of intersection between a plane, e.g., the reseau plane, and those rays that could be the reseau points themselves, can now be defined accurately in rectangular or polar coordinates. It is no longer necessary to distinguish between average radial distortion (which is rotational symmetrical and centered at the point of best symmetry), unsymmetrical radial distortion and so-called tangential distortion.

Although a comprehensive lens-distortion correction does not require the presence of a reseau, it is facilitated significantly by combining the lens distortion correction table with the table of calibrated reseau coordinates. Hence, the actual benefit lays in the combination of lens distortion and image deformation correction into one operation.

In addition, the principal point of autocollimation can now replace the point of best symmetry as reference point. The differentiation between these two points, however, is of significance only if exterior orientation data (rotations) are measured or required with a high accuracy.

REDUCTION OF FIELD WORK FOR THE DETERMINATION OF ELEVATIONS

Within the block, 22 flight lines of 60.7 km-length each are to be flown. Therefore, 45 profiles of 60.7 km-length have to be levelled and corrected with each other, which amounts to at least 2792 km of levelling. Improved positional accuracy translated into photography at a smaller scale, reduces this value by 121.4 km for each eliminated flight line.

According to the earlier given NCC specifications, spot elevations have to be determined so that the errors of 90 percent of all values are within 25 percent of the contour-line interval of, in general, 5 ft (1.5 m). As the 90 percent level is equal to 1.65 m_{el}, m_{el} being the *rms* error in elevation, an accuracy of

$$m_{el} = \frac{1.524 \text{ m} \cdot 0.25}{1.65 \cdot 15000} = 15.4 \ \mu\text{m}$$

in image scale would have to be obtained.

Meier[1] suggests an *rms* error for elevations of 13.5 μm in image scale. In order to determine spot elevation photogrammetrically with this accuracy, photography at a scale not smaller than 1:17,100 should be used. However, this *rms* value was obtained with non-reseau photography. Inasmuch as image deformation causes systematic elevation errors,[3] more accurate elevation values can be expected if reseau corrections are used. Thus, the levelling activity can be restricted to the determination of a control net.

IMPROVED FLIGHT PLANNING

Because of the levelling for the determination of the elevations of the orientation points, flights have so far been planned to follow the major road pattern of the area (Figure 7) which is not parallel to the map-sheet dividing lines. Therefore, up to six models contributed to one map sheet. Recently it was decided to plan the flight lines so that they are parallel to the map sheet dividing lines. If reseau corrections would yield a guaranteed maximum positional error of 32 μm instead of the specified 47 μm, photographs at the scale 1:21,700 could be taken so that one photograph would cover 4 map sheets or 1 model would cover 2 map sheets (see Figure 8). The orientation points would then ideally be located only in all map sheet corners. Assuming that the *rms* error for photogrammetrically determined elevations could be reduced to ±10.6 μm, all three coordinates could be obtained from photogrammetric operations.

The factor 2,500/21,700≈1/8.7 is well within the capabilities of first-order plotters.

DISADVANTAGES OF RESEAU PHOTOGRAPHY

If reseau has so many advantages, one wonders why only approximately one percent (estimated) of the aerial cameras in commercial use are reseau cameras. The main reason is the widespread belief that the use of a reseau must lead to an increased number of points to be measured. It was shown in this paper that this argument is not true unless inflexibility of existing regulations prevent the changes in flight planning which are the key to the economical application of reseau photography.

However, some disadvantages are connected with the use of reseau photography. Probably the most serious one is the initial investment: reseau cameras are significantly more expensive compared with non-reseau

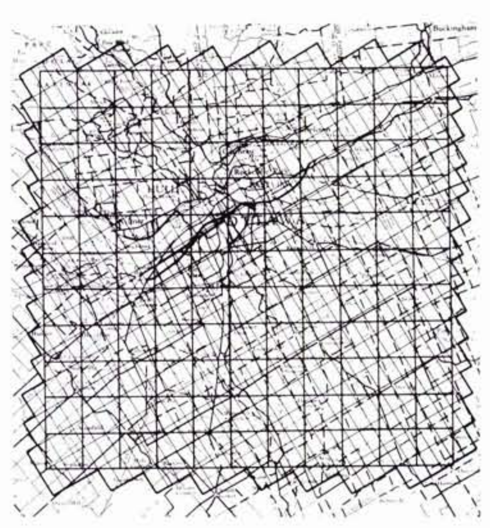

FIG. 7. Map of the Ottawa area showing location and size of models according to previous flight planning compared to the map sheet layout. (4 map sheets from one square).

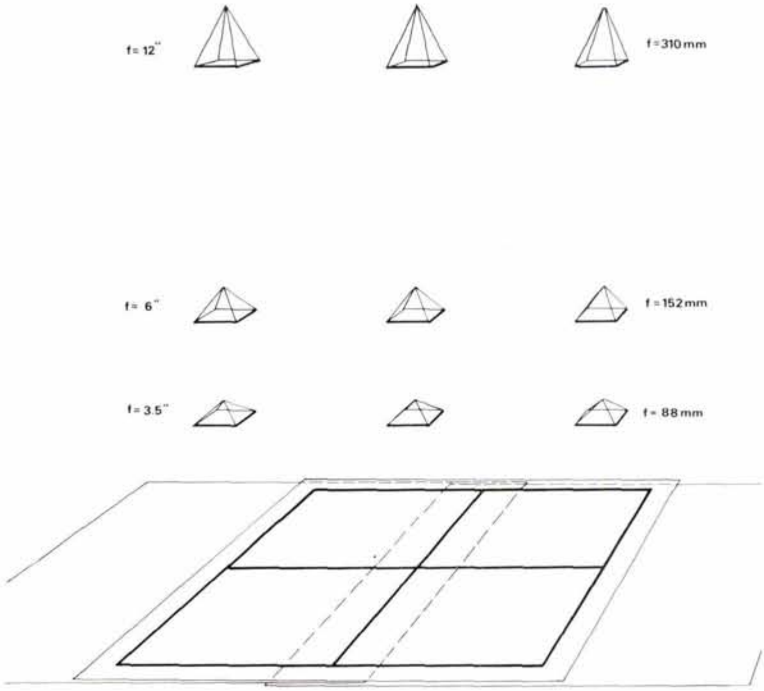

FIG. 8. Relationship between flying heights and focal length in the situation that one photograph covers four map sheets (60 percent forward overlap and 20 percent sidelap are assumed).

cameras. This additional expense seems to be an even more serious disadvantage in view of the fact that the reseau can be used practically only in analytical procedures.

Another disadvantage is the presence of the reseau crosses in the aerial photograph. Although no complaints from users (about loosing image detail due to the presence of the reseau) are known to the author, the reseau is a quite undesirable addition to any type of rectified photograph or photomap. Other unwanted additions to these photogrammetric products are scratches on the reseau plane projected into the photographic image.

The occurrence of scratches finally indicates higher maintenance costs for a reseau camera: a replacement of the register glass carrying the reseau will be necessary from time to time.

CONCLUSIONS

A reseau is more effective in reducing image deformation if each image point is corrected individually using the nearest reseau point or, better still, the four surrounding reseau points. In the latter case, the affine Transformation 2 and the bilinear Transformation 4 proved to be not only the most suitable but also the least expensive. The differences between a correction of an image point using one reseau point only and a correction based on the four surrounding reseau points are the smaller depending on the density of the reseau.

The computational effort for the correction based on one reseau point only is less than 1/250th of that needed for the use of four reseau points.

The application of individual corrections derived for each point from a dense reseau leads to a distribution of image deformation residuals which is close to normal because of the elimination of the large local deformations. Given a maximum positional error, an *rms*-error being a certain fraction of the maximum error (i.e., one third or, with a safety margin, somewhat smaller) can now be derived and the photography planned at the scale that will just yield this *rms* error. In other words, reseau permits an economical use of aerial photography because a smaller scale can now be selected due to a significant reduction in the safety factor used up to now. Not only does this reduce the overall number of points to be read in spite of an increased number of points per photograph, but it also guarantees the possibility of using analytical methods with any photography no matter how large its deformation might be.

The assistance of Mr. W. J. MacLean of the National Capital Commission in Ottawa is gratefully acknowledged.

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(*Continued from page 101*)

Mapping camera views such as this, supplemented by vertical photographs from the mapping and panoramic cameras and oblique telephotos with the Hasselblad cameras, will open a new chapter in the study of this part of the Moon. The spacecraft was 107 km above the surface when these views were exposed during the 71st revolution about the Moon. From that altitude the distance to the horizon is about 600 km.

Aristarchus and Herodotus (right), the two large craters in the middle distance, are about 170 km south of the ground track. The crater Marius, about 42 km in diameter and 530 km from the spacecraft, is the largest crater near the horizon. A long sinuous rille, Vallis Schroeteri, follows an arcuate course between the Cobra Head high on the plateau and the mare deposits of Oceanus Procellarum.