FRONTISPIECE. The cross-section of a plume.

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Extinction Coefficient

Determinations for smoke plumes with photogrammetric techniques has an average deviation of 10 percent relative to values from the airborne nephelometer.

(*Abstract on next page*)

INTRODUCTION

IN THE AUGUST 1970 issue of PHOTOGRAMMETRIC ENGINEERING, a photographic method was reported whereby a polluted air mass could be effectively photographed and mapped with *reasonable accuracy*. That report discussed various types of topographic mapping of visible industrial pollution and contained a short discussion about the "Possibilities for Quantitative Analysis."

This article is devoted to Quantitative Analysis, or obtaining numerical characteristics of a polluted air mass. The paper is written with the assumption that the physical dimensions of the polluted air mass are determined by any one of the methods previously described in the research project. The primary interest is to obtain quantitative information about the polluted air mass in general and its property with respect to the extinction of light in particular. The knowledge of the extinction of light is important because it is correlated to the mass concentration of atmospheric aerosol as shown by Charlson in Reference 2.

The present article is the result of a research project entitled, "A Study of Three Dimensional Extension of Polluted Air," supported by the National Center for Air Pollution Control, Bureau of Disease Prevention and Environmental Control (Project No. AP-00661-01).

QUANTITATIVE ANALYSIS OF PHOTOGRAPHS

The purpose of the aerial camera is to record as accurately as possible the location of a ground object as well as its luminance distribution. Unfortunately, the photographic recording alters both of these basic characteristics. The location is altered by the lens distortion, and the recording of the ground's luminance is altered by the film's characteristic upon development as well as by the atmosphere. This alteration is not a linear process. Consequently, if one wishes to use all the quantitative information given by a photograph, the reproduction characteristic of the system must be studied. The following well-known equation expresses the response of the film ot light intensity:

$$D = \log \frac{1}{T'} = \gamma \log E \qquad (1)$$

where D is the density of film, T' is transmittance of the negative (both are measurable with the microdensitometer), E is the exposure, and γ is the characteristic of the film. Exposure E is defined as:

$$E = \frac{2.7t(I_s R_g T_a + B_a)T_l}{(f_{\#})^2 F} \tag{2}$$

(Kohler's equation,[3] p. 85), where

E = exposure in meter-candle-seconds
t = exposure time in seconds
I_s = incidental horizontal plane luminance
R_g = ground object reflectance
T_a = atmospheric transmission
B_a = atmospheric luminance
T_l = lens transmission
$f_{\#}$ = relative aperture
F = filter factors.

ABSTRACT: *The average extinction coefficient at different places of an industrial plume can be determined by photogrammetric methods. The photographs, as a recording medium, were taken at an altitude of approximately 20,000 feet using polaroid filters which eliminated the effect of "background aerosol." These were scanned with a microdensitometer at various cross sections located at different distances from the source of the smoke. The microdensitometer reading are converted into an extinction coefficient by an approximate mathematical process. Coefficients required for this mathematical treatment were determined by actual nephelometer measurement. The method seems to be suitable to monitor polluted air over large geographical areas from a high-altitude camera platform. The disadvantage of the method is the use of polaroid filter which necessitates taking the photographs in the early morning or late afternoon. Further, in order to get the image of the ground through the unpolluted air, it was necessary to use the camera platform at a high altitude.*

In order to establish a quantitative relationship between the image of the polluted air mass and the measurable quantities D or T', a mathematical model must be developed between T_a and D or T'. Reflectance R_g of the ground object has a limited application in this respect because the polluted air mass can be regarded optically as a solid substance only in very high concentration. For industrial pollution this may hold only immediately at the source, i.e., around the chimney. The use of T_a, the atmospheric transmission, promises better results because the polluted air mass as a semi-transparent material affects the T_a, and as such it is detectable several miles from the source.

According to the Bouguer-Lambert Law the following equation can be written:

$$T_a = \frac{I}{I_0} = e^{-\sigma_a h} \tag{3}$$

(Johnson: "Physical Meteorology,"[4] p. 77, or Middleton, "Vision Through the Atmosphere,"[8] p. 14, or Kerker, "The Scattering of Light,"[5] p. 38), where I_0 is the intensity of light entering a layer of uniform atmosphere with thickness h, I is the intensity of light after passing through it, and σ_a is the extinction coefficient of the air and e is the exponential base.

Let us assume that an aerial photograph is under investigation, and a portion of the terrain shown is covered by uniformly distributed smoke of l thickness whereas the other portion shows the ground without any visible pollution. Then the atmospheric transmission over the polluted area may be expressed as:

$$T_p = e^{-\sigma_a'(h-l)} e^{-\sigma_s l} \tag{4}$$

where σ_s is the extinction coefficient of the uniformly distributed smoke, and σ_a' is the extinction coefficient of the air layer above the pollutant. Substituting Equations 4 and 3 into Equation 2 and then into Equation 1, the transmittance T_p' of the negative film can be obtained over the pollution covered as well as over the clearly visible ground T_g'. Mathematically, the following equations can be obtained assuming $\gamma = 1$:

$$\frac{1}{T_p'} = \frac{2.7tI_s R_g' T_l e^{-\sigma_a'(h-l)} e^{-\sigma_s l}}{(f\#)^2 F} + \frac{2.7tB_a' T_l}{(f\#)^2 F} \tag{5}$$

$$\frac{1}{T_g'} = \frac{2.7tI_s R_g T_l e^{-\sigma_a h}}{(f\#)^2 F} + \frac{2.7tB_a T_l}{(f\#)^2 F}\,. \tag{6}$$

It can be shown that if Equation 6 is divided by Equation 5, and the result is solved for the extinction coefficient σ_s of the pollutant, the following equation can be obtained:

$$\sigma_s = A + B\left(\frac{1}{l}\log_{10}\frac{T_g'}{T_p'}\right) \tag{7}$$

where A and B are constants composed of the second part of Formulas 5 and 6, and the conversion factor between natural and 10-based logarithms as well as from the coefficients of e.

The derivation resulting in Equation 7 can be done in more than one way; however, in every instance certain assumptions must be made which make Formula 7 approximate in nature. The first assumption is that the ground reflectance under the plume and over clear areas R_g is constant. This assumption necessitates the use of average ground reflectance obtained from the scanning of larger areas of clearly visible ground under a microdensitometer. A more significant assumption, however, is that

$$e^{-\sigma_a'(h-l)} = e^{-\sigma_a h}. \qquad (8)$$

This assumption can only be made if the thickness of the polluted layer is very small compared to the flying height and, what is even more important, if polarizing filter is used in the aerial camera. It was emphasized in the previous publication (Equation 8, Reference 10) that the effect of the "background aerosol," which is expressed by Formula 8, is greatly reduced by use of these filters, thereby making Equation 8 a justifiable assumption. This means that the flights must be made in the late afternoon to obtain the maximum polarization due to the nearly 90° angle between the camera axis and the incidental light from the sun.

In conclusion, it can be stated that it is possible to develop a mathematical relationship between the extinction coefficient of a polluted air layer and densitometric measurements, however approximative in nature. In order to obtain quantitative information from aerial photographs, a polarizing filter must be used, and the photograph must be taken when the polarization by light scattering is at its maximum, i.e., late afternoon or early morning. The constants A and B in Equation 7 must be determined by field calibration. If diapositive transparencies rather than negatives are used for densitometric measurements, the correct equation then is

$$\log_{10}\sigma_s = C + D\left(\frac{1}{l}\log\frac{T_g'}{T_p'}\right). \qquad (9)$$

The justification of this equation can readily be seen from a reproduction cycle of the film, for example, Chapter 9 in "Photographic Systems for Engineers," by F. M. Brown.[1]

PHOTOGRAPHIC CONSIDERATIONS

All the photographs for this research were taken with a Hassalblad 500C, 70-mm camera with Distagon 4/50 objective on Kodak Ektachrome ER 5257 film through a polarizing filter. The photographs were taken during late afternoon hours. The elevation of the sun over the apparent horizon was about 15°; the angle of observation between the camera axis and the incidental light was about 75°. At this observation angle, the degree of polarization approaches the maximum which means it can be as much as 70 percent depending on wave lengths and on the incidence of scattering particles, etc. (for more detail see H. E. Landsberg, "Advances in Geophysics,"[6] Vol. 10, p. 187, or J. C. Johnson, "Physical Meterology,"[4] p. 55). The low altitude of the sun further helped to reduce the contrast which minimized the adverse effect of approximations in Equation 9 to obtain a better average for the T_g' or indirectly for the ground reflectance.

FIG. 1. Aerial photograph taken with a Hasselblad camera from 10,000 feet during a 4-knot wind in color (shown here in black and white).

The flights were made at 10,000, 15,000 and 20,000 feet altitude resulting in photographs at the scale of 1:60,000, 1:90,000 and 1:120,000. The test areas were chosen to include an industrial area with multi-sources of pollution and an area with a single source. Port of Tacoma and Port Townsend were selected accordingly. A basic requirement for the evaluation of photographs was to cover polluted and unpolluted areas on a single picture.

First, the test area with a single source of pollutant was examined. For example, Figure 1 shows an aerial photograph taken from 10,000 feet altitude during a northerly wind of about four knots per hour at the scale of

1:60,000. These photographs may be judged as suitable for quantitative evaluation because only a small portion of the picture records the image of pollution. However, there are certain weather conditions such as an inversion, shown in Figure 2, where the wind is calm, and the smoke (even over a single source) mushrooms up covering the whole picture area. For this reason, it was concluded that 1:60,000 scale photographs were unsuitable for photointerpretation purposes in connection with industrial air pollution.

Figure 3 shows a photograph taken at 15,000 feet. The scale of the photograph is 1:90,000 over the test area of a multi-source pollutant which may be either inside or outside the area covered by the picture. The whole photographed area is almost entirely covered by a polluted layer, with the resulting conclusion that photographs taken at 15,000 feet altitude at 1:90,000 scale are not suitable for quantitative analysis. In Figure 4 a photograph was taken at 20,000 altitude over the same area as in Figure 1, but the scale of this picture was 1:120,000. The atmospheric haze, even at that altitude, was almost completely eliminated by the polarizing filter and the boundary of the plume can be easily recognized. It was concluded that the photographs at the scale of 1:120,000 are the most suitable for quantitative photointerpretation from the air pollution point of view. Further, the polarizing filter effectively reduces the effect of haze or background aerosol. This means that the theory presented in the previous study,[10] which considered this filtering effect can be regarded as justifiable.

These results indicate that small-scale images, particularly those which can be obtained from outer space, should be the most suitable for surveying or monitoring the visible air pollution.

PRACTICAL EXPERIMENTATION

The purpose of practical experimentation was to obtain a calibration method to determine the A and B or C and D constants in Equations 7 and 9, and also determine numerical data about the degree of reliability obtainable with this method.

The missions were flown by Walker and Associates of Seattle with their twin-engine Cessna 310. The aircraft was equipped with the Hassalblad camera and the University of Washington's nephelometer. The camera was mounted on a Wild R.C.8 camera suspension unit with an adapter.

The vertical photographs were taken with 60 percent overlap so that the stereo-image of the industrial plume could be established and, with it, the dimensions taken from the top of the plume and its horizontal extension.

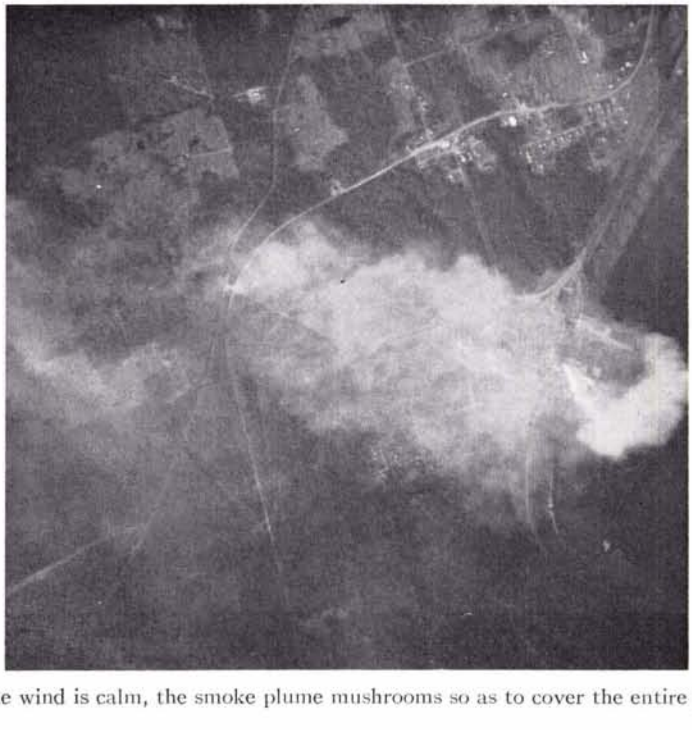

FIG. 2. If the wind is calm, the smoke plume mushrooms so as to cover the entire photograph.

FIG. 3. Taken from 15,000 feet and at a scale of 1:90,000, the photograph is not well suited to these studies because the plume covers the full extant of the photograph.

The slight change in the shape of the plume deteriorates somewhat the stereoscopic vision, but in spite of this handicap it is still possible to measure the stereo-image (see for instance stereograms published in NASA TM X-64546,[7] page 52).

Immediately after taking these photographs, the extinction coefficients were measured by the nephelometer. The nephelometer, which records continuously the extinction coefficients on a graph in $1\times10^{-4}\ m^{-1}$ unit, was flow through the smoke at several places. The locations of these nephelometer cross sections and profiles were identified so that vertical photographs could be taken with the Hassalblad camera at the beginning and at the end of each cross section. Later the location of these cross sections and profiles was identified on photomosaics by these vertical pictures. Such a mosaic and location of cross sections are given in Figure 5. Here the symbol S identifies the source, the cross sections were numbered 2 to 7, and No. 8 stands for the profile.

The airplane then took oblique photographs at the 4,000 feet altitude. The elevation of the lower and upper surface of the plume was determined from these oblique photographs. Analytical photogrammetric methods were used for this purpose, as reported in the previous publication.

Several such flights were made over the selected test areas. Consequently a large amount of data were collected this way under various weather conditions. The numerical evaluation began after the locations of the cross sections and profiles were identified. The same areas on the photographs were

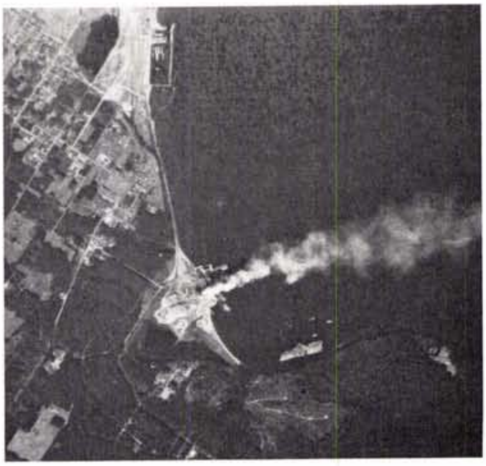

FIG. 4. This photograph was taken from 20,000 feet over the same area as Figure 1 but at a scale of 1:120,000; it is very useful for this application.

scanned under the University of Washington's microdensitometer. The output of the microdensitometer was obtained on a graph in the form of the film's transmissivity. Every cross section was shown on two photographs due to their 60 percent overlap; therefore, the relative transmissivity of the film was measured on both photographs, and averages of the two were used later in the computation. A typical output is given in Figure 6 for the right and left photographs. This figure shows the microdensitometer output for cross Section 3 in Figure 5.

The nephelometer extinction coefficients were then read from the graphs, and also the relative transmissivities of the images over polluted and clear areas were determined at the same points. During the course of this research, more than 700 sets of paired data were obtained for evaluation.

A least-squares adjustment was used for computation of calibration constants *C* and *D* in Equation 9. The observation method of adjustment was used where the observation equation for point *i* can be written as follows:

$$v_i = C + D \frac{1}{l} \log_{10} \frac{T_g'}{T_p'} - (\log_{10} \sigma_N) \qquad (10)$$

where v_i represents the deviations and σ_N is the extinction coefficient measured by the nephelometer. Besides the most probable values for the *C* and *D* constants the standard deviations were computed for the nephelometer observation as well as for the microdensitometer reading. Further, the use of the covariance matrix as a solution for normal equations in these adjustments permitted the computation of the statistical correlation coefficient (for a more detailed example of this type of computation see Chapter 8 in "Remote Sensing"[9]). The results are shown in Table 1 where the number of points or pairs of data are given in the first row; in the second row the standard deviation is given for the extinction coefficient, and in the third row the correlation coefficient. In the second portion of the table, the same data are given with the difference that, in the computation, the main thickness of the plume was used instead of the thickness being determined by each individual point such as in the first part of the table.

DISCUSSION OF RESULTS AND CONCLUSIONS

The overall reliability of the system is given by the correlation coefficient. The average correlation coefficient is 76 percent, the minimum is 61 percent, and the maximum is 92 percent of the theoretically possible 100

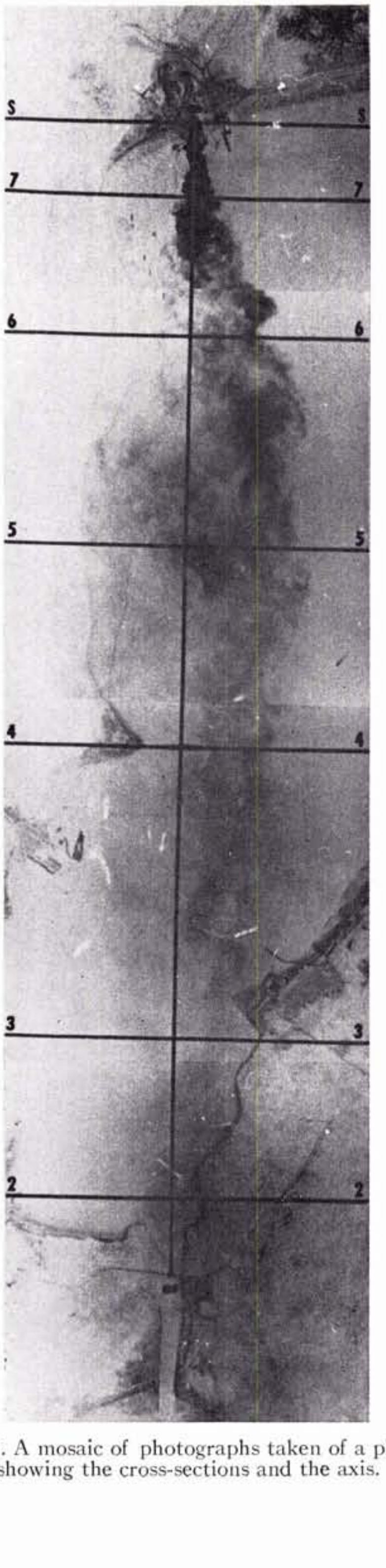

FIG. 5. A mosaic of photographs taken of a plume showing the cross-sections and the axis.

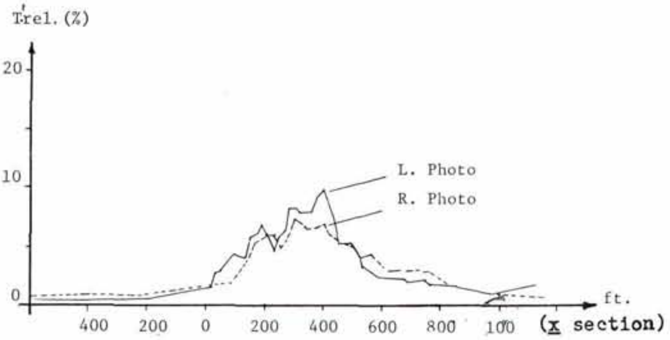

FIG. 6. The output of the microdensitometer for the left and right photographs of a cross-section of a plume.

percent. The performance of this method was therefore regarded as satisfactory.

The average deviation between the computed extinction coefficient and the one measured by the nephelometer is about 10 percent. A typical graphic comparison of this data is given in Figure 7 where the dashed line indicates the data obtained by computation and the solid line represents the extinction coefficient measured by the nephelometer. The stability of the system, namely the deviation factors involving different flight and film developing processes, has not been conclusively tested, but would require additional flights. However, from all indications, it seems that the variations between different flights are rather small if the exposure as well as film processing is done under controlled conditions.

One of the most time-consuming parts of this method is obtaining the thickness l of the plume at various points in a cross section. In order to evaluate the effect l on the results, computations were also performed using the average thickness of the plume at each cross section. It can be seen from the second part of Table 1 that this approximation is permissible without any deterioration of the results for the cross sections only. This is not practical for profiles as the last column of the table clearly shows. The correlation coefficient deteriorates to 36 percent which is far too low to be acceptable.

The calibration technique as described here needs considerable improvement. The density distribution, and with it the distribution of the extinction coefficient in a plume cone, produces a normal distribution; graphically it is a bell-shaped curve. If the plume is scanned with a microdensitometer, the average transmissivity, and with it the average extinction coefficient, is obtained and symbolized in the equation as σ_{si}. This concept can be expressed mathematically as:

$$\sigma_{si} = \sum_{i=0}^{i=l} \frac{\sigma_i}{l} \qquad (11)$$

where the $\sigma_{i=0}$ is the extinction coefficient at the ground level, and $\sigma_{i=l}$ is the extinction coefficient at the top of the plume. As a consequence, the σ_{si} determined photogrammetrically is measured in a vertical direction. The extinction coefficient σ_N is obtained by nephelometer measurements in a horizontal

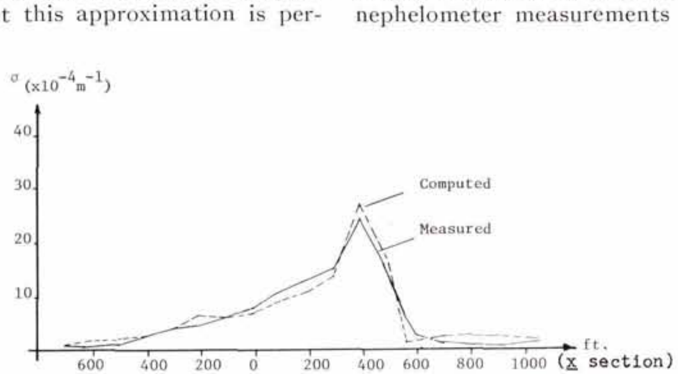

FIG. 7. The average deviation between the computed coefficient and the one measured by the nephelometer in the airplane is approximately 10 percent.

TABLE 1. RESULTS FROM THE LEAST-SQUARES SOLUTION AND THE VARIANCE-COVARIANCE DATA

No. of points used in comp.	54	61	106
Standard dev. of σ_N	$1.01\times10_{m}^{-4}-1$	$1.28\times10_{m}^{-4}-1$	$1.42\times10_{m}^{-4}-1$
Correlation coefficients	0.92	0.61	0.69

Data below computed using the main thickness of each cross section on the profile.

Standard dev. of σ_N	$1.00\times10_{m}^{-4}-1$	$1.28\times10_{m}^{-4}-1$	$1.02\times10_{m}^{-4}-1$
Correlation coefficients	0.92	0.64	0.73

direction. This concept is expressed by the Frontispiece where the normal distribution curves for both, namely the nephelometer and photogrammetric measurement, are indicated at an arbitrary cross section of the plume with the direction of flight marked by an arrow. The comparison is correct only if the nephelometer measurement perfectly corresponds to the place where the extinction coefficient is average. In view of this, the calibration would be better theoretically if the field measured data would also be shown vertically. This conclusion strongly recommends the replacement of the nephelometer with a spectrometer which could be synchronized with the aerial camera. Thus the operation of the spectrometer and the aerial camera would be done at the same time from the same altitude; for this reason, it would not be necessary for the airplane to make several crossings of the plume.

The theoretical limit of the applicability of the method can be found from the basic equation used; that is,

$$\frac{I}{I_0} = e^{-\sigma h}.$$

These are the limits of use, if

$$I = 0 \quad \text{and}$$
$$I = I_0.$$

Photographically, the $I_0=0$ slie at a close distance from the source. Where the plume photographically behaves as a solid body, no ground could be seen through the smoke. In other words, the intensity of reflected light from the ground is I_0, which does not pass through the smoke; thus $I=0$. The other boundary occurs at a considerable distance from the source of pollution where no measurable difference exists between T_p' and T_g' on the photograph. In these experiments it was found that the lower limit is about 1 to 3,000 feet from the source, and the upper limit is about 18,000 feet. A useful area for evaluation is about three miles long. It must be strongly emphasized that the length of the useful area depends greatly on the meterological conditions and on the rate of emission. Neither of these factors were taken into account in investigating the limit of useful area because they were considered to be outside of the scope of this project.

In general, it can be concluded from these experiments that recording polluted air masses by remote sensing, i.e., aerial photography, is possible using the outlined theory and techniques.

Considerably more extensive research would be required to obtain a better and more automated method of calibration in order to provide optimum results with optimum economy. This should be directed toward monitoring the changes in the polluted environment, either from earth's atmosphere or from space, by using an airplane or satellite.

The disadvantages of the method result partially from the photographic techniques. As mentioned earlier, the use of polaroid filter was required to eliminate the effect of *background aerosol*. This is possible only if the camera axis and the direction of incidental light are approximately perpendicular. This fact limits the timing of the photographs to either early morning or late afternoon.

Further microdensitometer readings should be taken over terrain which is not covered by a polluted air mass. This fact requires rather high-altitude airplanes, thus eliminating the use of smaller ones.

The method by its nature is best suited for recording the pollution over a large geographical area; for this reason, a high-altitude airplane (50,000 feet or higher) or a satellite would be needed as a camera platform. Consequently, the method would not replace local sampling but would augment it.

TABLE 1. (*Continued*)

72	118	239	84 (Profiles only)
$1.16\times10_m^{-4}-1$	$1.20\times10_m^{-4}-1$	$1.10\times10_m^{-4}-1$	$1.03\times10_m^{-4}-1$
0.91	0.92	0.61	0.65

Data below computed using the main thickness of each cross section on the profile.

$1.15\times10_m^{-4}-1$	$0.71\times10_m^{-4}-1$	$1.03\times10_m^{-4}-1$	$8.01\times10_m^{-4}-1$
0.92	0.93	0.70	0.36

ACKNOWLEDGEMENTS

The author wishes to express his deep appreciation to Dr. A. T. Rossano, Jr., Professor of Air Resources Engineering, and to Dr. R. J. Charlson, Associate Professor of Atmospheric Chemistry, both of the Department of Civil Engineering at the University of Washington. He also wishes to thank Mr. H. Moniwa and Mr. M. Nasu, Research Assistants, for their valuable contributions to this research project.

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