DR. PAUL ANDRÉ GAGNON
MICHEL PERRON
Laval University
Quebec 10, Canada

Experiments on Model Formation

An advantage is brought out for analytical aerial aerotriangulation for difficult models where only a reduced portion of the area can be used.

INTRODUCTION

IN THE USUAL production-oriented practice of photogrammetry, there are currently accepted norms relative to the number and disposition of points to be used for model formation. The validity of these norms has been confirmed by many experiments. Among the more recent works on the subject, can be named, for instance, those of Togliatti[4,5] Proctor, Robinson and Hull[3], and Yacoumelos.[6] Their investigations, however, do not consider at all, nor shed enough light on, some aspects of model formation which are highly susceptible to be of interest to the modern photogrammetrist who uses analytical means for model formation.* These aspects or questions refer to model stability and accuracy; they can be stated as follows:

- What correlation exists between residual *Y*-parallaxes and errors on ground controls computed from the models? Which is the same as to ask: is *Y*-parallax a good indication of the absolute accuracy of a model?
- To what degree can the accuracy of a model be affected by variations in the solution of the relative orientation, i.e., by variations of the elements of orientation? Again, the question can be stated in a different way: Is it possible that, for a given model, different solutions be considered to be good or acceptable?
- What part of the area of a model must be usable so that it can be formed? And then, what is the influence of the disposition of the points? What must be the disposition and number in the limit or critical cases?

ABSTRACT: *Past investigations on model formation leave room for further experiments on model stability and accuracy. Some questions are answered concerning the correlation between* Y*-parallax and absolute accuracy, stability of the solution of relative orientation, and limit cases for a solution.*

DESCRIPTION OF THE EXPERIMENTS

To answer these questions, experiments were made with two perfectly-controlled models (Figure 1). The control points are part of the test area of the National Research Council of Canada and their positions are known to within 5 cm. In both instances, the scale of photography was 1:40,000 and the camera a 6-inch-focal-length Wild RC-8. A Wild STK-1 was used for the observations, Gagnon's[3]* program to compute model coordinates and Schut's[6] program to transform model coordinates to ground coordinates. Photograph coordinates were corrected for radial distortion, earth curvature and atmospheric refraction by entering the appropriate constants in the specific subroutines of Gagnon's program.

Our experiments are divided in three groups:

★ In the first one, we consider that the total area of the model can be used to select relative-orientation points. We present six cases (Fig-

* Of course, analogical and analytical methods for model formation are based on the same geometry. However, on account of the possibilities of computers, the latter can be more rigorous and more flexible.

* This program computes a rigorous solution of relative orientation by solving the coplanarity condition and, from that, computes model and strip coordinates. The dependent method is used for model formation, and the left camera is considered as being fixed.

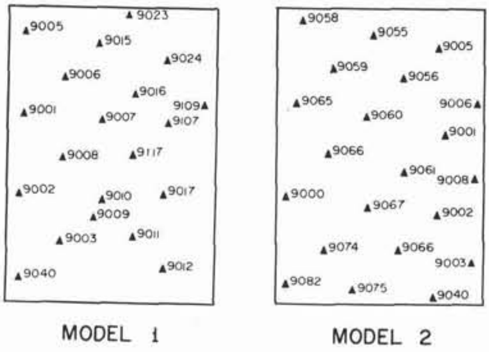

FIG. 1. Distribution of control points.

ure 2) differing by the number and disposition of these points.

★ Figure 3 illustrates the different cases of Group 2. It can be seen that the effective area is progressively reduced, the boundary of the usable part (white) being at 45° with the base. The sub-groups present variations in the number and disposition of relative-orientation points.

★ Group 3 (Figure 4) differs from Group 2 in that the boundary is kept parallel to the base.

In all the cases, the programs have computed, for all the control points of each model: (a) Y-parallax at the approximate scale of photography, and (b) X, Y and Z errors of the ground coordinates.

In all the cases, too, have been computed the elements of orientation ω, ϕ, κ, by and bz for the right camera.

RESULTS

GROUP 1

Tables 1 and 2 list the results of Group 1. Figure 5 is built from these results and completes them: it shows the graphs of the standard vertical and planimetric errors and the graph of the standard Y-parallax to which, for convenience, a scale factor has been applied. The inspection of these tables and figure leads to the conclusion that:

- Residual Y-parallaxes and standard residual Y-parallax seem to be a good indication of the absolute accuracy of a model.
- Good results can be achieved through slightly different solutions, that is, if there is a variation in the value of the elements of orientation.
- The use of more than six well-distributed points does not lead to a significant increase in accuracy (this, of course, is only a confirmation of what has been established by previous research). On the other hand, a loss of accuracy can result from using only five points.

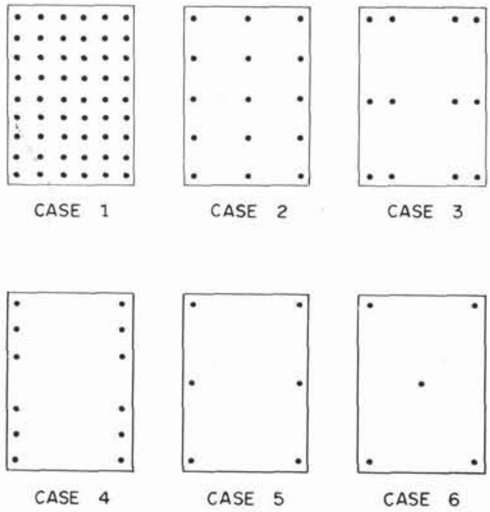

FIG. 2. Distribution of orientation points.

GROUP 2

Tables 3 and 4, and Figure 6 show the results of Group 2. From these, it can be seen that:

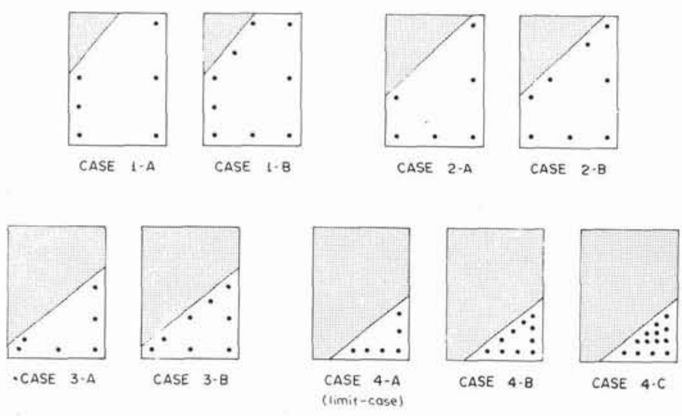

FIG. 3. Distribution of orientation points, Group 2.

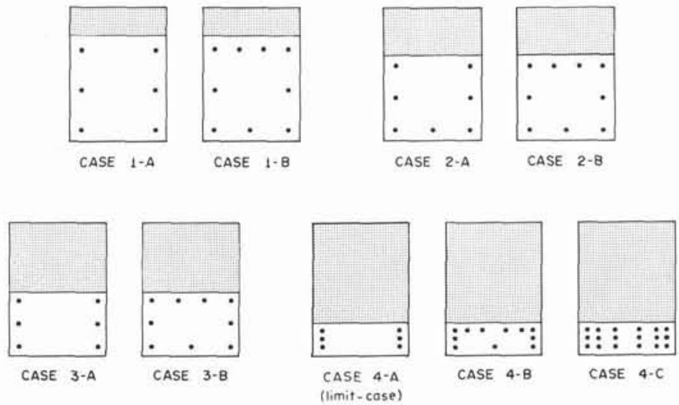

FIG. 4. Distribution of orientation points, Group 3.

- If fifty percent of the model area can be used, increasing the number of orientation points does not result necessarily in a gain of accuracy.
- The limitis reached when only about one-fourth of the model can be used. In that situation the number of points must be increased, and these points must be chosen on the periphery of the usable part. The results will then be about half as accurate as in the normal case.
- *Y*-parallax is a good indication of the absolute accuracy.

Besides, as a check on the computed value of the elements of orientation, they were introduced, in each case, on a Wild A-7. We found that:

TABLE 1. RESULTS FROM GROUP 1, MODEL 1

Cases	Standard Residual Y-parallax at photo-scale (μm)	Planimetric Accuracy m_X(m)	m_Y(m)	m_P(m)	Vertical Accuracy m_Z(m)	relative to flying height	Orientation Elements ω^o	ϕ^o	κ^o	bY(mm)	bZ(mm)
1	± 11	±.41	±.33	±.52	±.38	1/16 000	-1.96	1.14	-.95	-7.18	-6.82
2	± 11	±.41	±.33	±.52	±.38	1/16 000	-1.96	1.14	-.95	-7.18	-6.82
3	± 14	±.44	±.35	±.56	±.40	1/15 000	-1.95	1.15	-.95	-7.24	-6.84
4	± 10	±.42	±.33	±.53	±.39	1/15 000	-1.97	1.13	-.95	-7.27	-6.82
5	± 13	±.46	±.33	±.58	±.39	1/15 000	-1.95	1.14	-.95	-7.25	-6.83
6	± 15	±.41	±.35	±.57	±.39	1/15 000	-1.95	1.15	-.95	-7.23	-6.86

TABLE 2. RESULTS FROM GROUP 1, MODEL 2

Cases	Standard Residual Y-parallax at photo-scale (μm)	Planimetric Accuracy m_X(m)	m_Y(m)	m_P(m)	Vertical Accuracy m_Z(m)	relative to flying height	Orientation Elements ω^o	ϕ^o	κ^o	bY(mm)	bZ(mm)
1	± 15	±.53	±.50	±.72	±.37	1/16 000	-1.37	-.14	1.07	-7.40	-7.08
2	± 13	±.55	±.52	±.76	±.38	1/16 000	-1.36	-.13	1.07	-7.39	-7.08
3	± 17	±.62	±.58	±.85	±.41	1/15 000	-1.37	-.13	1.01	-7.31	-7.09
4	± 12	±.54	±.54	±.76	±.40	1/15 000	-1.38	-.15	1.01	-7.35	-7.08
5	± 17	±.62	±.58	±.85	±.38	1/16 000	-1.37	-.13	1.01	-7.29	-7.08
6	± 15	±.68	±.71	±.98	±.43	1/14 000	-1.37	-.13	1.00	-7.32	-7.07

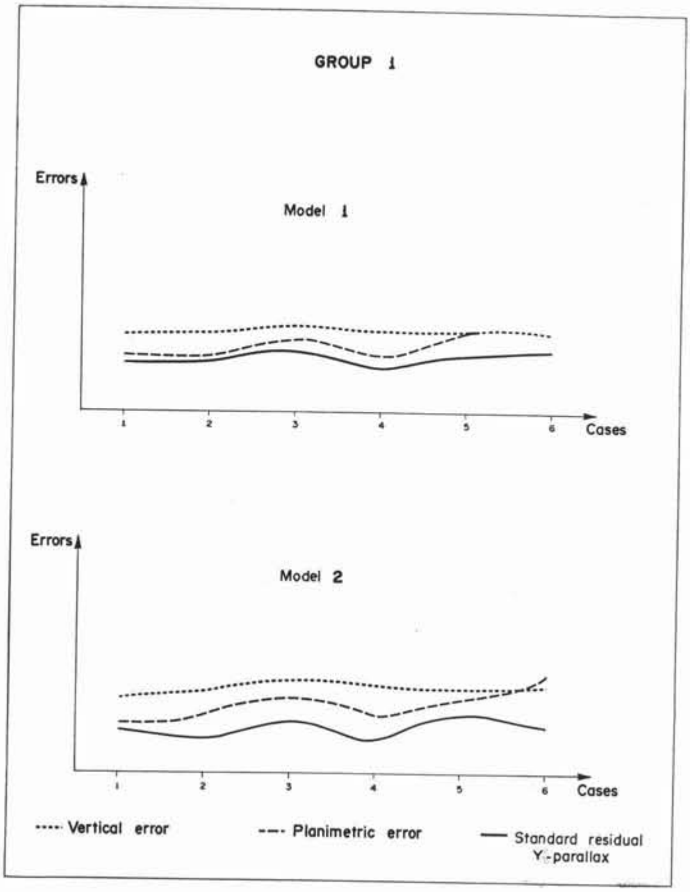

FIG. 5. Standard planimetric and vertical errors and standard residual *Y*-parallax, Group 1.

TABLE 3. RESULTS FROM GROUP 2, MODEL 1

Cases	Standard Residual Y-parallax at photo-scale (μm)	Planimetric Accuracy m_X(m)	m_Y(m)	m_P(m)	Vertical Accuracy m_Z(m)	relative to flying height	Orientation Elements ω^o	ϕ^o	κ^o	bY(mm)	bZ(mm)
1-A	± 13	±.42	±.30	± .51	±.44	1/14 000	-1.95	1.14	-.95	-7.14	-6.85
1-B	± 12	±.43	±.34	± .55	±.39	1/15 000	-1.96	1.14	-.95	-7.27	-6.85
2-A	± 28	±.46	±.40	± .61	±.65	1/9 000	-1.94	1.09	-.97	-7.21	-6.85
2-B	± 13	±.41	±.30	± .51	±.46	1/13 000	-1.95	1.14	-.96	-7.21	-6.85
3-A	± 18	±.42	±.51	± .66	±.64	1/9 000	-2.00	1.14	-.97	-7.34	-6.84
3-B	± 47	±.61	±.75	± .97	±.64	1/9 000	-2.00	1.24	-.92	-7.38	-6.89
4-A	- - - - - - - - - - - - - No solution -										
4-B	± 44	±.53	±.76	± .93	±.80	1/7 500	-2.10	1.13	-.94	-7.39	-6.79
4-C	± 55	±.60	±.91	±1.09	±.99	1/6 000	-2.08	1.11	-.96	-7.40	-6.74

TABLE 4. RESULTS FROM GROUP 2, MODEL 2

Cases	Standard Residual Y-parallax at photo-scale (μm)	Planimetric Accuracy m_X(m)	m_Y(m)	m_P(m)	Vertical Accuracy m_Z(m)	relative to flying height	Orientation Elements ω^o	ϕ^o	κ^o	bY(mm)	bZ(mm)
1-A	± 19	±.54	± .53	± .76	± .44	1/14 000	-1.37	-.13	1.07	-7.40	-7.08
1-B	± 18	±.61	± .58	± .84	± .39	1/15 000	-1.36	-.13	1.01	-7.30	-7.08
2-A	± 17	±.64	± .62	± .89	± .35	1/17 000	-1.37	-.14	1.01	-7.30	-7.08
2-B	± 17	±.60	± .59	± .84	± .40	1/15 000	-1.37	-.13	1.01	-7.32	-7.07
3-A	± 29	±.72	± .65	± .97	± .49	1/12 000	-1.37	-.12	1.01	-7.34	-7.11
3-B	± 22	±.57	± .46	± .73	± .80	1/7 500	-1.37	-.07	1.03	-7.31	-7.11
4-A	No solution										
4-B	± 83	±.86	±1.19	±1.46	±1.38	1/4 000	-1.46	-.13	1.03	-7.51	-6.96
4-C	± 34	±.62	± .89	±1.08	± .90	1/7 000	-1.42	-.15	1.01	-7.45	-7.02

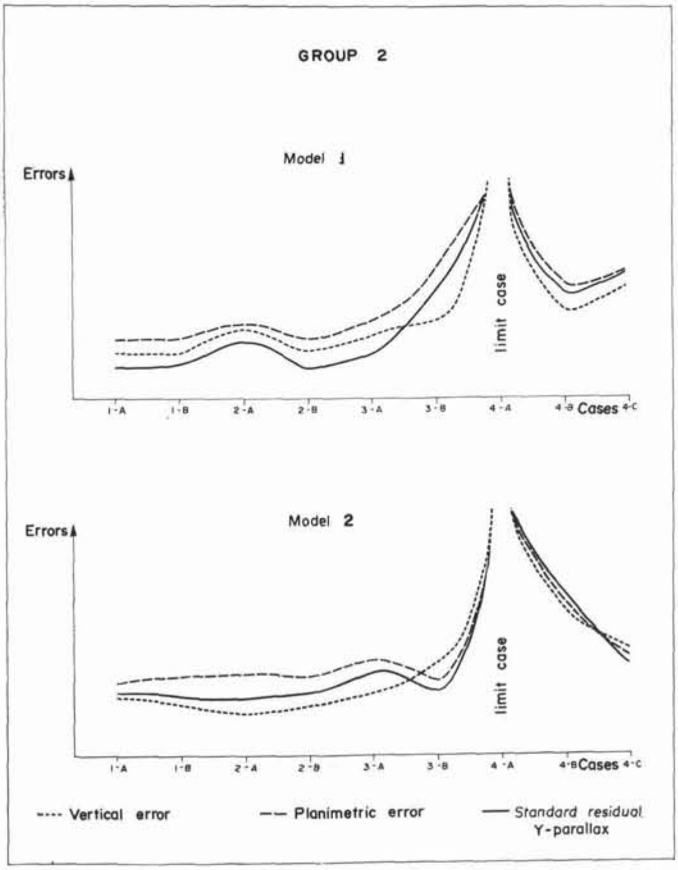

FIG. 6. Standard planimetric and vertical errors and standard residual Y-parallax, Group 2.

TABLE 5. RESULTS FROM GROUP 3, MODEL 1

Cases	Standard Residual Y-parallax at photo-scale (μm)	Planimetric Accuracy m_X(m)	m_Y(m)	m_P(m)	Vertical Accuracy m_Z(m)	relative to flying height	Orientation Elements ω^o	ϕ^o	κ^o	bY(mm)	bZ(mm)
1-A	± 13	±.41	± .29	± .50	± .45	1/13 000	-1.92	1.14	-.96	-7.13	-6.85
1-B	± 13	±.40	± .30	± .50	± .44	1/13 000	-1.93	1.14	-.96	-7.15	-6.84
2-A	± 24	±.46	± .50	± .67	± .62	1/10 000	-1.99	1.17	-.93	-7.37	-6.89
2-B	± 26	±.46	± .51	± .68	± .63	1/10 000	-2.00	1.17	-.93	-7.37	-6.88
3-A	± 97	±.90	±1.37	±1.64	±1.75	1/3 000	-2.15	1.10	-.94	-7.56	-6.71
3-B	± 104	±.93	±1.44	±1.71	±1.82	1/3 000	-2.14	1.15	-.92	-7.59	-6.78
4-A	- - - - - -	- - - -	- - - -	- - - -	No solution	- - - -	- - - -	- - - -	- - - -	- - - -	- - - -
4-B	± 64	±.64	± .61	± .88	± .59	1/10 000	-1.79	1.20	-.94	-7.27	-7.03
4-C	± 108	±.95	±1.43	±1.71	±1.43	1/4 000	-2.00	1.17	-.93	-7.21	-6.76

◊ The models formed with the first four sets of values looked perfectly clear and free from *Y*-parallax.
◊ The size of the observed *Y*-parallax, in cases 3-A and 3-B, was about half a floating-mark.
◊ In the limit case, where only one fourth of the model was used, the observed *Y*-parallax was about 1.5 floating-marks; however, the operator found that it could be greatly reduced by a small *bz* correction.

GROUP 3

Tables 5 and 6, and Figure 7 show the results of Group 3. They lead to practically the same findings as were derived from the experiments of Group 2. The only difference is a small weakening of the accuracy.

As before, the elements of orientation were introduced on an A-7 and the observed *Y*-parallax was found to be a little larger than for Group 2.

CONCLUSION

The results of these experiments give an answer to the questions stated at the beginning:

- They show that *Y*-parallax is a reliable indication of the accuracy of a model.
- They indicate what can be considered as a limit case.
- They fix the procedure, concerning the number and disposition of the points if the model approaches such a case.

These experiments bring out also an advantage of analytical aerial triangulation in the cases of difficult models where only a

TABLE 6. RESULTS FROM GROUP 3, MODEL 2

Cases	Standard Residual Y-parallax at photo-scale (μm)	Planimetric Accuracy m_X(m)	m_Y(m)	m_P(m)	Vertical Accuracy m_Z(m)	relative to flying height	Orientation Elements ω^o	ϕ^o	κ^o	bY(mm)	bZ(mm)
1-A	± 22	± .66	± .56	± .87	± .55	1/11 000	-1.35	-.12	1.07	-7.34	-7.09
1-B	± 21	± .64	± .56	± .85	± .52	1/11 000	-1.55	-.12	1.07	-7.34	-7.09
2-A	± 21	± .58	± .46	± .74	± .72	1/8 000	-1.37	-.08	1.02	-7.30	-7.11
2-B	± 19	± .55	± .45	± .71	± .71	1/8 000	-1.37	-.08	1.03	-7.32	-7.10
3-A	± 54	± .65	± .86	±1.07	±1.13	1/5 000	-1.44	-.14	1.02	-7.48	-7.01
3-B	± 73	± .68	± .85	±1.08	±1.11	1/5 000	-1.42	-.08	1.04	-7.42	-7.00
4-A	- - - - - -	- - - -	- - - -	- - - -	No solution	- - - -	- - - -	- - - -	- - - -	- - - -	- - - -
4-B	± 173	±1.84	±2.17	±2.85	±2.05	1/3 000	-1.24	-.05	1.04	-7.08	-7.37
4-C	± 96	±1.15	±1.23	±1.68	±1.16	1/5 000	-1.31	-.07	1.03	-7.22	-7.25

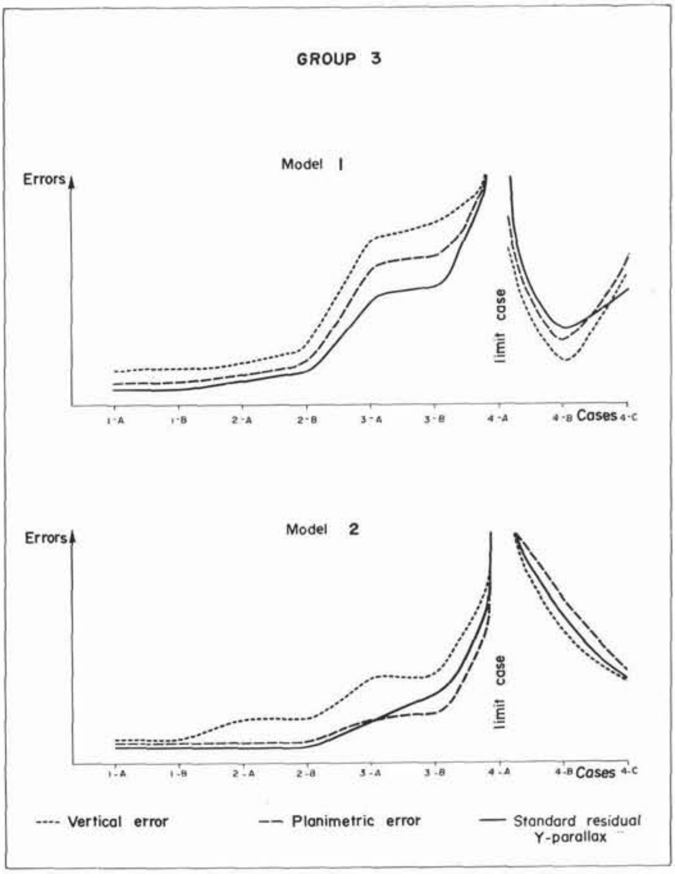

FIG. 7. Standard planimetric and vertical errors and standard residual *Y*-parallax, Group 3.

reduced part of the area can be used. It is evident that, by conventional methods without the help of analytics, the models could not have been formed properly in the plotter (in experiments 4 of Groups 2 and 3). Besides, if the models are normal, the plotting process is speeded up considerably if the analytical treatment includes the automatic determination of all elements that are needed for the complete orientation of a pair of photographs in a stereoplotter.[1]

These experiments, of course, do not cover everything on model formation, stability and accuracy. First, conclusions based on only two models need to be confirmed. Second, these conclusions apply only to two general configurations given to the effective model. For these reasons it would be very interesting and useful to see other investigations follow and complete these simple tests.

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