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Film Deformation Investigation

Considerations include the separation of measurement error from film deformation using systematic trend statistics, immaterial transformations, and orthogonal polynomials.

ABSTRACT: *In some of the previous studies of film deformation little or no distinction had been made between those residual image coordinate errors that were due the measuring process from those that were strictly deformational. Data from a prior study are used to illustrate the ideas. The application of systematic trends, immaterial transformations and orthogonal polynomials are considered in an effort to derive valid corrections to improve analytic aerotriangulation.*

INTRODUCTION

OF ALL THE research necessary for the improvement of photogrammetric methods of measurement, the investigation of film deformation is most elemental—elemental in the sense of being amenable to strict experimental and data reduction controls. It is the purpose of this paper to outline considerations to be used for reducing the measurements obtained in film deformation studies. These considerations were brought to light through a study[1] conducted on data collected by Brock and Faulds.[2]

Three separate objectives seem to be valid for film deformation studies:

- Standardization of Film Characteristics,
- Standardization of Film Handling, and
- Determination of *Corrections* to be applied to photogrammetric images.

The first named objective, Standardization of Film Characteristics, is of concern primarily to the film manufacturer who must determine the characteristics of his film for the purpose of product improvement. The second objective, Standardization of Film Handling, has as its purpose the maintenance of quality control over the development, drying, and storage of film and glass plates.

In these first two objectives the investigator is not particularly interested in the exact amount of deformation for each area of each and every piece of film; it is more usual that the interest is in guaranteeing that the film, in general, conforms to certain pre-selected criteria. In this sense the third objective, Determination of *Corrections* to be Applied to Photogrammetric Images, differs from the first two. For in the third objective one wants to know not only that the film deformation does not exceed a given magnitude, but also how much a given image on the film must be moved (corrected) in order to nullify the affect of deformation. This paper is directed to this last objective.

Two methods of data collection are in current use: the grid method, and the moiré fringe method. As the grid method seems to be the more appropriate for the numerical determination of corrections, the present discussion is confined to this method.

MEASUREMENT ERROR CONFOUNDED WITH FILM DEFORMATION

In determining film deformation one applies an algebraic polynomial generally of first, second, or third degree. The only theoretical condition placed on the deformation is that it be representable by such an algebraic polynomial, and it is usual to add terms until one has what he considers to be a reasonable fit of the film to the grid. The polynomial

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model is, therefore, *open-ended*. As a consequence of this, some definite criteria must be applied to the judgment of such a polynomial model.

A first criterion is the comparison of two variances. Both of these variances represent the errors of measurement of the individual grid intersections, but they differ in the manner in which they are computed. The first variance is estimated from the discrepancies of repeated measurements of the grid intersections and is not influenced in any way by the choice of the polynomial model. The second variance is computed from the sums of squares of residuals after a least-squares fit of the model; it is influenced by (1) errors of measurement, and (2) the inadequacy of the chosen model to represent the film deformation. If a model is found for which these two variances are equal (within some limit), one has the tendency to assume that the model fit is good.

The difficulty with applying this criterion is that the first variance—that is, the variance as determined by repeated measurements of the grid intersections—can be highly unreliable. In the analysis of the Brock-and-Faulds data it was found that a sizable bias existed between the measurements of the same grid as performed by two different operators. This bias was as great as 3.3 micrometers on some grids. Thus if the variance of the mean is obtained by pooling the observations, computing the variance of a single observation, and dividing by the number of observations, the variance of the mean may be seriously overestimated. A more reliable way should be to compute the means for each operator and then obtain the variance of the overall mean from the variance of the differences of the means of each operator. If $\overline{X}_A$, $\overline{X}_B$ represent the means of a grid intersection as determined by operator A and operator B respectively, then the overall mean is:

$$\overline{X} = \frac{\overline{X}_A + \overline{X}_B}{2}$$

and the variance is (neglecting correlation):

$$\sigma_{\overline{X}}^2 = \tfrac{1}{4}(\sigma_A^2 + \sigma_B^2).$$

But the variance of the function $(\overline{X}_A - \overline{X}_B)$ is equal to the variance of the function $(\overline{X}_A + \overline{X}_B)$ and so the variance of the overall mean can be computed from the variance of the differences $(\overline{X}_A - \overline{X}_B)$. This method of computing the variance is independent of the number of measurements inherent in $\overline{X}_A$ and $\overline{X}_B$.

A second criterion which should be, but seldom is, used is that of determination of systematic trend.[3] For this criterion one forms the ratio δ^2/σ^2 where δ^2 is the variance of successive differences of residuals and σ^2 is the variance of the residuals. If *no* trend exists, the expectation of δ^2/σ^2 is 2; if a trend *does* exist, the ratio will be *less* than 2. This test does not require knowledge of the variance of the measurements—it is applied strictly to the residuals as obtained from model fit.

Tables 1 and 2 give the variances and systematic trends for two types of film for different ages after their development, and for two different exposures. Each age and exposure was fit to five different polynomial models. The data used were from Brock and Faulds. One sees that in general as the variance is reduced, the systematic trend is also reduced (i.e., the numerical value is raised). An *a priori* variance of 5.7 micrometers was computed from the measurements. It is evident that this *a priori* variance is too great; because generally if the variance after fit is at, or is slightly below, 5.7 micrometers, a considerable amount of systematic trend still exists.

AVERAGE SHRINKAGE

It is the purpose of this section to compare two different methods of determining average shrinkage. The first method was given by Brock and Faulds. They found the shrinkage between two grid intersections by measuring the distances on the control grid and on the film grid between consecutive intersections. The difference of these two distances is the shrinkage. The average shrinkage is the average of the shrinkages over the whole plate. The second method is a least-squares fit of an affine transformation; the average shrinkage was then determined from the parameters of the transformation. Table 3 gives the average shrinkages and standard deviations of both methods. There are two grids, 1 and 4, each grid measured at four different ages. The average shrinkages, regardless of age, agree to a remarkable degree! Round off error is sufficient to account for the small differences. The standard deviations do not agree so well. At the younger age the first method *under*estimates variance relative to an affine transformation. As the film ages and becomes more nearly affine (see Table 1) the first method *over*estimates variance.

IMMATERIAL TRANSFORMATIONS

An immaterial transformation is a transformation between the control grid and the

TABLE 1. VARIANCES, FIDUCIAL ARRANGEMENT 1, 81 RESIDUALS, 81-POINT FIT*

	Acetate Base						*Estar Base*	
Age	*11 Days*		*45 Days*		*109 Days*		*7 Days*	
Grid No.	*1*	*4*	*1*	*4*	*1*	*4*	*1*	*4*
Similarity								
S_{xx}	122.7	143.7	8.7	39.9	6.3	10.4	27.8	27.7
S_{yy}	99.1	155.6	10.8	39.1	7.5	30.1	28.7	31.5
Affine								
S_{xx}	45.1	22.6	4.6	10.4	3.7	5.3	2.6	2.2
S_{yy}	21.5	34.8	6.7	9.7	4.9	25.0	3.4	5.9
Projective								
S_{xx}	48.3	26.2	4.6	9.9	2.8	9.5	2.6	2.0
S_{yy}	13.1	20.2	5.6	6.1	4.6	9.8	3.2	5.7
Hyperbolic								
S_{xx}	44.8	22.4	4.1	10.0	3.7	4.9	2.3	2.2
S_{yy}	10.4	17.3	5.7	6.0	4.7	7.1	2.9	5.7
Higher								
S_{xx}	2.5	4.2	2.2	7.9	1.7	3.6	1.8	1.8
S_{yy}	4.6	14.3	4.1	5.9	3.8	5.8	2.4	3.6

* Note: This Table is taken from Reference 1.

TABLE 2. SYSTEMATIC TRENDS, FIDUCIAL ARRANGEMENT 1, 81 RESIDUALS, 81-POINT FIT*

	Acetate Base						*Estar Base*	
Age	*11 Days*		*45 Days*		*109 Days*		*7 Days*	
Grid No.	*1*	*4*	*1*	*4*	*1*	*4*	*1*	*4*
Similarity								
X	.22	.12	1.08	.31	.69	1.13	.54	.17
Y	.94	.73	1.06	1.17	1.25	.87	.80	1.02
Affine								
X	.60	.56	.98	.50	.90	.89	1.39	1.54
Y	1.07	1.09	1.64	1.29	1.70	1.05	1.96	1.84
Projective								
X	.56	.48	1.03	.43	1.07	.43	1.45	1.66
Y	1.15	1.37	1.83	1.70	1.85	1.42	2.03	1.91
Hyperbolic								
X	.60	.57	1.07	.44	.87	.82	1.69	1.54
Y	1.16	1.56	1.84	1.73	1.78	1.58	2.18	1.91
Higher								
X	1.47	1.19	1.42	1.27	1.36	.93	1.83	1.91
Y	2.17	1.69	2.31	1.79	2.14	1.87	2.50	2.30

* Note: This Table is taken from Reference 1.

TABLE 3. AVERAGE SHRINKAGE

	Brock and Faulds				*Affine Transformation*			
	Average Shrinkage		*Standard Deviation*		*Average Shrinkage*		*Standard Deviation*	
	X	Y	S_X	S_Y	X	Y	S_X	S_Y
Mar 1	73	66	3.26	3.47	73.1	66.4	6.7	4.5
4	75	68	3.29	4.33	75.4	67.2	4.8	5.9
Apr 1	40	40	2.18	2.85	40.5	40.3	2.1	2.6
4	48	45	3.72	2.88	49.1	45.4	3.2	3.1
Jun 1	31	32	2.70	2.61	31.3	32.5	1.9	2.2
4	33	33	2.37	3.53	33.3	33.6	2.3	5.0
Aug 1	9	12	2.07	2.69	9.3	12.2	1.6	1.8
4	5	9	2.00	3.05	5.1	8.7	1.5	2.4

* Note: This Table is taken from Reference 1.

film grid in which no deformation is involved. In general only the similarity transformation (perhaps with an inversion) is an immaterial transformation. For example, if one measures the control grid and then removes it, replacing it with the film grid which in turn is measured, then it is unlikely that the film grid will occupy exactly the same position on the comparator as did the control grid. The rigid motion required to bring the two grids to the same coordinate system is then immaterial; it in no way reflects film deformation. Some authors (Lampton and Umbach[4], Ziemann[5]) have apparently considered the similarity transformation inclusive of scale changes as immaterial. This is justified provided focal length is modified to counteract the scale change of the film.

In at least one instance one may arrive at an erroneous conclusion by not recognizing the existence of an immaterial transformation. Let us suppose that it is desired to find the relative efficiency of using four fiducial marks for determining film shrinkage. If one does this by comparing the residual variance as obtained by a least-squares fit over the whole grid plate to the residual variance obtained over the whole plate by a least-squares fit to only the four fiducials, it is possible that those intersections not used in the four fiducial least-squares fit will contain among them a significant immaterial transformation. The existence of the immaterial transformation will cause the computed variance to be greater than it ought to be. The way to overcome this problem is to fit the immaterial transformation to the intersections which have not been used in the primary adjustment. The resulting residuals and variance will be reduced accordingly.

Orthogonal Polynomials

Kheyfets[6] has shown us that we can apply two-dimensional orthogonal polynomials to topographic surfaces. His ideas are easily applicable to film deformation studies, especially if equally spaced grids are used. The basic advantage of orthogonal polynomials is that the normal equations are strictly diagonal. Thus one may add terms to the polynomial without influencing the previously computed terms. Consequently it is very simple to assess the influence of each term with regard to variance and systematic trend. It is also comparatively simple to study the effect of different grid spacings.

Summary

Three objectives pertain to film deformation studies. This paper has discussed several items one ought to consider in dealing with the third objective, i.e., the determination of corrections to be applied to photogrammetric images. These considerations include the separation of measurement error from film deformation by means of a systematic trend statistic, the idea of the immaterial transformation, and the possibility of using orthogonal polynomials.

References

1. Bender, L. and J. Tremlett, "Photogrammetric Film Shrinkage Transformations," Rome Air Development Center *Technical Report* No. RADC-TR-67-553, Dec. 1967.

2. Brock, R. and A. Faulds, "Investigation of Atmospheric Refraction and Film Shrinkage," Rome Air Development Center *Technical Report* No. RADC-TR-61-301, Nov. 1961.
3. Crow, Davis, and Maxfield, *Statistic Manual*, Dover Publication, 1960, page 62.
4. Lampton and Umbach, "Film Distortion Compensation Effectiveness," *Photogrammetric Engineering*, Nov. 1966, page 1035.
5. Ziemann, H., "Is the Request for Eight Fiducial Marks Justified?," *Photogrammetric Engineering*, Jan. 1971, page 67.
6. Kheyfets, B. S., "Approximation of a Topographic Surface by Chebyshev Orthogonal Polynomials," *Geodesy and Aerophotography*, No. 2, 1964, page 104.

Errata

In the article, "Psychophysics," by Mr. J. C. Trinder, the expressions in the second column of page 465, May 1971, should read:

$$\text{exponent} = -\ 0.322\ (\text{width})^{0.272}$$

$$S = 12(\Delta D - \Delta D_0)^{-0.322\ (\text{width})0.282}.$$

In Table 6 at the bottom of the same page, all the numbers on the right of the expressions are exponents:

$13.5(\Delta D - .03)^{-.297}$
$11.2(\Delta D - .01)^{-.403}$
$12.0(\Delta D - .0035)^{-.499}$
$18.8(\Delta D - .04)^{-.123}$
$7.3(\Delta D - .015)^{-.52}$
$12.3(\Delta D - .005)^{-.52}$
$9.0(\Delta D - .015)^{-.607}$
$13.7(\Delta D - .025)^{-.555}$
$11.0(\Delta D - .003)^{-.415}.$