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Accuracy of X-Parallax Clearance

Improvements in pointing can be obtained on instruments based on binocular viewing of symmetrical targets if stereoscopic viewing is artificially introduced by optical means.

ABSTRACT: *A method of investigating the accuracy of x-parallax clearance for sharp and blurred circular targets has been given. For sharp targets it was found to be independent of annulus width for annuli between 0.3 and 10 mrad, whereas it was a function of annulus width for widths smaller than 0.3 mrad. For blurred targets the accuracy deteriorated rapidly for targets with a grade of the density profile less than 0.3 $\Delta D/mrad$. A comparison is made between these results and accuracies for similar tasks of monocular and binocular pointing.*

INTRODUCTION

IN RECENT YEARS considerable research has been conducted on accuracies of monocular and binocular pointing to sharp and blurred targets. Very few investigations however, have been performed on the capabilities of stereoscopic pointing, or more particularly x-parallax clearance, to such targets.

Palmer (1960) investigated the accuracy of stereoscopic pointing for photographic details with particular reference to personal or systematic errors. He found that systematic errors increased as photograph image quality deteriorated. This is similar to the pattern of results obtained by Trinder (1971a) for monocular pointing to blurred targets. Investigating stereoscopic acuity under a number of different conditions, Anson (1959) found that the visual system can tolerate considerable blur before acuity noticeably deteriorates.

There are however, few quantitative results describing capabilities of stereoscopic tasks in photogrammetry. Investigations described in this paper have attempted to compare monocular, binocular and stereoscopic pointing accuracies for similar tasks, and the influence of image blur on these results. A distinction is made between binocular pointing, i.e., placing a measuring mark laterally in the center of a target using two eyes, and stereoscopic pointing, i.e., placing a floating mark in the plane of a target.

Research into stereoscopic pointing in photogrammetry may cover a very wide range of target types and sizes, and also target background conditions. To commence with, work in this field will concentrate on the accuracy of placing a circular black floating mark in the same plane as a bright circular sharp or blurred target with a uniformly dark background. These experiments are designed specifically to give a direct comparison between the accuracy of x-parallax clearance and results of monocular and binocular pointing to such targets. It is possible (O'Connor, 1967, 110) that there may be a relationship between binocular and stereoscopic pointing results "since the stereoscopic situation results from two vernier situations." The information reaching the two eyes is almost the same.

For the stereoscopic case however, the measuring marks move in relation to the targets in a different manner to the measuring mark movements in the binocular case. More particularly, stereoscopic viewing depends on the stereoscopic fusion of the two similar images. Therefore, there is a fundamental difference between binocular and stereoscopic viewing. Experiments presented in this paper indicate the influence of such differences on the pointing results.

EXPERIMENTAL PROCEDURE

Experimental equipment used for binocular and monocular pointing research (Trinder, 1971a, b) could be ideally suited to investigations on x-parallax clearance. To create the stereoscopic image, a dove prism was used to rotate through 180° the target and measuring mark viewed by one eye. As the target and measuring mark were both symmetrical, the images received by the two eyes could be fused to give the stereoscopic image. If the measuring mark was moved off-center, for one eye it appeared to the left, and for the other, to the right. The overall x-parallax was seen as a displacement of the fused measuring marks or floating mark in depth, in relation to the plane of the target.

To investigate the accuracy of x-parallax clearance, 10 sets of 10 observations were taken by subjectively placing the floating mark in the plane of the target using the average error method (Guilford, 1954). The standard deviation of these observations represented the pointing accuracy. The floating mark was always moved away from the observer in order that the task of placing the floating mark on the ground from above in a photogrammetric plotter was reproduced as closely as possible. Both sharp and blurred targets were observed, most of which were the same targets as observed previously.

Some aspects of stereopsis are discussed before presenting the results.

SOME ASPECTS OF STEREOPSIS

STEREOSCOPIC ACUITY

It is generally recommended (Ogle, 1962, 287) that stereoscopic acuity should be tested using the constant-stimulus method (Guilford, 1954), although this does not preclude the use of the average-error method if special circumstances warrant its use. As was encountered in (Trinder, 1971b), different results would be expected from the two methods. To reproduce specifically the observational methods in photogrammetry as closely as possible, the average-error method has been used.

Stereoscopic acuity depends on many factors, particularly the type of test target and observation conditions. There are similarities between visual and stereoscopic acuities in this regard and, although correlation seems to exist between them in some respects, information on this point is not conclusive. Attempts are made to relate visual and stereoscopic acuity results in this paper.

The stereoscopic threshold or acuity under a given set of conditions is the least difference in saggital distance that can be discriminated (Ogle, 1962, 286). As a general rule it is best described in terms of the disparity between images in the two eyes of two objects expressed in terms of difference in convergence angle. Ogle expresses it as:

$$\eta = 2a\Delta b/b^2$$

where $2a$ is the interocular distance, Δb is the stereoscopic threshold distance between the two objects, and b is the fixation distance.

For this study, inasmuch as x-parallaxes are to be observed, acuity may be expressed as

$$\eta = px/b$$

where px is the x-parallax between the two observed measuring marks, and b is the fixation distance.

OBSERVATION DISTANCE

Ogle (1958) maintains that stereoscopic acuity remains constant for observation distances ranging from 0.5 m to 10 m, provided secondary stereoscopic cues are not present to affect the results. Some variation seems evident in acuity; however, for observation distances less than 2m (Amigo, 1963). The observation distance of 10 m used in Trinder (1971a) was maintained for these experiments. Apart from the instrument and adaptation screen in front of the instrument, no other objects were present to provide secondary cues.

LEVEL OF ILLUMINATION

Ogle (1962, 294) states that like visual acuity, stereoscopic acuity increases with retinal illuminances and tends to approach a limiting value at high photopic levels. Over the range of luminances used in photogrammetric practice, it may generally be assumed that stereoscopic acuity will be close to its limiting value. A luminance level of 12 mL was considered satisfactory for these investigations.

OBJECT CONTRAST

It has been proved that contrast of sharp and blurred targets does not affect monocular or binocular pointing accuracies (O'Connor, 1967), (Trinder, 1971a) except for targets with very small annuli. Similar results have been indicated by (Ogle and Weil, 1958) in experiments on stereoscopy for 1/25 second exposure targets with a wide range in contrasts. Ample evidence is therefore available for assuming that contrast does not appreciably affect stereoscopic acuity except for targets near the visibility threshold.

UNEQUAL LUMINANCE FOR EACH EYE

For stationary objects it seems that stereoscopic acuity is not markedly affected even if luminances of the two images are substantially different (Ogle and Groch, 1956). An associated effect of unequal luminances however, is that for some types of objects such as squares erected at right angles to the line of sight (Ogle, 1962, 302), the object may appear to be rotated about a vertical axis through the fixation point. The target presented to the observer in these investigations was similar to such objects, and therefore the possibility of such a rotation should not be ruled out. No significant rotation was noted in these investigations however.

The Pulfrich stereophenomenon (Ogle, 1962) may occur if moving objects are viewed such that illumination reaching the two eyes is unequal. As the measuring mark in these investigations moved only very slowly, and finally came to rest, this phenomenon was not expected to affect the results.

UNEQUAL MAGNIFICATIONS OF THE TWO IMAGES

It is known (Ogle, 1962), (Julesz, 1963) and (Palmer, 1960), that stereopsis can still be maintained even though overall differences in magnification between the two images may be as much as 15–20 percent. No figures are available on the extent to which acuity may be affected however. If differences in magnification exist in one meridian only, distortions may appear in the object. Ogle (1962, 350) claims that image disparities caused by meridional differences in magnification of as little as 0.12 to 0.25 percent can be detected.

Effect of Use of Dove Prism

The dove prism used in the experiments was recovered from a military dial sight. A check on the quality of the prism revealed the following information. The angles of the two inclined faces with the prism base were found to be equal within the measuring accuracy of approximately 30 seconds of the spectrometer. Flatness tests using an optical flat and sodium light, revealed no significant deviations from flatness. The prism was therefore of high quality, and rays projected through it were free from significant distortions.

Rays transmitted through dove prisms undergo two refractions at the external faces and one internal reflection. No magnification is therefore introduced provided the prism has been accurately ground. The overall loss of light at the refraction and reflection surfaces of the prism would be approximately 10 percent. Therefore the prism acts in this regard similar to a neutral density filter of 0.05 density. Such a filter would not be expected to affect the stereoscopic acuity significantly. Although no rotation was detected in the target, marginal systematic errors in the estimation of the true depth may have resulted. As this aspect was not investigated in this study, the possibility of such errors was considered unimportant.

The observations were divided into 2 groups of 50 observations with the dove prism in front of a different eye for each set. This was done to detect any residual differences in acuity which may have been caused by unequal luminances, or small unequal magnifications in the two images. Such differences in acuity may have occurred if the behaviour of the visual system varied depending on which eye viewed through the dove prism. On no occasion did the two groups of observations prove to be significantly different, and therefore they were combined to give the standard deviation.

Observers

The one principal observer available to carry out the observations was *JCT*, who had performed the majority of observations in the previous investigations. The vision of *JCT* was tested in the Department of Optometry at the University of N.S.W. and found to be satisfactory. One other observer, *LB*, who was an experienced photogrammetric observer but not skilled at this specific task, was employed to check the results of *JCT*. A total of 40 observations were taken per target by *LB* on a sample of sharp targets.

Results of Observations

Results of observations to sharp and blurred targets are presented in Tables 1 and 2. All observations have been made at 10 m viewing distance, except the 10 mrad annulus sharp target, which was viewed at 5 m. For sharp targets the parameter is annulus width in radians subtended at the eye, whereas for the blurred targets the parameter is grade-of-blur of the density profile expressed as $\Delta D/mrad$, i.e., change in density per milliradian subtended at the eye. Background density of all targets was 0.3 except for the sharp targets with annulus width less than 200 μrad, which were of high contrast. It was found, similar to O'Connor (1967), that pointing to such small targets of low contrast gave very poor results.

TABLE 1. STANDARD DEVIATIONS DERIVED BY STEREOSCOPIC OBSERVATIONS TO SHARP TARGETS

Annulus Width (mrad)	*Standard Deviation (μrad)*	
	Observer	
	JCT	*LB*
0.05	2.2	
0.11	3.8	
0.20	5.6	
0.50	7.8	27.8
1.0	8.4	
2.0	7.3	18.6
5.0	8.4	21.8
10.0	8.0	

TABLE 2. STANDARD DEVIATIONS DERIVED BY STEREOSCOPIC OBSERVATIONS TO BLURRED TARGETS. ANNULUS WIDTH OF TARGETS WAS 2.0 MRAD

Grade of Blur $\Delta D/mrad$	*Observer JCT Standard Deviation μrad*
0.031	86.6
0.037	54.0
0.046	52.7
0.065	43.3
0.082	28.0
0.12	18.0
0.195	14.8

The blurred target results in Table 2 all had an annulus width of 2 mrad. Inasmuch as Table 1 indicates that annulus width does not affect pointing accuracies for widths from 0.3 and 10 mrad, it was considered unnecessary to observe blurred targets with different annulus widths.

A displacement of the measuring mark on the target is seen as a displacement in opposite directions by the observer's two eyes. The effective x-parallax is therefore twice the displacement actually recorded. As the standard deviations observed may be considered as representing the Difference Limen (*DL*, Trinder, 1971b), the *DL*'s of x-parallax clearance are twice the measured standard deviations in Tables 1 and 2. This procedure gives values identical to the stereoscopic acuity formulas previously defined. The values in Tables 1 and 2 multiplied by 2 are presented in Figures 1 and 2. Results of *LB* have not been plotted in Figure 1, but they generally confirm the findings made on observations by *JCT* in the next section. Monocular pointing accuracies obtained by O'Connor (1962, 1967) and Trinder (1971a) have been included in these figures.

Stevens' psychophysical formula (Stevens, 1962) which was successfully applied to results of monocular pointing (Trinder, 1971b) has also been applied to the accuracies in Figure 2 and presented in Figure 3. Corresponding graphs obtained for monocular pointing have been included.

DISCUSSION

The results in Figure 1 of pointing accuracies for sharp targets give a direct comparison between monocular, binocular and stereoscopic observations. It has been proved (Roger *et al*, 1969) (Trinder, 1971b) that similar accuracies are obtained by monocular and binocular pointing. The accuracies for monocular pointing presented in Figures 1 and 2 may therefore be assumed to apply also to binocular pointing to similar targets. For stereoscopic observations in Figure 1 almost identical results are obtained for annuli less than 1 mrad, but there are deviations from binocular accuracies for annuli greater than 1 mrad. The comparison over the range of annulus widths less than 1 mrad indicates that similar criteria may be used by the visual system for pointing over this range. Attempts to explain this section of the graph for monocular pointing by investigating the shape of the luminance profile actually seen by the observer, have been made in Trinder (1971c).

For annuli from 1 mrad to 10 mrad in Figure 1, no correlation occurs between stereoscopic and binocular pointing accuracies, although the information received by the visual system is very similar in both instances. However, in the stereoscopic situation the two measuring marks move in apparently opposite directions.

From Table 1 it may be deduced that for observation systems based on binocular pointing to sharp symmetrical targets with uniform background, greatly improved results can be obtained if a dove prism is introduced into the observation system, such that an artificial stereoscopic observation is made. Indeed, the pointing accuracies obtained by such a system are presented in Table 1 and result in improvements of stereoscopic over binocular viewing, ranging from a factor 2 for small annuli to 10 to 12 for a 10 mrad target.

For blurred targets, stereoscopic pointing accuracies are substantially different from

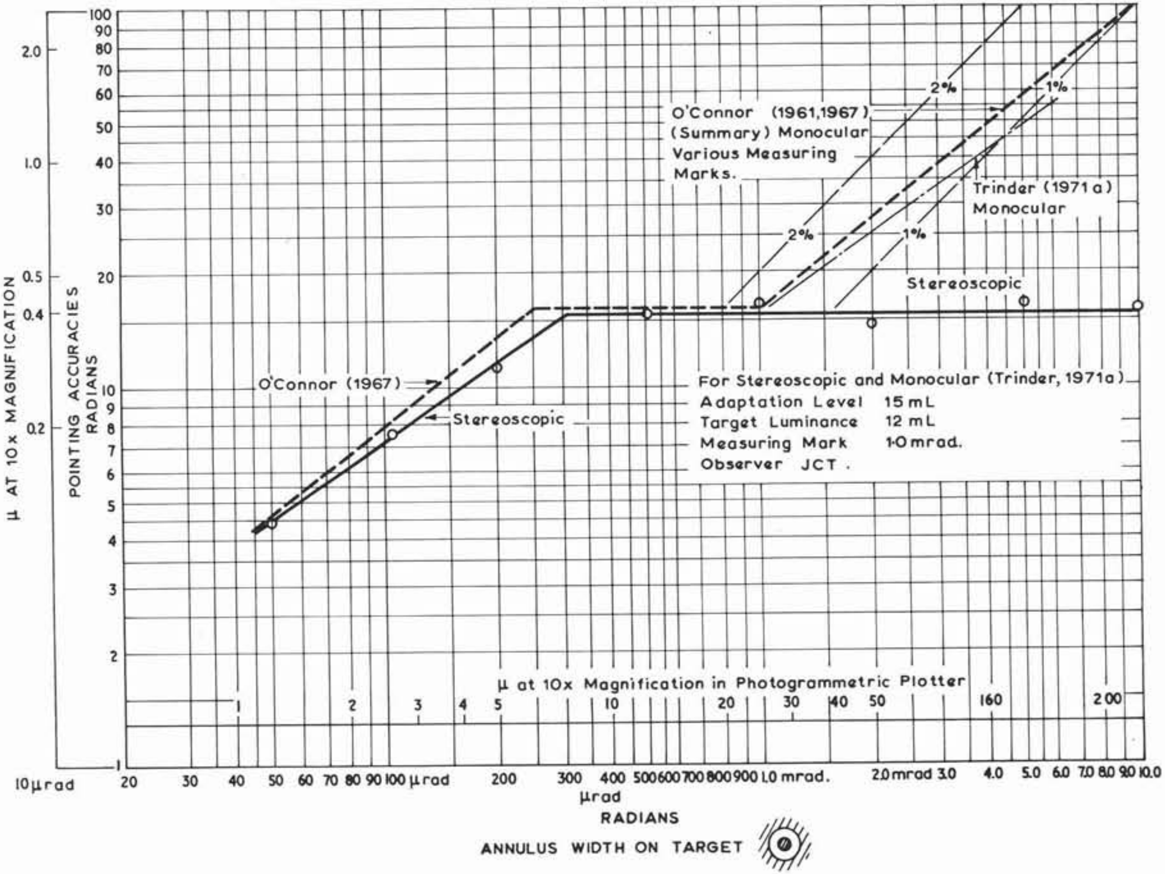

FIG 1. Stereoscopic pointing accuracies for sharp targets in terms of annulus widths. A summary of monocular pointing accuracies derived by O'Connor (1967) and Trinder (1971a) have been added for comparison.

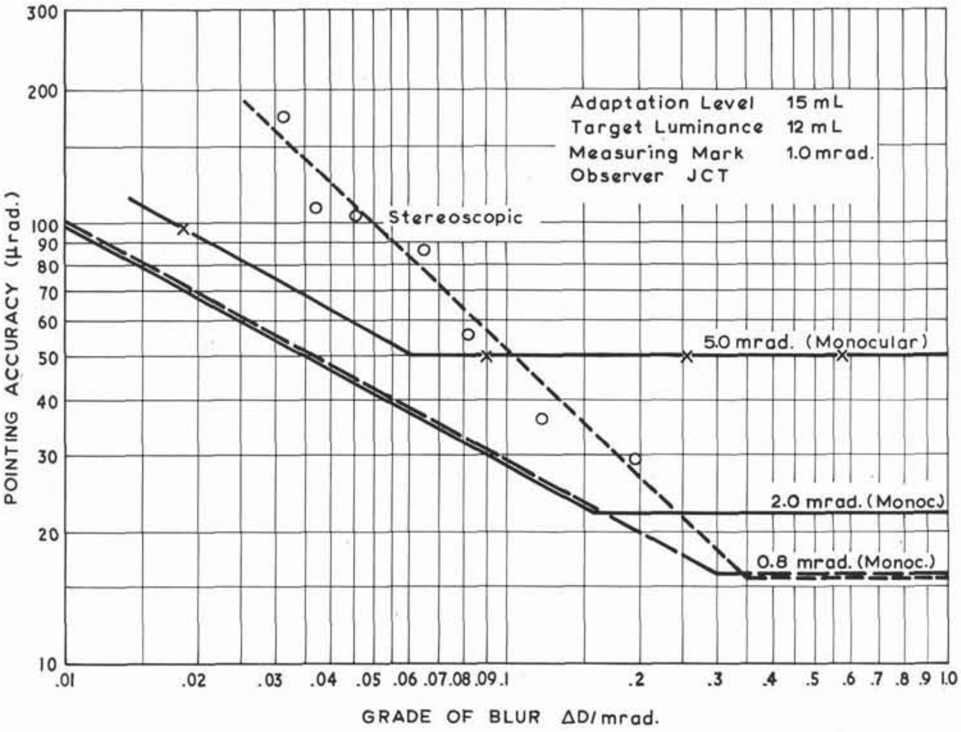

FIG. 2. Stereoscopic pointing accuracies for blurred targets against the grade of the target density profile, expressed in terms of *ΔD/mrad.* Monocular pointing accuracies obtained on similar targets (Trinder, 1971a) with annulus widths as shown have also been added.

monocular and binocular pointing accuracies for grades of blur less than 0.3 ΔD/mrad. Although the coefficient of Stevens' formula in Figure 3 is very similar, the exponent is smaller, indicating the greater difficulty encountered in the stereoscopic observation. In Figure 2 however, approximate correlation occurs between binocular and stereoscopic results in the general shape of the lines, and the point at which accuracies deteriorate from values that were obtained for sharp targets. Although the shapes of the curves are similar, however, the magnitudes are significantly different.

Anson (1959), investigating stereoscopic acuity, found that a considerable loss in resolution of the target can be tolerated before stereoscopic acuity deteriorates provided the two images viewed are alike. The level of target blur at which stereoscopic acuity deteriorates according to Figure 2, is approximately 0.3 ΔD/mrad. The grade of target density profiles given by Hempenius (1964) range from 0.15 to 0.32 ΔD/mrad if the targets are viewed in an instrument at 10× magnification with perfect optics. Though Hempenius' targets are only a few specific examples, indications are that stereoscopic acuity for such targets used in practice will remain close to its optimum value. On the other hand if image qualities are significantly worse than about 0.2 ΔD/mrad, substantial reductions in accuracy can be expected.

Explanations of the accuracies obtained in Figures 1 to 3 are not clear. A comparison of results of stereoscopic and binocular observations highlights the very complex nature of the visual system. The only information given to the observer in both tasks of stereoscopic and binocular pointing are the edges of the measuring mark and target. The existence of contours subjectively viewed at such edges, or changes in luminance, has been well established. O'Connor (1967) and Roger *et al* (1969) have discussed their characteristics under different viewing conditions, and the neural processes which may lead to their formation. However, as research on such processes is still in its early stages, it is not possible to deduce the significance of such contours on binocular or stereoscopic viewing. On no occasions were Mach bands actually seen in these observations.

In the instances of binocular and stereo-

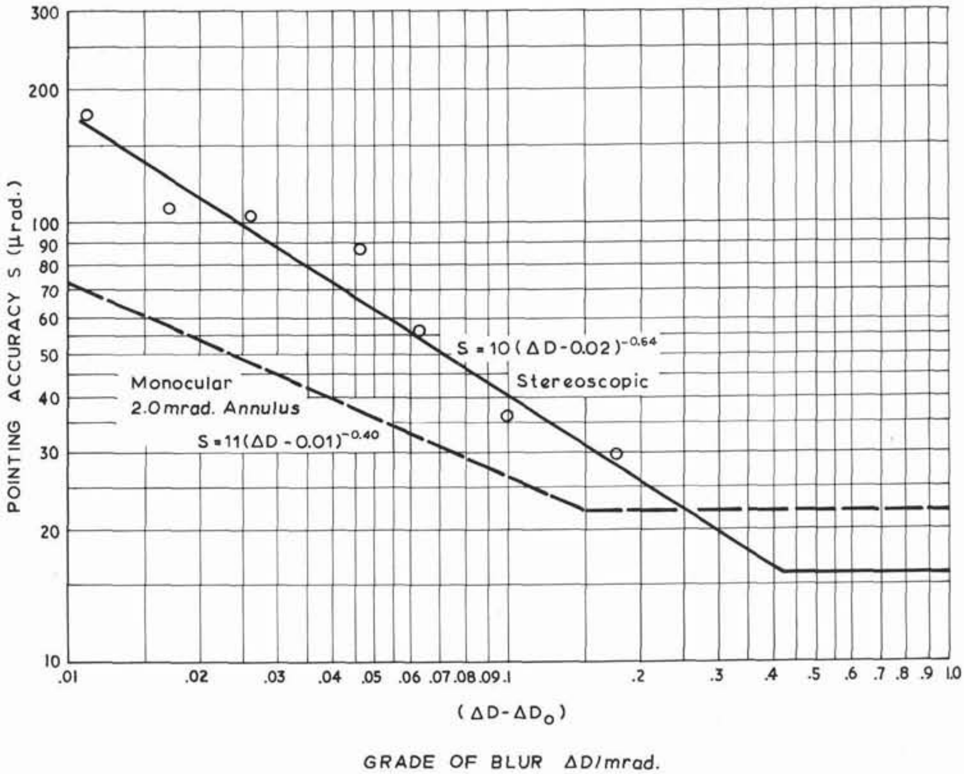

FIG. 3. Stereoscopic pointing accuracies S for blurred targets expressed in terms of $(\Delta D - \Delta D_0)$ as derived by Stevens' formula (Stevens, 1962). ΔD is the grade of target density profile, ΔD_0 is the threshold level of blur derived by experience with near-threshold targets.

scopic viewing, the same subjective stimulus patterns would presumably exist because the same objects are viewed in both cases. These stimulus patterns however are used by the visual system in different ways. For binocular pointing they are used to obtain measurements of the distance between the measuring mark and target edges except over the range of annuli smaller than 0.3 mrad where pointing seems to depend on a different criterion (Trinder, 1971c). For stereoscopic pointing, the edges are used as the basic information for determining depth. The visual process involved is the elimination of retinal disparities between corresponding object points.

The capabilities of these two different functions of the visual system vary depending on the target characteristics. Approximate correlation between the capabilities of the two functions would be expected, but the wide variations are due to the inherent differences between the two processes. Whereas the stereoscopic processes of elimination of retinal disparities are superior to the measuring processes of binocular pointing for most sharp targets, they become inferior for blurred targets. This is not surprising because for stereoscopic pointing to blurred targets, corresponding blurred edges must be located on each object and disparities between them eliminated. For binocular pointing, one sharp measuring mark edge and one blurred target edge only must be located to evaluate the width of the annuli on each side of the measuring mark.

The size of sharp targets at which pointing accuracies deteriorate from the level of 16 mrad in Figure 1 has not been found, because targets larger than 11 cm diameter could not be placed in the instrument. The 10 mrad target was viewed at 5 meters in an attempt to test this point. Although Ogle (1962) maintains that stereoscopic acuity is constant for observation distances between 0.5 m and 10 m, it was considered inadvisable to reduce the observation distance more than by a factor of 2. In addition, a further reduction in viewing distance would render the instrument insufficiently accurate.

O'Connor (1967, 97) found that monocular pointing accuracies remained constant if narrow target annuli were separated by a measuring mark up to at least 40 mrad in diameter. This indicates that pointing accuracies seem to remain constant over a significant width on the fovea although the involuntary micro-nystagmus of the eyes is also likely to play a role. The constancy of stereoscopic pointing to sharp targets for annulus widths of 0.3 to 10 mrad is likely to be related to O'Connor's findings. Stereoscopic pointing accuracies for sharp targets therefore may remain constant up to an annulus width of at least 20 mrad.

The general theories of Andersen and Weymouth (1923) to explain acuity results still seem to be the most satisfactory available, as proposed in Trinder (1971a). The exact manner in which the target and measuring mark positions are resolved by the visual system, however, is as yet unknown, since explanations of Andersen and Weymouth only indicate the way in which the visual system may behave generally, and not the complex neural processes which combine to produce this behaviour. Without further knowledge on the visual system it is impossible to give more detailed explanations of the results obtained in this paper.

CONCLUSIONS

★ The relationship between stereoscopic and visual acuities for sharp and blurred targets vary depending on target characteristics.

★ Stereoscopic pointing results remain constant and independent of target annulus width for annuli greater than 0.3 mrad, at least up to 10 mrad annuli. For annuli smaller than 0.3 mrad, stereoscopic pointing accuracies increase, being substantially the same as monocular and binocular pointing results.

★ Stereoscopic pointing accuracies to blurred targets deteriorate more rapidly for increasing blur than do monocular or binocular pointing accuracies.

★ Substantial improvements in pointing accuracies can be obtained on instruments based on binocular viewing to symmetrical targets if stereoscopic viewing is artificially introduced by optical means so that the observation is made stereoscopically.

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