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# Calibration of a Comparator

**A computer program, applying least squares and weighting the master grid values, finds the errors of the instrument as well as the observational accuracy.**

## INTRODUCTION

PHOTO COORDINATES are an essential requirement for both analytical aerial triangulation and calibration of survey cameras. The simultaneous adjustment procedure also requires the standard error of the photo coordinates. Generally photo coordinates are measured using stereo or mono comparators and all measuring instruments are frequently calibrated to check their stability.

A comparator is calibrated in order to determine the precision of observations, the standard error in $x$ and $y$ coordinates, the systematic errors and the stability of the comparator readings over a period of time.

Generally, comparators are calibrated using standard grid plates. The coordinates of the grid plates are subject to errors. In the methods described by Prof. B. Hallert[1] and Mr. G. H. Rosenfield[2] the errors originating in the grid plate coordinates cannot be eliminated.

ABSTRACT: *The objective of this paper is to describe a procedure of calibrating a comparator using two grid plates with known standard error and then completing a simultaneous adjustment carrying the comparator and grid coordinates as unknown parameters. The paper also describes a method of obtaining the standard error of the photo coordinates using the standard error of the grid coordinates. In the existing literature on this subject no method of correcting the errors originating in grid plates is offered. Further, existing literature describe only procedures of obtaining the standard error of the precision of observations, but not the standard error of the photo coordinates.*

The objective of this paper is to describe a procedure for calibrating a comparator using two grid plates of known standard errors and to perform a simultaneous adjustment, carrying the comparator and grid coordinates as unknown parameters. The paper also describes a method of obtaining the standard error of the photo coordinates using the standard error of the grid coordinates.

The method described in this paper is used to calibrate the Zeiss PSK stereocomparator of the Ohio State University and the results given below are from this calibration.

The first section of the paper explains the theory and the method of obtaining the precision of the comparator observations. The second section deals with the theory of obtaining standard error in $x$ and $y$ coordinates using simultaneous adjustment procedures. The third section analyses the results obtained in the calibration of the PSK comparator. The final section gives a flow chart of the computer program used for computation.

## PRECISION OF OBSERVATIONS

### METHOD

The selected points on the grid plates are numbered consecutively. The plates are placed on both sides of the comparator with a slight Kappa rotation with respect to the comparator grid. The readings are then taken on both plates for all points. This may be repeated any number of times, e.g., $L$ times

### THEORY

In order to compute the precision of observation, the mean and residuals $v$ of the coordinates on each point and on each plate were computed. The standard error was computed for each point from

$$\sigma_i = \sqrt{[\Sigma(V_x^2 + V_y^2)/(L-1)]}$$

where $L$ is the number of observations at a point $i$, and $V_x$, $V_y$ being residuals in $x$ and $y$ coordinates respectively. Hence the standard error of observation

$$\sigma = \sqrt{[\Sigma\sigma_i^2/4N]}$$

$N$ being the total number of points. The number of degrees of freedom is $4N$ because $x$, $y$ coordinates are measured in two plates.

## STANDARD ERROR IN X AND Y COORDINATES

### THEORY

In Figure 1 suppose $x$, $y$ refers to grid-plate coordinate system, and $x'$, $y'$ refers to comparator coordinate system. Ideally the two systems must be related to each other by a rigid-body transformation and change of scale (isogonal affine transformation),

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = S\begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ c' \end{bmatrix} = \begin{bmatrix} Ax + By + c \\ -Bx + Ay + c' \end{bmatrix}$$

where $S$ is a common scale factor, $\varphi$ is the angle shown in Figure 1, and$A$, $B$, $c$, $c'$ are constant transformation parameters. Using the above equations it is possible to make a least square adjustment and obtain values of the parameters $A$, $B$, $c$, $c'$. The variance of unit weight is then given by

$$\sigma^2 = \Sigma\{(x' - x'a)^2 + (y' - y'a)^2\}/(2N - 4)$$

where $x'a$, $y'a$ are the adjusted values.

In practice we have the following points to consider:

- The grid coordinates are subject to errors.
- The scale of both the comparator and grid plate in $x$ and $y$ directions may not be the same.
- The angle between the $x$ and $y$ axes of both the comparator and grid plate may not be at right angles.
- Mechanical errors in the comparator, namely, backlashes, rounding-off errors, etc.

An observer has no control over the errors due to the last item except that he can perform the observation in a systematic sequence and control the maximum limits of these errors.

Manufacturers of grid plates do not, in general, give errors on the orthogonality of the $x$ and $y$ axes. However, the standard error of $x$ and $y$ coordinates are available. Thus we will assume that the grid plate axes are perpendicular to each other with zero error.

In Figure 2, suppose $s$ and $s'$ are the scale factors along $x'$ and $y'$ axes. If the $x'$ axis makes an angle $\varphi$ with $x$ axis, and the $y'$ axis makes an angle $\varphi'$ with the $x$ axis, then we have

$$x' = 1/s(x\cos\phi + y\sin\phi) + c_1$$
$$y' = 1/s'(x\cos\phi' + y\sin\phi') + c_2 \tag{1}$$

which can be written as (a general affine transformation)

$$x' = A_1x + B_1y + c_1$$
$$y' = A_2x + B_2y + c_2 \tag{2}$$

where $A_1$, $B_1$, etc., are the transformation parameters.

Now suppose we treat the grid coordinates as observed quantities. We will have the following equations in our mathematical model for adjustments:

$$x' - (A_1x_a + B_1y_a + c_1) = 0$$
$$y' - (A_2x_a + B_2y_a + c_2) = 0$$
$$x - x_a = 0$$
$$y - y_a = 0$$

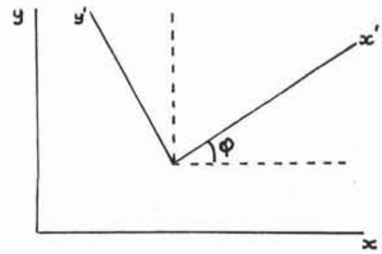


FIG. 1. Coordinate transformation: $x$, $y$ refer to grid-plate system, and $x'$, $y'$ to the comparator.

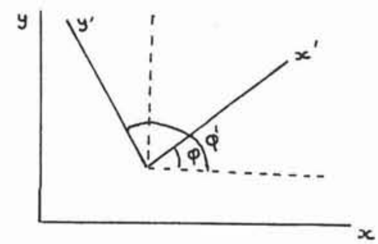


FIG. 2. Definition of the lack of orthogonality $\phi$ between the grid-plate and comparator coordinate systems.

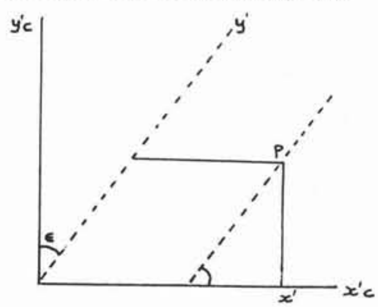


FIG. 3. The angle $\epsilon$ is the corrected direction of the comparator $y'$-axis.

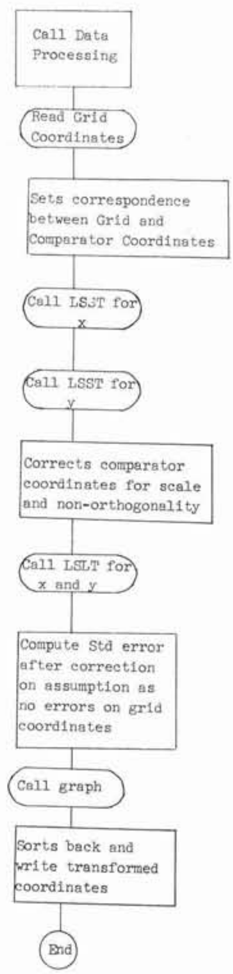


FIG. 4. Flow chart of the main computer program.

where $x_a$, $y_a$ are the approximate values of $x$, $y$. From this we will have the following observation equations:

$$\begin{pmatrix} V_{x'} \\ V_{y'} \\ V_x \\ V_y \end{pmatrix} - \begin{pmatrix} x_a & y_a & 1 & 0 & 0 & 0 & A_1 & B_1 \\ 0 & 0 & 0 & x_a & y_a & 1 & A_2 & B_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta A1 \\ \Delta B_1 \\ \Delta C_1 \\ \Delta A_2 \\ \Delta B_2 \\ \Delta C_2 \\ \Delta x \\ \Delta y \end{pmatrix} + \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{pmatrix} = 0$$

where

$$W_1 = x' - (A_1 x_a + B_1 y_a + c_1)$$
$$W_2 = y' - (A_2 x_a + B_2 y_a + c_2)$$
$$W_3 = x - x_a$$
$$W_4 = y - y_a.$$

At this stage we have to consider the weights. The standard error of the grid coordinates are known (unlike the standard error of the comparator coordinates), and the standard error of the precision of observation of the comparator observations is also known. Using this and the fact that a weight is $\alpha 1/\sigma^2$, we can estimate the weights. Also notice that the adjustments of observations of $x'$ and $y'$ can be handled independently.

Suppose we adjust using $x'$ only and suppose $Px$ and $Pcs$ are the weights on comparator and grid coordinates respectively. We can then rewrite the observation equations as

$$\begin{aligned} -V_{x'} + A1\delta + A2\ddot{\delta} &= x' - (A_1 x_a + B_1 ya + c_1) \\ -V_x + (1, 0)\ddot{\delta} &= x - x_a \\ -V_y + (0, 1)\ddot{\delta} &= y - y_a \end{aligned}$$

where $\dot{\delta} = (\Delta A, \Delta B, \Delta c)$ and $\ddot{\delta} = (\Delta x, \Delta y)$, and $A1$ and $A2$ are corresponding vectors. Thus we have the normal equations as

$$A1^t Px A1\delta + A1^t Px A2\ddot{\delta} = A1^t Px E1$$
$$A2^t Px A1\delta + (A2^t Px A2 + \overline{P}cx)\ddot{\delta} = A2^t Px E1 + \overline{P}cx\overline{E}_2$$

where

$$E1 = x - (A_1 x_a + B_1 y_a + c_1)$$
$$\overline{E}_2 = \begin{pmatrix} x - x_a \\ y - y_a \end{pmatrix}$$
$$\overline{P}_{cx} = \begin{pmatrix} Pcx & 0 \\ 0 & Pcx \end{pmatrix}.$$

We can write the normal equations as

$$N\begin{pmatrix} \delta \\ \ddot{\delta} \end{pmatrix} = W$$

and

$$\begin{pmatrix}\dot{\delta}\\ \ddot{\delta}\end{pmatrix} = N^{-1}W$$

are the corrections for the parameters
where

$$N = \begin{pmatrix} A_1{}^tPxA_1 & A_1{}^tPxA2 \\ A_2{}^tPxA_1 & (A2^tPxA2 + \overline{P}cx) \end{pmatrix}$$

$$W = \begin{pmatrix} A1^tPxE1 \\ A2^tPxE1 + \overline{P}cx\overline{E}2 \end{pmatrix}.$$

Using $[\dot{\delta}, \ddot{\delta}]^T$, the adjusted values of $A_1$, $B_1$, $c_1$, $x_a$, $y_a$ can be obtained. If necessary an iterative procedure can be used. In general two or three iterations will be sufficient.

From the adjusted values, the standard error of unit weight can be obtained in the usual way by computing the residuals. Let the standard error of unit weight be $\sigma_0$. Now if our weighting is correct (i.e. our hypothesis is correct) then the given grid standard error $\sigma_x{}^2=\sigma_0{}^2/Pcx$ can be tested using the chi-square test. If it does not satisfy the chi-square test we can estimate a new value of $Pcx$ using $\sigma_0{}^2$ and $\sigma_x{}^2$. Using the new value of $Pcx$ and old $Px$ we can recompute a new value of $\sigma_0{}^2$ which should satisfy the chi-square test on $\sigma_x{}^2=\sigma_0{}^2/Pcx$. It should be possible to do so with one or two trials. Thus, knowing $\sigma_0{}^2$, the standard error in the comparator $x$ coordinate will be given by $\sigma_{x'}{}^2=\sigma_0{}^2/P_x$.

Similar procedures can be used for estimating $\sigma_{y'}{}^2$. The adjustment procedure also gives the values of $A_1$, $B_1$, $C_1$, etc. Thus, using Equations 1 and 2, we have

$$1/S^2 = A_1{}^2 + B_1{}^2; \quad 1/S'^2 = A_2{}^2 + B_2{}^2$$

i.e.,

$$S = 1/\sqrt{A_1{}^2 + B_1{}^2}; \quad S' = 1/\sqrt{A_2{}^2 + B_2{}^2}$$

$$\tan\phi = B_1/A_1 \text{ and } \tan\phi' = B_2/A_2$$

$$\phi = \tan^{-1}(B_1/A_1) \text{ and } \phi' = \tan^{-1}(B_2/A_2).$$

Ideally, $\varphi'-\varphi=\pi/2$ but, due to mechanical errors, etc., in practice we will have $(\varphi'-\varphi)=\pi/2-\epsilon$ (say).

These values of $S'$, $S$ and $\epsilon$ may be used for future work as follows. From Figure 3 it is evident that the corrected value of comparator coordinate is given by

$$x_c' = Sx' + S'y' \sin\epsilon$$

$$y_c' = S'y' \cos\epsilon$$

where $x_c'$, $y_c'$ are the corrected values of $x'$, $y'$ with a standard error of $\sigma_x'$, $\sigma_y'$ as computed earlier. However, the parameters $A$, $B$, etc., can be used directly.

## Analysis of the Results

Based on the above theory, the PSK comparator was calibrated and the results are as follows.

The standard error of precision of one observation is obtained as 1.5 $\mu$m. On weighting $Px:Pcx=1:2$, $\sigma\hat{x}$ becomes 0.8 $\mu$m, and on weighting $Px:Pcx=1:3$, $\sigma\hat{x}$ becomes 0.7 $\mu$m, which means that the chi-square test will be satisfied as the given $\sigma\hat{x}=0.7$ $\mu$m.

Using these results, the standard error in both $x$ and $y$ comparator coordinates is obtained as 1.2 $\mu$m. This is to be expected as we are using the mean values of the ob-

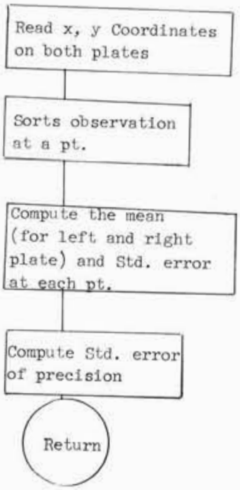


FIG. 5. Flow chart for the subroutine for processing the data for $N$ points.

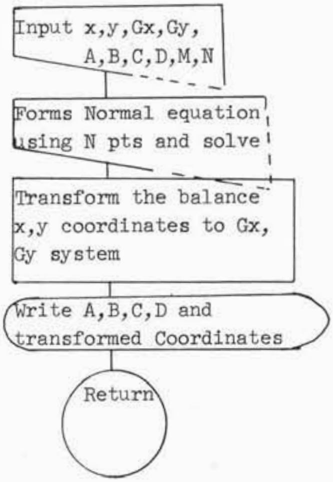


FIG. 6. Flow chart for Subroutine LSLT, applying Equations 2, using least squares, where $M$ is the number of $x$, $y$ points and $N$ is the number of $Gx$ and $Gy$ points.

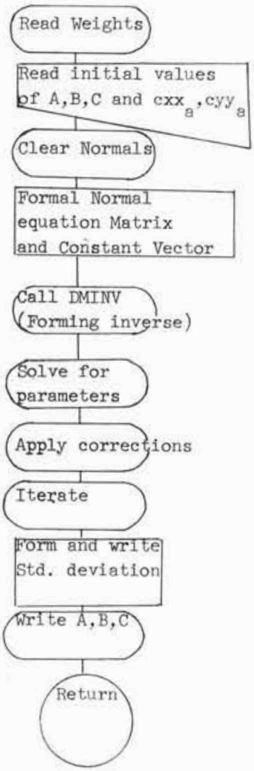


FIG. 7. Flow chart for Subroutine LSST.

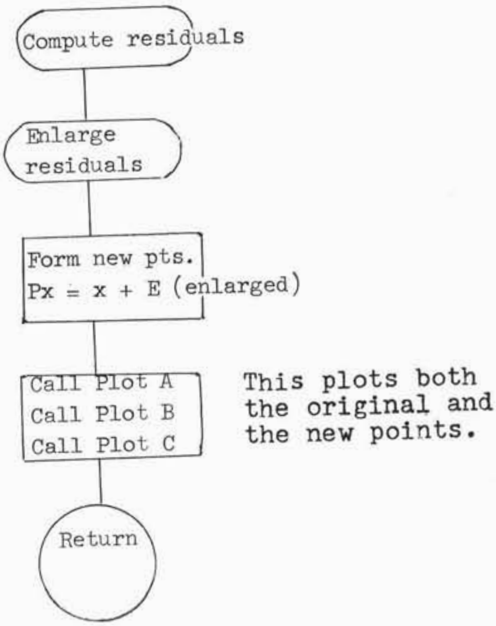


FIG. 8. Flow chart for Subroutine Graph.

served coordinates at each point. In other words, the standard error of precision of the mean should be $1.5/\sqrt{L}$, $L$ being the number of observations.

On performing a rigid-body transformation, together with a scale factor, the standard error in $x$ and $y$ is obtained as 1.65 μm, on the assumption that: the scale in both $x$ and $y$ directions are the same, $x$ and $y$ axes are at right angles and there are no errors in the grid coordinates.

Using the scale factor and the correction for non orthogonality given in the rigorous adjustment, and performing the transformation as before on the assumption that the grid coordinates are not subject to error, the standard error is obtained as 1.5 μm; which shows that 0.15 μm error is due to a systematic error in scale and nonorthogonality and the residual of 0.3 μm is due to errors in grid coordinates.

## Computation and Programming

Based on the theory stated above, a computer program was written in Fortran for IBM 360/75 to perform the necessary computations. Flow charts of the program are shown in Figures 4–8.

## References


1. B. Hallert. "Test measurements in comparators and tolerance for such instruments," *Photogrammetric Engineering*, Vol. 29, No. 2, pp 301–314, 1963.
2. G. H. Rosenfield. "Calibration of a precision coordinate comparator," *Photogrammetric Engineering*, Vol. 29, No. 1, 1963.