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Pointing Precision, Spread Functions and MTF

If a suitable relationship can be demonstrated for pointing precision to isolated objects using modulation sensitivity, it may widen the application of the modulation transfer function.

Introduction

The precision of pointing to sharp and blurred circular targets has been presented to date, in terms of target parameters of width, grade of the density profile, and background density characteristics (O'Connor, 1967a), (Trinder, 1971a). These results have been obtained on specially designed instruments with very high accuracies. Pointing precisions obtained by these workers are therefore not influenced significantly by the measuring instrument, and hence describe the pointing capabilities of skilled observers. Small variations may be expected for different individuals however.

Abstract: *The spatial frequency components of circular blurred targets observed in a previous work have been derived theoretically by convolving the MTF of different imaging systems, and the Fourier Transform of circular ground targets. A relationship may then be derived between the pointing precisions of the computed targets, the spread function, the ground target size and contrast, and frequency limit as determined from the contrast sensitivity of the visual system to repetitive sine-wave targets. Simple relationships may then be derived between ground target size and the spread function on the one hand, and ground target size and Modulation Transfer Function on the other, to give optimum pointing precision for a given imaging system.*

The significance of systematic errors in pointing should be emphasized. Precision in this paper is expressed as the standard deviation of repeated observations. Systematic error is the difference between the observed mean and the correct central position of the target. Pointing to circular targets can reach extremely high precisions for targets with small annuli. However, for both small and large targets, systematic errors may be as large as 5 times the standard deviation. In addition, they may change from day to day, or even from hour to hour, erratically. For a high measuring accuracy, the systematic errors must be controlled or eliminated. One technique of eliminating systematic errors has been given in Trinder (1972). The present article is restricted to a discussion of pointing precision, but the possible existence of systematic errors should be kept in mind.

Pointing precisions have not been analyzed in terms of the target spatial frequency components computed from the Fourier Transform of the target, nor have they been analysed in terms of the Modulation Transfer Function (MTF) or granularity in photographs. A study of these aspects will be undertaken in this paper.

In a technique derived by Hempenius (1964) for analyzing the influence of MTF's of the different components of imaging systems, the behavior of the visual system was presented as the contrast sensitivity of the visual system to repetitive sine-wave targets of different spatial frequencies, or modulation sensitivity—in future abbreviated to MS. Granularity was introduced by reducing the MS of the observer, as a function of granularity and modulation of the targets. A similar approach to that of Hempenius was adopted for this study.

The first step in these investigations was to analyze the relationship between the spatial frequency components of blurred targets and pointing precision, where the influence of the visual system was introduced as MS (Hempenius, 1964). That is, isolated details were analyzed using the visual response curve for continuous sine-wave targets. This operation may not be strictly valid (Hempenius, 1964), (Brock, 1964) because the visual system may not perform the same for isolated details as it does for repetitive targets. However, this step was taken in order to determine whether indeed a relationship could be derived. The MS of the visual system has been determined by a number of researchers. No similar relationship is available for isolated targets because they are composed of a number of spatial frequencies at different intensities. This is probably one of the reasons why MTF's have not been used more extensively in the past. If a suitable relationship can be demonstrated for pointing precision to isolated objects using MS, it may widen the application of MTF's in photogrammetry.

The problem in Step 1 may therefore be formulated as follows:

- Is there a relationship between pointing precision, spatial frequency spectra of targets, and MS?
- If such a relationship exists, is the size of the circular ground target, which will result in the highest pointing precision, a function of MS?

In Step 2 of these investigations the optimum target size was analysed in terms of MTF's and point spread functions (PSF), viz:

- Is there a relationship between optimum ground target size, and the PSF and MTF of the imaging system?
- If so, what accuracies can be expected from such targets?

Although these questions may not be answered in this article for all imaging systems, considerable progress has been made in treating imaging systems whose PSF's, in practice, approximate to Gaussian functions (Hempenius, 1969), (Wolfe and Tuccio, 1960). It may also be possible to use these results for non-Gaussian PSF's although this step has not been tested.

Pointing precisions used have been derived in Trinder, (1971a) by observing in the x-direction, i.e., parallel to the eye-base. It has been established (O'Connor, 1967b) that precisions in the y-direction are approximately 30% larger than in the x-direction. Therefore, for precisions in the y-direction, values quoted in this paper should be increased by 30%.

Results are derived and presented in terms of angular subtense in radians. For conversion to linear dimensions at a given magnification the following step can be taken:

$$\text{Linear Dimension}(\mu m) = \frac{\text{Angular Subtense (mrad)} \times (250)}{\text{Magnification}}.$$

Magnification can be considered as an additional parameter in the figures presented, and indeed an optimum magnification as well as ground target size may be envisaged (Hempenius, 1969, 230); it is not presented as a variable in this article, however.

Computation of Blurred Target Density Profiles and Frequency Spectra

The circular blurred targets observed in Trinder (1971a) were photographed in a simple light box by optically convolving original sharp ground targets, and Gaussian PSF's with various σ-values, representing imaging systems of different qualities. Here *imaging system* includes filter camera, film, reproduction equipment, plotting instrument, etc. This operation theoretically gave images equivalent to blurred targets on aerial photographs viewed in a plotter. Although the density profiles of the resulting blurred targets were accurately measured by a microdensitometer, indicating a close agreement with the shape of a Gaussian intensity profile, accurate estimates of the σ-values were not possible. The slope of the profile, which was accurately measured on the density profile, was used as the parameter for blurred targets (expressed as $\Delta D/mrad$).

The size of ground target and σ of the Gaussian PSF can be more accurately controlled by *computing* the convolution. The density profile, which can be transformed from the intensity profile using methods indicated by Hempenius (1969, 13), and its slope, can then be related to pointing results. This computation can be performed directly in the spatial domain, using the standard convolution formula, but the process is often very cumbersome and time consuming. Alternatively, it can be performed in the frequency domain, by computing the Fourier Transforms of both PSF and ground target, and multiplying the two functions at corresponding frequency values. The blurred-target intensity profile can then be derived by computing the inverse Fourier Transform of this product. The Fourier Transform of the PSF is the MTF, assuming no phase shift in the imaging system, and it is a Bessel Function for the ground target.

Not only is the operation in the frequency domain more suitable for the computation, but it also provides the spatial frequency spectrum of the blurred target. Computations for this article were performed in the frequency domain using the Fast Fourier Transform (FFT) scientific subroutine of IBM, and profiles in both the spatial and frequency domains were plotted on a Calcomp plotter.

(In the following discussion the term *ground target contrast* refers to the contrast of a ground point at the camera lens. The relationship between this contrast and the actual contrast on the ground may be derived from the formula given by Brock, et al. (1966). *Blurred target* refers to the image of the target on the photograph seen through the optics of a stereoplotter.)

Ground target sizes from 1 mrad to 10 mrad were chosen, together with PSF's with σ's varying from 0.1 to 5 mrad, depending on the size of ground target. The contrast of an isolated detail is defined as *Imax/Imin* or *Imax/Ibackground.* Contrast may also be converted into an equivalent density difference across the target from $\Delta D = \log Imax - \log Imin$, or deduced from the graphs of Hempenius (1969, 38). Slopes of the density profiles of each blurred target were computed for ground target contrasts shown in Table 1. An equivalent modulation computed from $(Imax-Imin)/(Imax+Imin)$ of a continuous sine-wave target for the three contrasts is also shown.

The widths of the blurred targets and hence the annuli assuming a measuring mark (MM) of 1 mrad were derived on the computed density profiles at levels varying from 50 to 20 percent, depending on the blurred target contrasts and observer in Trinder (1971a). Slopes of the density profiles in terms of ground target size and PSF proved to be in fair agreement with similar curves derived for Trinder (1971a). Such curves were based on a limited number of ground targets and PSF's, particularly at high contrast, and hence were subject to some error.

Substituting the slopes of the density pro-

TABLE 1.

Object Contrast	$\frac{Imax}{Imin}$	1.2	2.5	5.6
Equivalent Modulation	$\frac{Imax-Imin}{Imax+Imin}$	0.10	0.44	0.70
Density Difference	ΔD	0.10	0.40	0.74

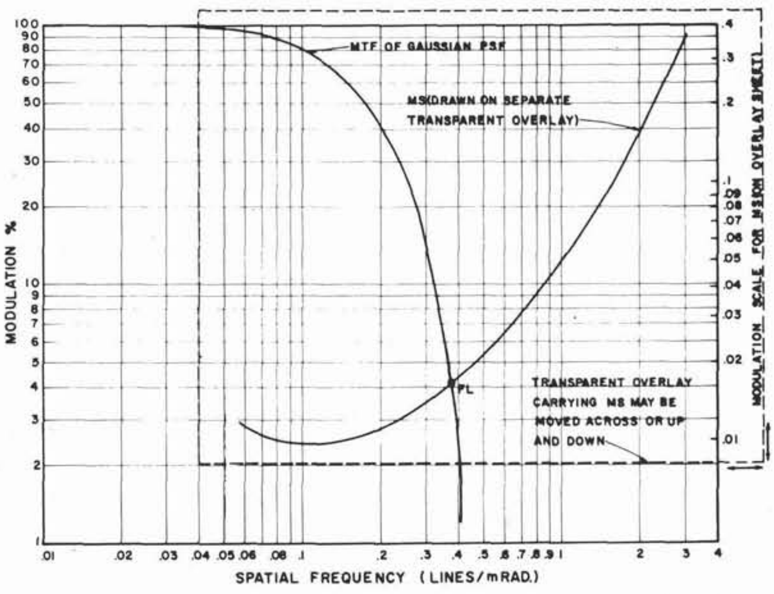

FIG. 1. Replica of Modulation Transfer Board of Hempenius (1964). The modulation sensitivity of the visual system (MS) on a separate transparent overlay, is introduced with the contrast of the target against 100 percent modulation. The MTF of an imaging system is entered as shown. Intersection of the two curves gives the frequency limit, FL.

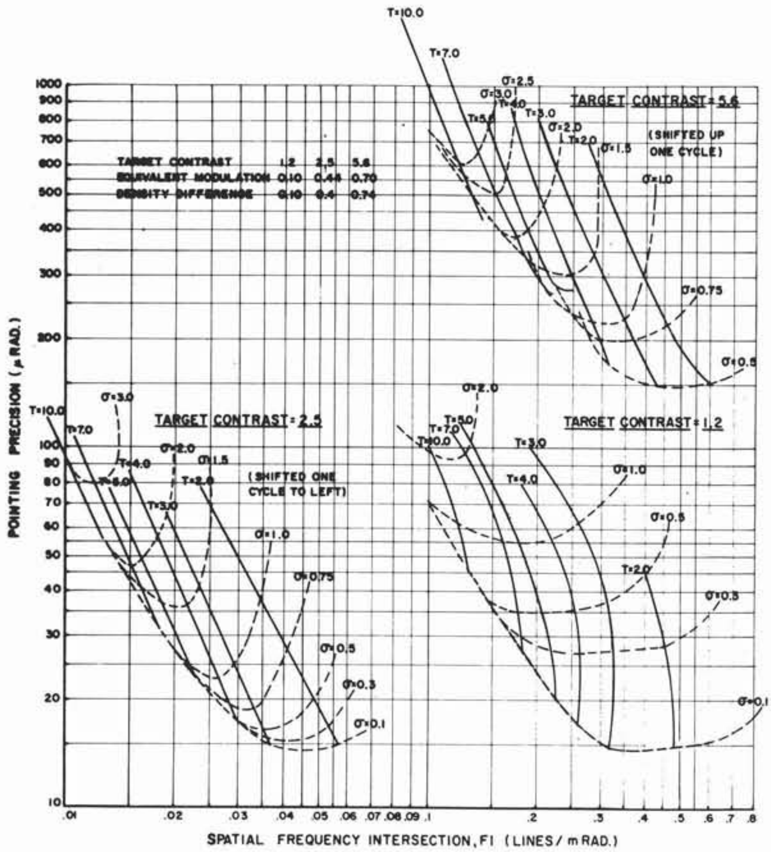

FIG. 2. Spatial Frequency Intersections for circular ground targets of diameter T and Gaussian PSF σ-values, against pointing precision (MM = 1.0 mrad, observer JCT). The curves for target contrasts of 2.5 and 5.6 have been displaced as shown.

files and annulus sizes obtained from these computations into the figures in Trinder (1971a), precisions were determined for each ground target and PSF combination. These results are plotted in Figures 2 and 3 for two observers along with the spatial frequency intersections (FI) of each blurred target, which are described in the following sections.

MTFs in Relation to MS of Visual System

Methods proposed for the interpretation of MTF's in relation to the visual system have been given by Charman and Olin (1965) and Hempenius (1964). Charman and Olin have used a Threshold Quality Factor (TQF) which is computed as the area between the MTF curve and the MS curve under specific situations. They maintain that the TQF correlates well with the resolving power obtained with the same photographs. These investigations were based on repetitive targets.

Hempenius (1964) approaches the problem in a similar manner, but his technique is more versatile in that any target contrast (i.e., modulation) may be included as well as granularity and optical magnification of an instrument. The criterion used by Hempenius was the limiting spatial frequency or Frequency Limit, FL, as shown in Figure 1. The FL is the frequency at the intersection of the MTF curve of the photographic system and the MS of the visual system. It corresponds with resolving power which is normally measured at high contrast on square-wave targets.

A similar technique is used by Brock, et al. (1966), Scott (1966) and others, for assessing resolving power, based on three-bar square-wave resolution targets. The effect of the visual system is introduced as a threshold modulation (TM) curve, which is experimentally determined for each film. This approach could also be used in Hempenius' method where an MS curve for a specific film,

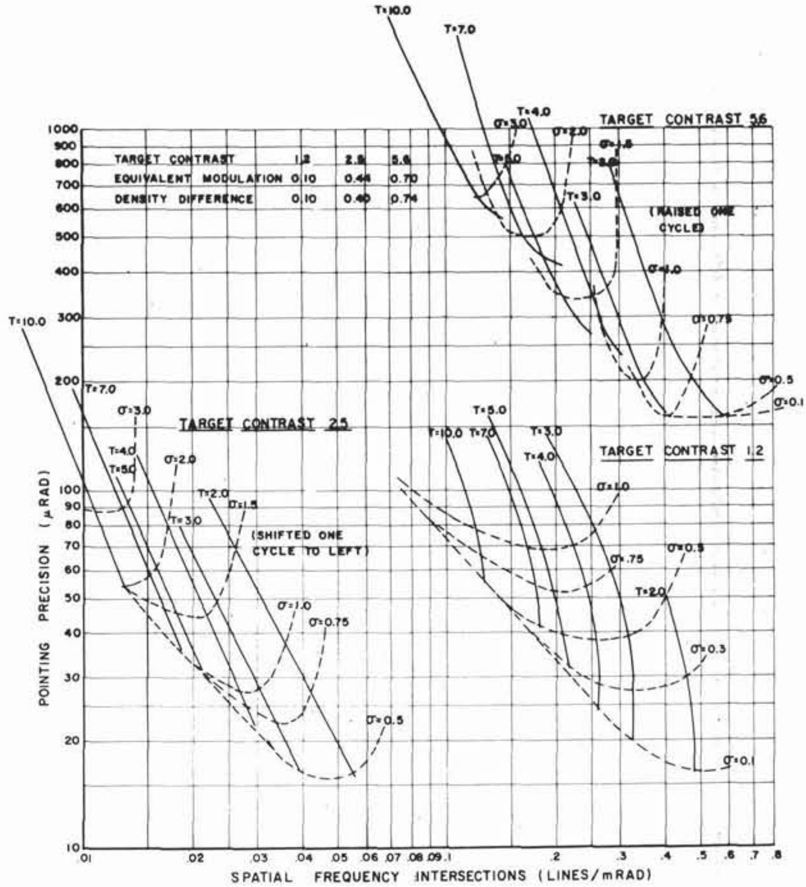

FIG. 3. Spatial Frequency Intersections for circular ground targets of diameter T and Gaussian PSF σ-values, against pointing precision (MM = 1.0 mrad, observer AHC). The curves for target contrasts of 2.5 and 5.6 have been displaced as shown.

development and granularity is used. However, provided assumptions on linearity of the photographic system, and also the behavior of the visual system with regards granularity are justified under most circumstances, the generality of Hempenius' method seems desirable. Further work is required to compare the two approaches.

The FL may not be the best criterion for assessing image quality. Other factors of the MTF curve may include more information on the shape of the MTF curve, and hence describe image quality better. Charman and Olin's TQF does include information on the whole curve, but in the form of an area, which does not specifically describe the shape. Linfoot (1965) proposes the statistical mean information content, where the criterion for evaluation is the statistical mean structural power spectrum. However as yet there is no indication that the visual system reacts predictably in terms of power spectra of spatial frequencies.

As the FL of Hempenius is relatively straightforward to derive for research and also practical situations, it was the first item to be investigated. If a Gaussian PSF is assumed, the σ-value uniquely describes both the PSF and MTF, and hence the shape of the MTF is known. Therefore, given the σ-value and the FL, the characteristics of the imaging system are prescribed. For this reason, it was emphasized earlier that this research applies specifically to imaging systems with Gaussian PSF's, although the results might also be useful for other imaging systems.

Application of MTF Techniques to Blurred Targets

The spatial frequency components of

blurred targets depend on those of both the MTF and the ground target. As pointed out in the first section, the MS is used herein to interpret the behavior of the visual system to the frequency spectra of isolated details, in particular, the blurred targets observed in Trinder (1971a). Techniques adopted by Hempenius described in the previous section are therefore used on the spatial frequency spectra computed in the second section. The spectra are inserted in exactly the same way as MTF's, the intersection of the spectra and the MS give a Frequency Intersection (FI). It is termed FI because it is different from the FL derived from the MTF.

The spatial-frequency spectrum of a target plotted on logarithmic scales was entered on a small replica of the Modulation Transfer Board designed by Hempenius (1964) as an MTF. As both modulation and spatial frequency scales are plotted on logarithmic scales, the curves maintain their shape if moved along either axis. Location of the MS curve was set to read *lines/mrad* on the frequency scale. The curve for MS was that used by Hempenius (1964) and is an average curve determined from a number of investigations. Some differences can be expected between these results and those derived from other MS curves. Although the overall effect of the differences is not expected to be significant, tests should be conducted to check this.

Object contrast was converted to modulation, (or $\gamma \times$ modulation, if γ is not equal to *1*, (Hempenius, 1964), where γ is the slope of characteristic curve of the photographic emulsion) and introduced onto the board by placing that modulation value on the MS curve, against 100 percent modulation. Intersection of the MS curve and the target frequency spectrum gave a Frequency Intersection, FI. FI's determined from the range of ground target sizes and PSF's are given in Figures 2 and 3 for two observers. In addition two larger MM's of 2.0 mrad and 4.0 mrad have been investigated at a contrast of 2.5, but not presented. A discussion of these curves in Figures 2 and 3 follows.

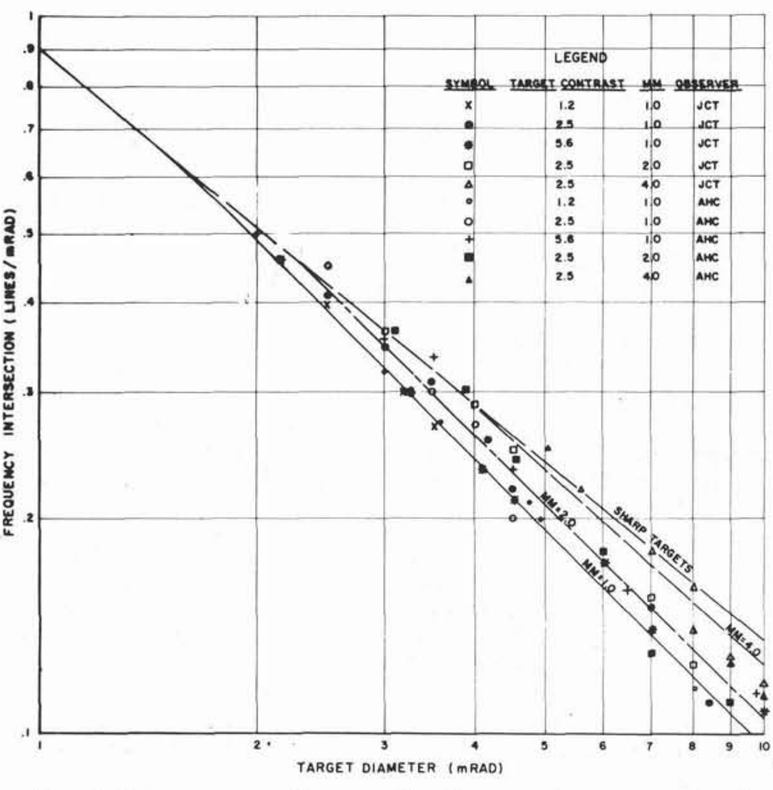

FIG. 4. Optimum target diameter plotted against the corresponding frequency intersections, FI, as derived from Figures 2 and 3. FI's derived from frequency spectra of sharp targets have also been plotted.

Analysis of Density Profiles and Frequency Spectra of Blurred Targets

The relationship between pointing precision, the FI, PSF σ, and ground target sizes at three contrasts for two observers is shown in Figures 2 and 3. The pattern of results for σ and target size are discussed first.

As the contrast of the ground target increases, so does the grade of the density profile of the blurred target. Pointing precision likewise improves. The curves for pointing precision against PSF σ for each ground-target contrast are similar in shape to parabolas. In general for small ground targets, the grade of blur of the density profile is shallow and hence low pointing precision will result. As the ground target becomes larger, the grade of the density profile increases and therefore precision improves. However, as the ground target increases, so does the annulus for a given MM. Because the annulus also affects pointing precision, precision for a given PSF σ improves, reaches a maximum as the ground target size increases, (*i.e.*, minimum standard deviation) and then commences to deteriorate. The turning point on the curve gives the optimum ground target size for a given PSF, and in addition a FI and a resulting pointing precision. These results are plotted in Figures 4, 5 and 6. The turning points in Figures 2 and 3 are not critical and therefore some variation exists in the determination of the optimum ground-target size and FI.

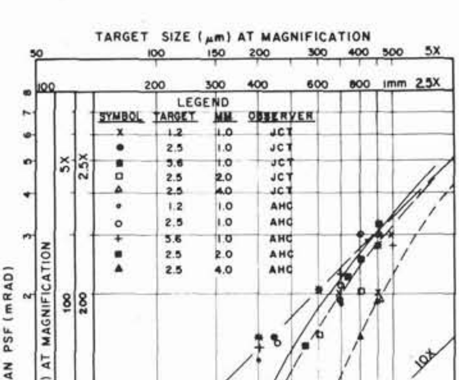

FIG. 5. Optimum target size against Gaussian PSF σ, for three measuring marks (MM) as shown. Equivalent linear dimensions at two magnifications in a photogrammetric instrument have been introduced on secondary axes.

Referring to the results for FI in Figures 2 and 3, a linear or near-linear relationship exists between pointing precision and FI for each target as derived from the MS. Such linear or near-linear relationships on logarithmic scales are typical of the performance of the visual system, and also other aspects of psychophysics (Stevens, 1962). It seems that a psychophysical relationship such as Stevens' (1962) formula could be applied to these results, as was possible with the results of pointing (Trinder, 1971b) if expressed in terms of grade of the density profile. The application of MS to frequency spectra of blurred targets therefore gives an acceptable relationship, even though MS is based on repetitive sine-wave targets.

The figure for optimum ground target size, as expressed in terms of FI in Figure 4 is also linear, although this figure is, strictly speaking, a derived relationship from Figures 2 and 3. In Figure 4 *the optimum ground target size is independent of contrast*, but is dependent on MM size. The use of MM's of 2 mrad or 4 mrad requires ground targets which are larger than those used for a 1 mrad MM by ½ (MM size in use − 1 mrad MM). The choice of a suitable sized MM has been treated in some detail by Hempenius (1969, 211).

The FI for 2.5 contrast *sharp* targets plotted against ground-target diameter is also given in Figure 4. To plot the relationships for other contrasts, the FI at contrasts 1.2 and 5.6 for a 1.2 mrad target are 0.76 lines/mrad and 1.0 lines/mrad, respectively, and for a 10 mrad target they are 0.14 lines/mrad and 0.13 lines/mrad, respectively. The variations in FI for different contrasts seems to be minimal for blurred targets, and therefore is probably negligible for sharp targets except for the targets with small annuli (O'Connor, 1967). This linear relationship on logarithmic scales simplifies the pattern of pointing precisions for sharp targets, as it has been generally presented. Considerable effort has been spent on analyzing pointing precisions for sharp targets based on their intensity profiles (Hempenius, 1969), (Trinder, 1971c).

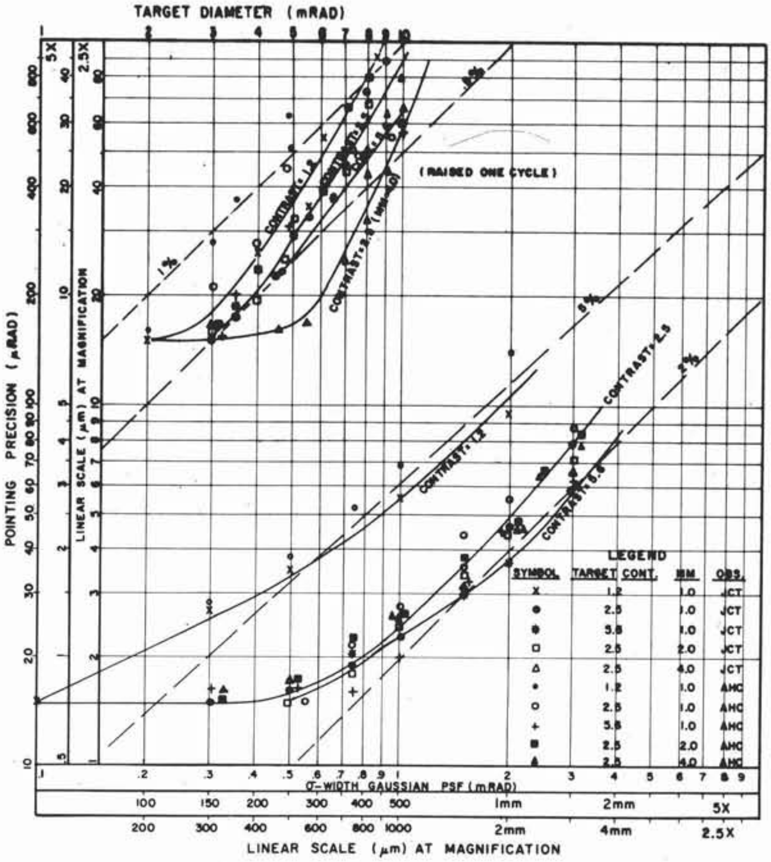

FIG. 6. Pointing Precisions obtained with optimum ground targets as specified in Figure 5, expressed in terms of PSF σ and ground-target size. Curves for which target size is the argument have been raised 1 cycle. The curves have been interpolated from the results of observer JCT, whereas actual plotted points for AHC have been added. The linear scale (in μm) above applies to target diameter, but on the lower scale it refers to PSF σ.

Figure 4 should provide additional information on this subject.

Formulation of the optimum ground target sizes and accuracies obtained follows.

Prediction of Optimum Ground Target Sizes in Terms of PSF

The optimum ground target sizes in terms of PSF σ are shown in Figure 5. They are *independent of ground target contrast,* and no significant variation occurs between the two observers. No simple formula can be used to predict the size of target as was attempted in Trinder (1971a). However, over the range of σ between 1.0 mrad and 3.0 mrad, the optimum target size is between 4× and 3× the PSF σ for a MM of 1 mrad. The figure of 2.5 σ given in Trinder (1971a) was therefore a reasonable estimate considering the approximations involved in its determination. For σ less than 1 mrad, and MM of 1 mrad, the target size should be $K\sigma$ where $K = 10$ for $\sigma = 0.2$ mrad and approaches infinity as σ approaches zero. That is, target size should approach MM size. It can be seen from Figure 5 that the annulus of blurred targets in practice, will be greater than the recommended minimum annulus width of 1 mrad in Trinder (1971a). Larger targets are required if larger MM's are used (see previous section).

Pointing precisions which would be obtained from these targets are given in Figure 6 in terms of PSF σ and target size. *Contrast affects the precision if expressed in terms of target diameter, as does the* MM *size, whereas* MM *does not affect the precision when expressed in terms of* PSF σ.

For ground targets greater than 2.5 mrad and σ greater than 0.5 mrad, pointing precision is 2 to 3 percent of PSF σ for medium to high contrast, although as ground-target contrast decreases, the precision also drops. For σ less than 0.5 mrad, precision approachs a constant value of 15 μrad, depending on the target contrast. At this stage it is no longer satisfactory to quote precisions in terms of σ because other factors of the observed blurred target will affect pointing precision, i.e., target size and contrast (see O'Connor (1967), Hempenius (1969) for the relationship for sharp targets).

The precision in terms of target size depends on contrast and MM size. However, for MM's less than 2 mrad the pointing precision is between 0.5 and 1 percent of the ground-target size for target sizes greater than approximately 3 mrad. This is similar to the pattern for pointing precisions on the same sized sharp targets. The conclusion may be reached that the optimum-sized targets if viewed by an observer will be fairly sharp. For smaller targets than 3 mrad, precision approaches 15 μrad, and may decrease further if the target is of very high contrast, and extremely sharp. As with the situation of precision against σ, the sharp-target figure should be consulted for targets less than 1.5 mrad.

If the PSF varies across the photograph, a ground-target size chosen for one part of the photograph may not be satisfactory for another. Hempenius (1969, 211) therefore maintains that the image quality, i.e., PSF σ, should not vary across the photograph by more than a factor of 2 if optimum pointing conditions are to be maintained. From Figures 2 and 3 this criterion would allow for variations in precision across the photograph of approximately 20 to 30 percent, depending on ground-target contrast.

As emphasized in the first section, precisions of pointing given in Figure 6 are those only of the observer, and do not include loss of precision caused by the measuring instru-

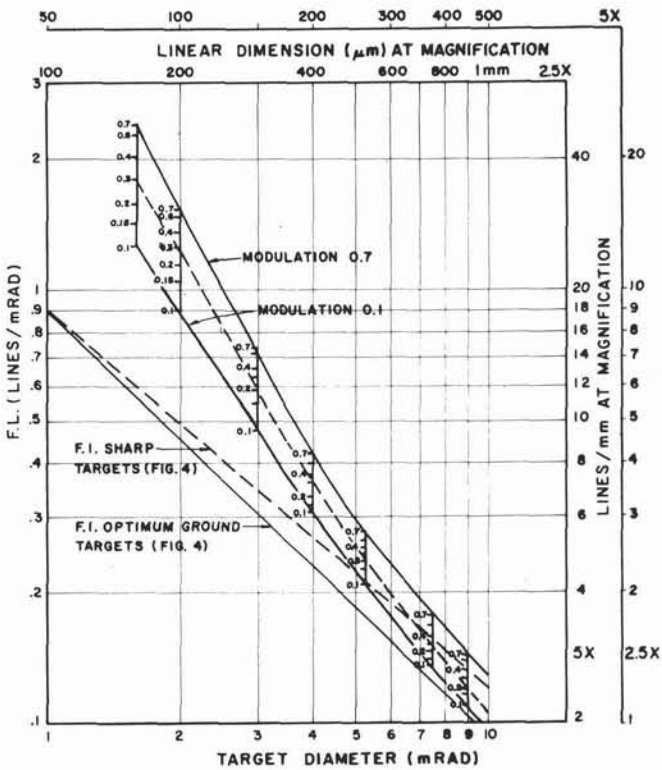

FIG. 7. Optimum target size in terms of the Frequency Limit (FL) derived from the intersection of the MTF curve and MS. An object contrast, expressed as modulation, has been inserted at regular intervals along the figure. Equivalent linear dimensions and frequencies (in lines/mm) at two magnifications in a photogrammetric instrument, have been added. FI's as given in Figure 4 for optimum size of target and sharp targets, have also been shown.

ment. The standard deviation obtained in a comparator may be larger than the corresponding value derived from Figure 6. This factor probably accounts for the larger estimates of precision, i.e., 10 percent of σ given by Hempenius (1969).

Figures 5 and 6 give simple relationships for choice of target and the expected precisions of pointing. If a simple rule exists for optimum ground target size in terms of PSF as in Figure 5, a similar rule should also exist between the MTF and target size. This is described in the following section.

Optimum Ground-Target Size in Terms of MTF

Because MTF's were to be analyzed a Frequency Limit, FL (as demonstrated by Hempenius) was derived for each PSF σ. This FL was then related to optimum ground-target size, with contrast as an additional variable, as shown in Figure 7, for an MM of 1 mrad. For larger MM's, the rule given earlier applies. It is noticed that Figure 7 is more complex than Figure 4, which relates FI to optimum target size. Contrast is not a variable in Figure 4, whereas it becomes a variable if FL is expressed in terms of ground-target size.

Given an imaging system of known MTF, using the Modulation Transfer Board of Hempenius, an FL can be derived from the MTF. This FL can then be entered into Figure 7, together with the expected contrast of the ground target, expressed as a modulation, to derive the ground-target size needed. The predicted precision will then be obtained from Figure 6.

Granularity

The Modulation Transfer Board of Hempenius introduced the effects of emulsion granularity by reducing the MS of the observer as a function of the granularity. This was done by raising the MS relative to the modulation scale. The amount of upward movement was controlled by the granularity board. There is good reason to believe that this additional step of including granularity may also be used in conjunction with pointing precision although no pointing results are available to check this proposal. If this technique is valid, the optimum ground target size required for a given MTF and granularity would be obtained by referring to the Modulation Transfer Board, given an equivalent modulation based on the ground target contrast and the granularity, to find a FL. This FL would then be substituted into Figure 7.

One experiment (Kelly, 1968) which was conducted on grainy photographs, derives a Dimensionless Quality Reduction Factor (DQRF) by subjectively matching a grainy photograph of certain MTF, with a grainless photograph of different MTF. $DQRF = \kappa / \kappa'$ where κ is the characteristic spatial frequency (at 61-percent modulation on the MTF curve) of the grainless photograph which has the same subjective quality as the grainy photograph, and κ' is the characteristic spatial frequency of photograph with grain. The DQRF follows a simple relationship when expressed in terms of the product $\kappa \cdot \sigma(D)$.

If the MTF's of the matched photographs of Scott, together with the corresponding granularity, give the same FL, some justification exists for using the Granularity Board of Hempenius for introducing the effects of granularity on pointing. If tested on the Modulation Transfer Board, the FL for each pair of matched grainy and grainless photographs given in Table 2 did agree very closely at modulations of 0.4 and 0.7. Poor agreement was obtained for the modulation of 0.1, but the influence of granularity in this instance was so great as to almost obscure the MTF. These results should be treated with caution, however, because the results of Scott are based on interpretation, i.e., a *recognition* task, whereas pointing is based on an *acuity* task. It is believed, however, that the visual system should behave in a similar manner for the two tasks.

As the pairs of matched grainy and grain-

Table 2.

κ' *of grainy photograph*	σ_D *RMS density variations about 1.0 density for Scanning Aperture of 24 μm*	κ *of grainless photograph*
0.65	0.75	0.43
1.14	0.48	0.86
2.0	0.75	0.86
2.0	0.48	1.37

less photographs do indeed give approximately the same FL, evidence exists that the Transfer Board may be used to derive a FL of grainy blurred photographs in order to compute the optimum ground-target size. Additional research is required on granularity to substantiate this proposal, particularly in regard to the effects of granularity on the MS, and possible variations in granularity with emulsion γ, contrast, exposure and the many other aspects of the photographic process.

Conclusions

★ The Frequency Intersections, FI, derived from the frequency spectra of blurred targets, and the MS follow a regular pattern if expressed in terms of pointing precision and target diameter. This pattern can be related to a known psychophysical formula. It therefore seems that MTF techniques are applicable to the isolated targets investigated in this study.

★ Relationship between FI and optimum target sizes follows a simple pattern independent of target contrast, and is much simpler than the corresponding relationship of FL (derived from MTF's) against optimum target size.

★ The FI in terms of target size for sharp targets follows a simple relationship, and may render further information on the behavior of the visual system to such targets.

★ For a MM of 1.0 mrad, optimum ground-target size should be 3 to 4 times the PSF σ, for σ greater than 1 mrad, and approaches the size of the MM as σ ideally approaches zero. Additional aspects of the blurred target will affect the choice of ground target as σ approaches zero however. For MM's larger than 1 mrad, add half the difference between that MM and a 1 mrad MM. The choice of optimum target size is independent of ground target contrast.

★ The precision of pointing in the x-direction is 2 to 3 percent PSF σ for ground target contrast greater than approximately 2.5, and decreases to 5 percent of σ for contrast of 1.2. This is independent of MM size. Precision may also be expressed as 0.5 to 1 percent of ground-target size for a MM size of 1.0 mrad. Precision of pointing in the y-direction is approximately 30 percent lower, i.e., larger standard deviation.

★ Optimum ground target size may be computed from MTF and the corresponding frequency limit (FL). It is evident that this technique can be further expanded to include the influence of granularity in the same manner as proposed by Hempenius in the use of the Modulation Transfer Board. These conclusions should be checked against other determinations of MS, and also in terms of variations of granularity with different photographic conditions.

★ The precisions quoted apply only to the performance of an observer and do not include the inaccuracies introduced by the measuring instrument. In addition, if the determination of the true center of the blurred target is important, the possible existence of systematic errors should be kept in mind.

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