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Radar Shadow Frequency

Cumulative frequency curves of terrain slopes provide a potentially powerful tool for landform analysis.

ABSTRACT: *To date the geoscientist has only made limited use of radar shadows for collecting quantitative geomorphic data. Although methods for calculating relative relief from the length of a radar shadow have been described, the relationship between the occurrence of radar shadows and terrain slope angle has not been previously utilized. As a terrain feature will shadow only if the backslope of the feature is greater than the depression angle at which it is imaged, slope information is available from the frequency of radar shadowing. By sampling across the range of the radar image, a cumulative frequency curve of slope distribution can be constructed. The radar shadow frequency method for determining terrain slope information was tested in six regions in the United States where both radar imagery and topographic maps were available. The results indicate that the radar-derived slope curves are comparable to topographic map-derived slope distribution curves and apparently are more realistic in mountainous regions.*

INTRODUCTION TO RADAR SHADOWING

THE PARAMETERS that determine whether or not a terrain function will produce radar shadow are depression angle β and the terrain slope of the slope facing away from the radar beam α_b. The relationship between β and α_b is such that three cases are possible: (1) the backfacing slope is fully illuminated and no shadow results; (2) the backslope is partially illuminated, producing a condition analogous to the twilight zone, commonly referred to as grazing; and (3) the backslope is obscured from the impinging beam causing a no-return area of radar shadow (Figure 1). More definitely, the conditions for the three cases can be expressed as a function of β and α_b (Table 1). If the backslope is illuminated (no shadow), the angle of the backfacing slope is less than the depression angle ($\alpha_b < \beta$); whereas, if grazing occurs, the two angles are equal ($\alpha_b = \beta$). Radar shadow, on the other hand, is exhibited if the terrain slope is greater than depression angle ($\alpha_b > \beta$), and it increases in both a probabilistic frequency occurrence and length as β decreases (Figure 1). As would be expected, radar shadowing is more intensive in the far range, that is, at the lower depression angles.

The condition necessary for radar shadowing as defined in Table 1 is valid only if the strike of the crestline is perpendicular to the propagational vector of the radar wavefront or parallel to the flight line. This arises because as the angle described between the flight line and the strike of the crestline θ increases, the angle at which α_b will shadow at a given depression angle also increases. The effect of θ on the angle at which α_b will shadow at a given β is given in Figure 2. As the figure illustrates, if a terrain backslope is positioned in the range where $\beta = 40°$ and the orientation of the crestline is such that $\theta = 40°$, then the backslope will not shadow unless $\alpha_b > 47.5°$. The absolute amount of error related to selected values for β and θ is given in Figure 3. For example, where $\theta < 30°$ the absolute error encountered (regardless of the value for β) is less than 5°. Where $\theta \geqslant 50°$, the amount of absolute error

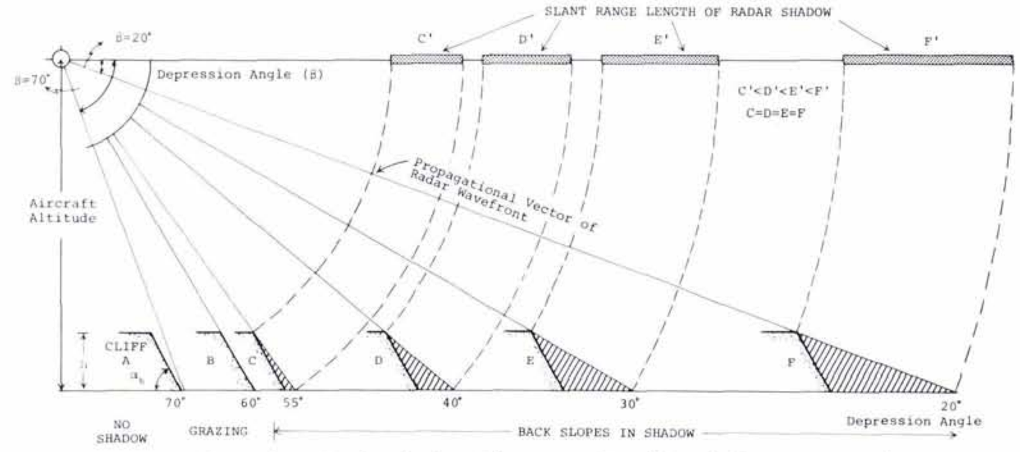

FIG. 1. Relationship of radar shadow (slant range length) with depression angle β.

skyrockets, especially if β is between 15 and 45°.

SLOPES DETERMINATION FROM RADAR SHADOWS

INDIVIDUAL SLOPES

Utilizing the conditions necessary for radar shadowing, i.e., $\alpha_b > \beta$ in conjunction with Figure 2, a semi-quantitative value can be assigned to a given terrain slope on radar imagery. For example, if a terrain slope is located at a $\beta = 40°$ and has a $\theta = 40°$ and exhibits radar shadowing, then the terrain slope α_b is greater than 47.5°. Therefore, Figure 2, an expression of $b = f(\theta)$, is also a nomogram that provides for rapid determination of the specific terrain slope angle.

REGIONAL SLOPE

Regional slope can be defined by sampling either as a single value such as the mean, median, or mode or as a range or distribution of values as in a histogram or cumulative frequency curve. To date only the former type of regional slope information, i.e., a single value, has been abstracted from radar imagery. For example, McCoy (1967, p. 2) suggests that a population of 25 to 30 slope measurements determined from radar imagery using radar foreshortening is a sufficiently large sample set to provide an accurate expression of the mean slope value for a single region if compared to topographic map data. McCoy also alludes to another means of obtaining mean slope value for a given geomorphic region utilizing grazing, a

TABLE 1. RELATIONSHIP OF BACKFACING TERRAIN SLOPE α_b WITH DEPRESSION ANGLE β TO THE OCCURRENCE OF RADAR SHADOW

Case	*Condition*
No Shadow	$\alpha_b < \beta$
Grazing	$\alpha_b = \beta$
Shadow	$\alpha_b > \beta$

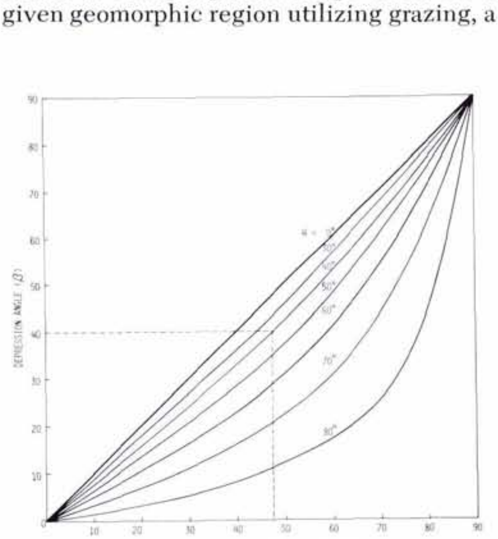

FIG. 2. Relationship of θ, the angle described by the flight direction and strike of the crestline, with α_b, the true backslope angle of terrain in shadow. This is also a nomogram for correcting the apparent backslope angle to true backslope angle. For example, if a slope at a 40° depression angle β and the angle between the flight path and strike of the crestline θ is 40° then the true backslope must be greater than 47.5°.

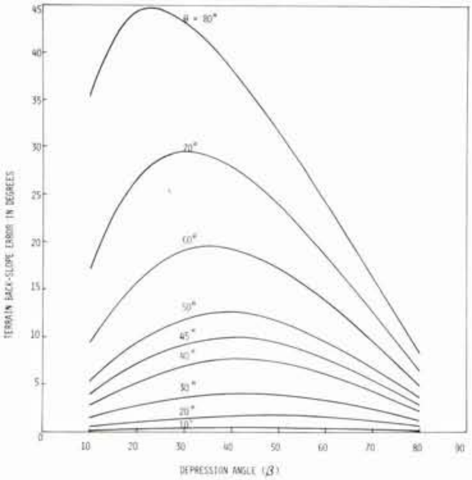

FIG. 3. Absolute error in terrain backslope α_b as a function of depression angle β and the angle described by the flightline and the strike of the crestline θ.

characteristic of radar if the backslope angle of the terrain α_b is equal to the depression angle β. By determining the depression angle where grazing occurs, the mean regional slope can be estimated provided that the landforms on both the near-range and far-range side of grazing are homogeneous.

The assumptions involved in using the grazing method if only one look-direction is available are that (1) the imaged terrain approaches a saw-tooth landform model, and (2) a random distribution of slope angles exists in the imaged area. As the number of look-directions increases, the importance of the assumptions is diminished because the bias of uni-directional sampling is progressively eliminated.

Several problems arise if the grazing method is employed, the most formidable of which are (1) the difficulty of delimiting areas of grazing, and (2) the high probability that the area exhibiting grazing will be lost in radar shadow unless the angle of the terrain slopes above the point of grazing decreases so that shadowing does not occur.

By utilizing the conditions for and the occurrence of radar shadows instead of grazing, some of the problems and limitations of the grazing method can be reduced. By changing to a shadow-no shadow decision along a crestline, the decision making is not only easier and more definitive but also expands the radar-derived data across the complete range dimension of the imagery and increases the sampling area within the delimited landform region. The most natural manipulated data output from this method is a cumulative frequency curve of slope values of the defined landform region from which both a histogram of slope values, or the mean regional slope value, can be derived.

The use of radar shadowing on a yes-no decision basis has several advantages over most methods used for determining regional slope values: (1) the increased speed of determining the regional slope values of large areas, (2) the discrimination of landform regions within large areas based on the plots of percent crestlines in shadow, (3) the ease with which the method lends itself to automatic methods of pattern recognition and measurement, and (4) the apparently high degree of accuracy.

As the assumptions involved in the shadow-frequency method are the same as those associated with the grazing method, the restrictions related to having a homogeneous area and a random distribution of slopes are relevant. It is up to the user of the method to satisfy the assumptions. This is accomplished primarily by a qualitative interpretation of the radar imagery that results in the visual (1) delineation of homogeneous landform regions, and (2) confirmation of random slope distribution.

RADAR SHADOW FREQUENCY METHOD

In order to test the utilization of radar shadow frequency as a tool for producing cumulative frequency curves of a given landform region, several areas in the United States, were selected where both topographic coverage and radar imagery were available. Table 2 lists the test areas and the scale, data, and contour interval of the topographic maps used. Most of the areas selected were mountanous regions with high terrain slopes since they were the most logical areas where the method would be used.

METHODOLOGY

Because map-derived and radar-derived cumulative frequency slope data were to be compared, two separate methods of data collection were established. The method used for collecting map-derived slope data was the method described by Strahler (1956) whereby a regularly divided rectangular grid overlay was placed on a topographic map of the test area and 100 samples were selected at random. The slope angles were then computed by determining the elevation

TABLE 2. TEST AREAS FOR RADAR SHADOW FREQUENCY METHOD FOR DETERMINING CUMULATIVE FREQUENCY SLOPE CURVES

Area	*Scale of Map (Date)*	*Contour Interval*
Annamoriah, W. Virginia	1:24,000 (1966)	20 ft.
Humbolt Range I, Nevada	1:62,500 (1956)	40 ft.
Humbolt Range II Nevada	1:62,500 (1956)	40 ft.
Stansbury Mts., Utah	1:62,500 (1957)	20 ft.-40 ft.
Chrome Ridge-Onion Mt., Oregon	1:62,500 (1961)	50 ft.
Seven Mile Peak, Oregon	1:62,500 (1960)	80 ft.

change from the contours for a given horizontal distance orthogonal to the contours passing through the randomly selected sample point and with approximately equal distance falling upslope and downslope from the point. The horizontal distance depended on the nature of the sampled slope. For example if the slope was very small, or the selected sample point was close to the crest-line, the *standard* horizontal distance used was 250 feet on either side of the sample point or a total of 500 horizontal feet.[1] On occasion, however, a 100-foot horizontal distance on either side of the sample was used.

As no standard procedure for collecting cumulative frequency slope data from radar exists, the methodology evolved as the study progressed and different problems were encountered. The first problem encountered was the sampling procedure. Although a larger number of sampling zones across the image would increase the number of points for plotting the cumulative frequency curve, a smaller data sample for each point would result. It was therefore decided that the best trade-off between a continuous source of data for plotting and a large enough sample to be significant was the division of the imagery into eight areas of equal ground range. The use of zones of equal ground range was important as it provided zones of *potentially* equal sample size. The eight zones were defined in terms of slant range depression angles[2] and an overlay prepared to help distinguish the zones and make the collection of radar data more practical.

The radar image was then enlarged optically or photographically by a factor of two in order to reduce the percent error in measuring the length of the crestlines and shadows. The data were collected for each zone and categorized according to the orientation of the crestline and then, on the basis of the angle described by the orientation of the crestline with the flight line, the proper correction factor applied to the data.

The proper correction factor applied was determined by (1) the depression angle width of the zone, and (2) the magnitude of the error in degrees that is introduced by crestline orientation. In the four zones in the far range two corrections were used: (1) if the orientation of the crestline with the flight path θ described an angle greater than 50°, the sample was not used; and (2) if θ was less than 50°, the data sample was used without correction. This decision was based on the 10° depression angle width of the four zones in the far range and an absolute error of around 10° if $\theta = 50°$ (Figure 3). Three corrections were considered in the four zones in the near range (1) if θ was greater than 50° the magnitude of error was too great and the data not used; (2) if θ was between 30° and 50°, the sample was tabulated with the data

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[1] The selection of the *standard* horizontal scale used was based on testing horizontal scales of different sizes (2,640 ft., 1,000 ft., and 500 ft.) in the Humbolt Mt. and the Chrome Ridge-Onion Mt. areas and it was found that the larger the horizontal scale, the greater the bias towards lower slope values due to averaging.

[2] The range depression angle β boundary of each of the eight zones were 19°, 22°, 25°, 29°, 36°, 46°, 59°, and 78°, progressing from far to near range. These depression angles were determined mathematically.

in the next zone closer to the near range[3]; and (3) where θ was less than 30°, the effect of θ on the backslope angle was less than the width (depression angle variation) of the sampling zone; therefore, no correction factor was applied.

After the correction for crestline orientation was taken into account, the percentage of crestline length in shadow was calculated for each of the eight zones. The values were subtracted from 100 percent to find the percentage of terrain backslopes less than the highest depression angle of the zone and a smoothed cumulative frequency curve plotted on the basis of data from each of the eight zones across the radar imagery.[4]

RESULTS

A visual comparison of the map- and radar-derived cumulative frequency curves of terrain slope α indicates that in five of the six areas tested the cumulative frequency curves from radar shadow data are representative of terrain slope data from topographic maps (Figures 4 to 9). It is also evident that the correlation between map- and radar-derived cumulative frequency curves increases as the map detail increases, i.e., as the scale becomes larger and the contour interval smaller. This might be expected because the radar shadow frequency method favors the sampling of higher terrain slope angles more than the map method,[5] a phenomenon also encountered in map-derived data if (1) the map scale is increased, (2) the countour interval is decreased, or (3) the horizontal dimension of the slope length sampled is decreased.

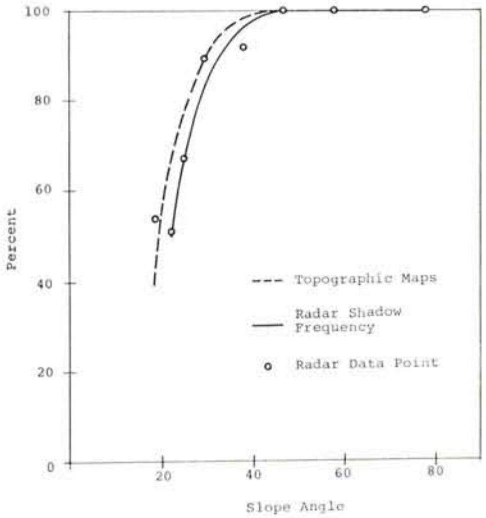

FIG. 4. Cumulative frequency curves of map- and radar-derived terrain slope α data. (Annamoriah, West Virginia).

Therefore, the sampling bias may, in fact, be in the map- rather than radar-derived data. This seems to be apparent in Stansbury Mts., Utah (Figure 7), where the radar-derived cumulative frequency curve seems more realistic (representative of the area) than the map-derived curve.[6] The cumulative frequency curves of Humbolt Range, Nevada (Figure 5), and Chrome Ridge-Onion Mt., Oregon (Figure 8), also show that a greater proportion of the high slope angles (>25°) are sampled from radar imagery than from topographic maps. The reliability of the radar data in detecting the high-slope angles is substantiated by radar geometry which prescribes that if the terrain backslope angle is greater than the depression angle at which it is imaged, radar shadow has to result. Therefore, a slope exhibiting radar sha-

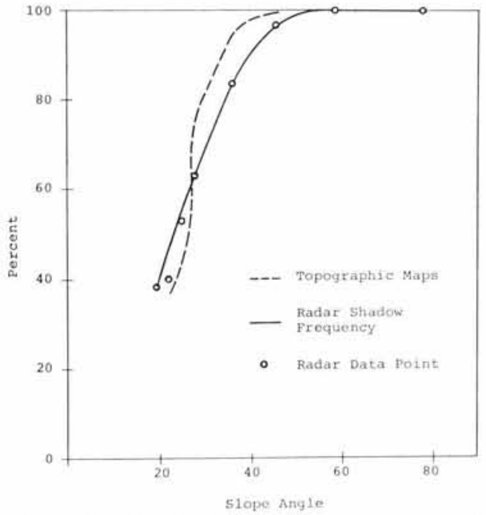

FIG. 5. Cumulative frequency curves of map- and radar-derived terrain slope α data. (Humbolt Range I, Nevada).

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[3] This effectively adds to the backslope angle 3° to 5°, which corresponds to the magnitude of error involved.

[4] The map-derived slope data was also plotted using the same eight zones used on the radar plot.

[5] Especially evident in the terrain slope (depression) angles above 25° to 30°.

[6] By selecting and sampling, the individual slopes that were producing shadows in the near range, it was documented on the topographic maps that slope angles in excess of 50° are found in the region but were not sampled by using 100 random sample points.

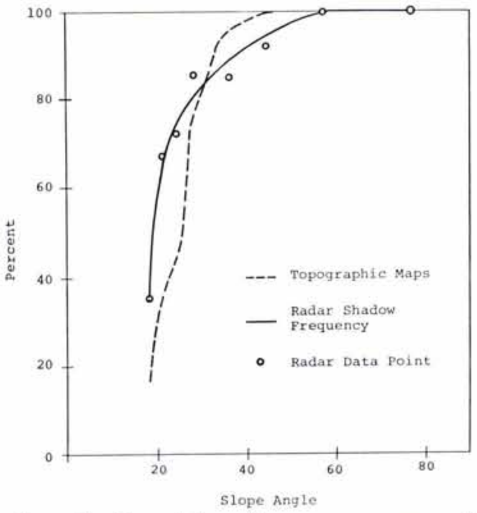

FIG. 6. Cumulative frequency curves of map- and radar-derived terrain slope α data. (Humbolt Range II, Nevada).

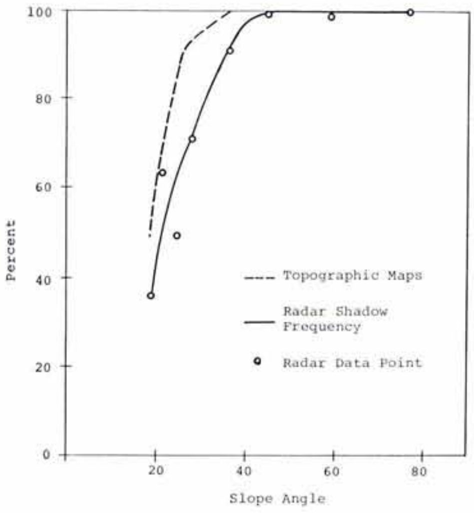

FIG. 8. Cumulative frequency curves of map- and radar-derived terrain slope α data. (Chrome Ridge-Onion Mtn., Oregon).

dowing at 60° depression angle *must* have a backslope greater than 60°. The sampling of this high terrain-slope angle is favored by the radar method because the shadowed slope is more than likely part of a well-defined crest and as such a prime sampling source.

Although it is theoretically impossible that the percentage of crestlines in shadow decreases progressing from the far range (low-depression angles) to near range (high-depression angles), examples were experienced in all but one test area (Humbolt Mt. I, see Figure 5). In several instances, the decrease in percentage was small and of little importance to the construction of the curve; however, in the Stansbury Mts. (Figure 7) and Chrome Ridge-Onion Mt., (Figure 8) either the rate of change in the percentage of crestlines in shadow varied drastically, or the size of the decrease in the percentage of shadowing between sampling zones was of such magnitude that it warranted further consideration.

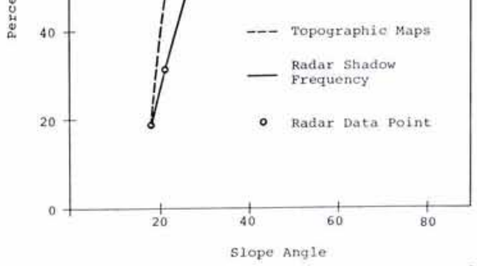

FIG. 7. Cumulative frequency curves of map- and radar-derived terrains slope α data. (Stansbury Mts., Utah).

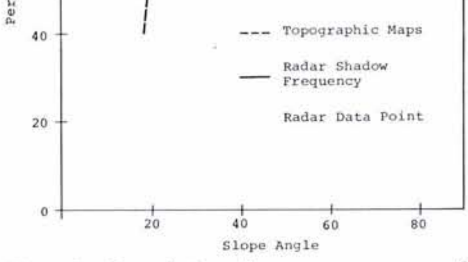

FIG. 9. Cumulative frequency curves of map- and radar-derived terrain slope α data. (Seven Miles Peak, Oregon).

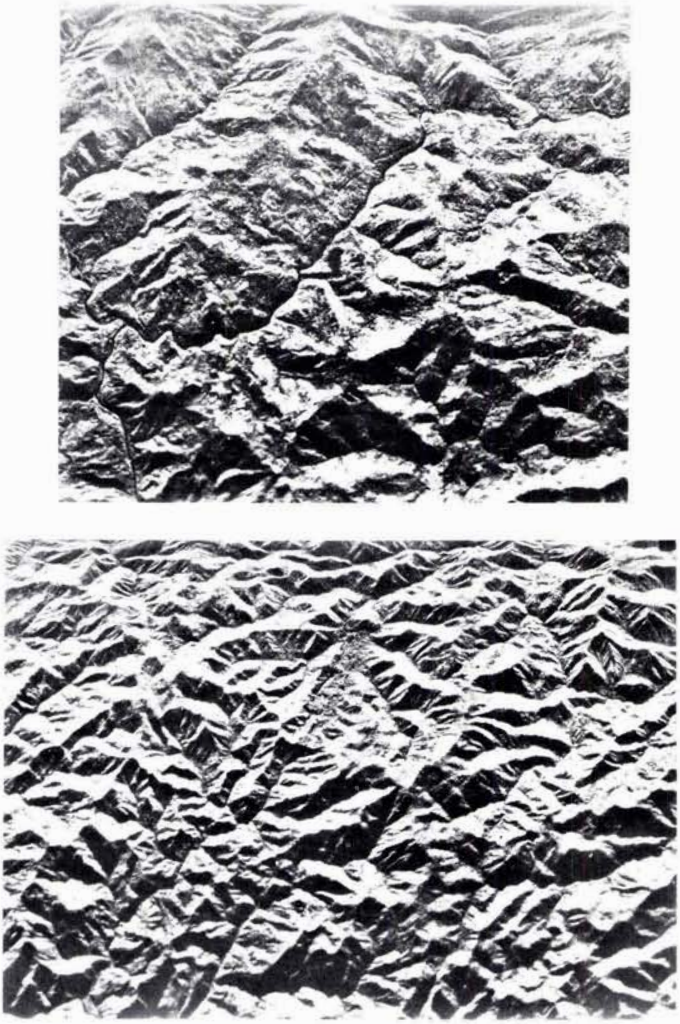

FIG. 10. Radar imagery of Seven Miles Peak (*above*) and Chrome Ridge (*below*) (Onion Mtn., Oregon). Seven Miles Peak area exhibits rounded, difficult-to-define crests, whereas the crests in the Chrome Ridge-Onion Mtn. area are knife-like, easy-to-define crests similar to the type of crests found in the other test areas.

In both instances it was found that either a narrowing of the delimited landform region in the azimuth direction, or an exceptionally wide spacing between crests, resulted in a small sample (total length of crestlines) which may have distorted the results.

The major problem area was Seven Mile Peak, Oregon (Figure 9) where the map and radar curves did not correlate, especially for the lower slope angles. For example, there is over a 30-percent discrepancy in the two data sources at the lowest end of the curves. A comparison of the radar imagery of the two test areas in Oregon provided at least a partial solution as to why such a large discrepancy exists in this one area. The major difference between Seven Mile Peak and Chrome Ridge-Onion Mt., Oregon, is topographic character of the ridge crests, rounded in the former and knife-like in the latter (Figure 10). The broad, rounded crestlines in Seven Mile Peak make crestline definition extremely difficult. A poor sampling set, which evidently over-emphasizes the low terrain slope angles on the radar-derived curve, results.

ADVANTAGES AND LIMITATIONS OF RADAR SHADOW FREQUENCY METHOD

Several advantages of using this method for producing cumulative frequency curves are (1) the method provides a more realistic cumulative frequency curve of slopes for areas of moderate to high slopes than topographic maps of a 1:62,500 scale; (2) large areas can be sampled and a cumulative frequency curve plotted in less time and with more accuracy than with topographic maps utilizing standard sampling techniques; and (3) the method works best in a mountainous type of terrain where the poorest topographic coverage exists and, as such, the radar-derived data provides a valuable complement to topographic maps.

The limitations involved in using the method are: (1) For maximum data retrieval, the landform region must extend completely across the range of the image, or there must be enough multiple coverage of the region to provide radar coverage in all portions of the range. The width of the region traversing the range should also be wide enough to provide a good sample. This means regions approximately 50 to 100 square miles in area. (2) The region must be homogeneous across the range and also have no preferred orientation of slope values. The assumption must be made, as previously mentioned, that the landforms are saw-toothed and the crestlines well defined. (3) The lowest slope value that can be discriminated depends on the far-range depression angle of the radar imaging system, a value in the range of 15°. This means that no discrimination of slope angles occurs below approximately 15°. As most of the critical slope angles for land use, terrain mobility, etc., are within the range of 0° to 15°, this is a serious limitation to the use of radar shadow frequency. This is a limitation of the imaging system rather than the method, however, and could be corrected in part with system modification. The expansion of the far range all the way to 1° depression angle is a possible solution, especially with a synthetic-aperture imaging radar system flown at a low altitude. Although such drastic modifications are not likely to take place, system modifications have lowered the far range depression angle several degrees ($\approx 5°$).

In summary, the ubiquitous nature of radar shadowing, plus the relatively straightforward relationships of radar shadowing to terrain slope and relative relief, provides not only the geomorphologist but also anyone engaged in terrain analysis with a potentially powerful tool for both functional and genetic landform analysis on both meso and macro scales.

ACKNOWLEDGEMENTS

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